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Optimal Design of Hydraulic Structure Using Simulation-Optimization Method

A Thesis

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in Partial Fulfillment of the Requirements for The Degree of Master of
Science in Infrastructure Engineering

By

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
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دَرَجَاتٍ وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ)
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Dedication

I dedicate this work to my father and mother, my husband, my brothers and sister, that have given me affection and love; I say to them: thank you for giving me life, hope, and the emergence passion for learning and knowledge.

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Abstract

Hydraulic structures such as gravity dams are classified as essential structures which need to high cost to construct it and have the vital role in providing strong and safe water resource. These structures are subjected to seepage problem which is considered to be a dangerous phenomenon that may generate uplift pressure, which may cause the dam to function improperly, in addition to the exit gradient that may cause piping if exceeded a safe value.

This research illustrates the application of a new Genetic Algorithm with Finite Difference Programming (GA-FDP) technique, A MATLAB code was used to perform the GA-FDP optimization model in order to find the optimal design for safe hydraulic structure. The objective was to minimize the construction cost function. The main constraints are those that satisfy a factor of safety against uplift pressure and factor of safety against piping. The proposed GA-FDP model has fulfilled the optimum design task into two stages. Firstly, Finite difference Programming (FDP) analyzed the seepage problem numerically after verification with GeoStudio(2018 R2)SEEP/W to obtain the uplift pressure and exit gradient and to determine other characteristics such as the pressure head, total head, discharge and total cost of the structure. Also, comparing numerical model SEEP/W of calculating uplift force and exit gradient under the dam with benchmark example in soil mechanics which shows a good agreement.

Secondly, the Genetic Algorithm (GA) was applied with finite difference programming to obtain the optimum location and depth for cutoffs needed for the preliminary design of the Dam which satisfied the factors of safety against piping and up lift pressure.

In FDP, the seepage problem was analyzed to define the effect of depth, location, number of cutoffs and isotropic degree on the value of uplift force and exit gradient. The results were observed that the relative head (H/B) had a significant effect on increasing the exit gradient and uplift force. Also, minimum exit gradient was noticed when the cutoff location ratio at the downstream is of ($x_1/b=1$) with a maximum relative depth of ($d_1/b=0.6$), while the minimum uplift pressure was observed when the cutoff location ratio at the upstream is of ($x_1/B=0$) with a minimum relative depth of ($d_1/B=0.1$).

Model result revealed that when a cut-off is at the upstream, the uplift pressure is decreased along the base of floor. Also, the uplift distribution decreases with increasing the depth of cut-off because the cut-off causes an increase in the length of creep, which increases the head loss. Whenever a cut-off is located a drop in the uplift pressure at that location is observed as expected. Also, the uplift pressure starts decreasing when using different lengths of cut-offs compared with the case of no cut-off; this behavior is reversed beyond the point where $x = 0.5b$.

In the Genetic algorithm-finite difference programming using the constrain of input variables with differential head (H/b)= 0.25,0.5,0.75, floor-length (B)=20m, depth of impervious layer ($D=30$), and ratio of permeability in x to y ($k_x/k_y = 1,2,4,8$). Six various depths ratio ($d/B=0.1:0.6$), for each depth ratio, various cutoff locations, ($b/B=0:1$) were used.

It is clear that the results were which obtained from GA-FDP are the optimum solution that's achieve the two constraints (safety against exit gradient and uplift force) with minimum cost. The optimum locations of cutoff 1 (X_1/B) varied from (0 to 0.33) B , (0 to 0.24) B , (0 to 0.1) B , and (0 to 0.18) B for K_r equal to (1,2,4, and 8) respectively. The optimum locations of cutoff2 (X_2/B) varied from

(0.875 to 1) B for various values of K_r . This behaviour is due to prevent increasing the uplift pressure and exit gradient of the structure. The optimum depth of cutoff 1 (d_1/B) varied from (0 to 0.35)B for K_r equal to (1, 2, 4), (0 to 0.6)B for K_r equal to 8) and the optimum depth of cutoff2 (d_2/B) varied from (0.15 to 0.5) B, (0.1 to 0.6)B, (0.2 to 0.7)B, and (0.3 to 0.7) B for K_r equal to (1,2,4 8) respectively.

It was found that the optimum solution reduced the cost by 81% ,72%, 66%, 53% from the cost of traditional solutions for K_r equal to (1,2,4, and 8) respectively.

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List of Symbols

<u>Symbol</u>	<u>Description</u>	<u>Dimensions</u>
A	The cross section area of a soil sample perpendicular to the flow path	L^{-2}
B	The length of floor for hydraulic structure	L
b1	The location of cutoff	L
c1	Cost coefficients for U/S cutoff per unit volume	IDL^{-3}
c2	Cost coefficients for U/S cutoff per unit volume	IDL^{-3}
D	average diameter of the soil particle	L
D	depth of impervious layer	L
d1	Depth of cutoff1	L
d2	Depth of cutoff2	L
e	The void ratio	-
Fuplift	Uplift force	MLT^{-2}
Fi	Fitness Function	$1/ID. L^{-3}$
Gs	Specific gravity of soil	-
Gs	Acceleration of gravity	LT^{-2}
H1	Head difference between U/S and D/S side	L
H2	The depth of water at upstream	L
h	The depth of water at downstream	L
Δh	Pressure head	L
I exit	The head difference between the two equipotential line	-
I cr	Exit gradient	-
K	Critical hydraulic gradient	LT^{-2}
Kr	Ratio of permeability in X direction to permeability in y direction	-
Kx	The hydraulic conductivity in X direction	LT^{-1}
Ky	The hydraulic conductivity in Y direction	LT^{-1}
Kz	The hydraulic conductivity in Z direction	$L T^{-1}$
L	Length of flow element(between two equipotential line)s	L
lw	Weighted creep distance	L
lh	The sum of the horizontal distances	L

lv	The sum of the vertical distances	L
μ	Absolute viscosity for water	$ML^{-1}T^{-1}$
Nf	Flow Channel	-
Nd	Potential Drop	-
n	porosity of the soil	-
pi	probability of selection	-
ρ	mass density of the water	ML^{-3}
q	The quantity of seepage for one element	$L^3T^{-1}L^{-1}$
qt	The total quantity of seepage per unit length	$L^3T^{-1}L^{-1}$
Rn	Reynolds number	-
t	The floor's thickness under the gate	L
t1	The thickness of U/S cutoff	L
t2	The thickness of D/S cutoff	L
u	Uplift pressure	$ML^{-1}T^{-2}$
v	The velocity of flow	L/T
γ_w	The unit weight of water	ML^2T^{-2}
X1	The location of cutoff1	L
X2	The location of cutoff1	L

Abbreviations

ANN	Neural artificial network
CGD	Concrete gravity dam
DFS	The downhill foundation slope
D/S	Downstream
FDP	Finite difference Programming
FDM	Finite difference method
FS	Factor of safety
GA	Genetic Algorithm
GAM	Genetic Algorithms Modeling
GA-FDP	Genetic algorithm and finite difference programming
PDEs	Partial differential equations
Pc	Probability of crossover
Pm	Probability of mutation
RWS	Roulette Wheel Selection
U/S	Upstream
UFS	The uphill foundation slope

Chapter One

Introduction

1.1 General

Dams are primarily built to retain water in reservoirs and run for long periods of time. Thus, the factors that can influence and reduce the functionality of these dams must be carefully studied to reduce its effects in order to achieve proper dam functionality. In contrast to other parts of the structure, the foundation of these structures must be given the highest priority in study and design, since failure of the foundation would cause the entire structure to collapse.

For hydraulic structures which built on penetrable soil foundations, the water flows through the soil and results uplifting forces, possibly carrying fine soil particles with it, contributing in erosion (piping phenomenon). some of control structures that minimize up lift pressure and seepage forces, such as cutoffs, sheet piles and concrete curtains at the base of the hydraulic structures that lengthen the seepage path. The upstream cutoffs minimize the uplift pressure and exit gradient in general. However, the uplift pressure decreases faster than the exit gradient. While downstream cutoff has direct impact on the exit gradient.

The two main critical points must be considered when designing a safe hydraulic structure against seepage:

First, uplift pressure is exerted on the hydraulic structure's floor as a result of water flowing below the structure. To avoid the structure from collapsing, this uplift force should be counterbalanced by the weight of the structure.

Second, the influence of the piping phenomena is determined by the exit gradient. This phenomenon will be happen when the value of the downstream exit

gradient exceeds the critical value. The exit gradient is called critical if the upward exerting force on the grain is equal to the submerged weight of the grain at the downstream point. To prevent piping, the seeping water's velocity must be decreased to a reasonable level. This can be done by employing sheet piles or cutoff walls, that lengthen the seepage line.

1.2 Statement of the problem

The seepage of water under the hydraulic structure represents the most important factor to be considered when planning and designing this structure, which produces uplift pressures and possibly carries fine particles of soil with it causing piping phenomenon. If seepage continues without being treated, the dam may eventually collapse, resulting in the loss of life and economic.

1.3 Aim of the study

The main aim of this study is to develop an optimization model to design a safe hydraulic structure against seepage which adopt a coupled simulation-optimization techniques by using finite difference model represented as subroutine and Genetic Algorithm with finite difference programming GA-FDP techniques on MATLAB (2020a) platform.

1.4 Methodology

The principle of flow in a porous media is concentrated on Laplace's equation of continuity, which represents the steady flow condition for a specific point in the soil mass, the following steps are followed to achieve the aim of this study:

- Calibrating GeoStudio SEEP/W software model with finite difference programming in Matlab.

- Utilizing the calibrated model to investigate the effect of multiple factors including differential head, isotropic degree, cutoff depth, cutoff location etc. on the uplift pressure and the exit gradient.
- Using the new technique of simulation optimization genetic algorithm-finite difference programming to find the optimal solutions required with minimize objective function , the factor of safety against uplift pressure, and the factor of safety against piping as a constraints.

1.5 Limitations and Assumptions

The limitations of this study are listed below:

- a) The cutoffs are vertical;
- b) The flow is two-dimensional, steady-state;
- c) Darcy's law is valid in the seepage domain;
- d) Depth of cutoffs $\leq 10\text{m}$.

1.6 Structure of this thesis:

Chapter one: this chapter gives an introduction and the statement of the problems. In addition, it specifies the objective, methodology, limitations and assumptions and the structure of this thesis.

Chapter two: gives the current understanding of seepage aspects. It also shows literature review which lists the related researches.

Chapter three: gives the theoretical aspects which includes the theory of seepage, finite difference method and the theory of Genetic Algorithm modeling.

Chapter four: this chapter introduces the details of model's development.

This chapter shows the optimization model genetic algorithm by Finite Difference Programming (objective function and constraints in this model) .

In addition, this chapter includes the details of parameters estimation of the optimization model.

Chapter five : this chapter introduces the analysis and discussion of results.

Chapter six : in this chapter, the conclusions and recommendations induced from this research were listed.

Chapter Two

Literature Review

2.1 Introduction

This chapter attempts to present a review of relevant literature which looks at many researchers for the seepage through permeable medium under hydraulic structures.

2.2 Previous Studies

Many researches have been carried out to investigate the seepage through permeable medium beneath hydraulic structures and the effect of the upstream and downstream cutoff on uplift pressure and exit gradient. Because of the difficulties encountered in analytical and experimental methods, numerical methods were used. Such methods provide accurate results that are comparable to the results of analytical solutions.

(Abbas, 1994) investigated a conformal mapping technique to find an exact solution for seepage flow under a structure built on infinitely permeable soil with an inclined cutoff at the downstream end. According to the study with an increase in cutoff inclination, the exit gradient significantly decreases over a distance beyond the floor end. For cutoff inclination of 10, 20, and 30°, the maximum exit gradient decreases, and for = 45°, it begins to increase. As the angle of cutoff's inclination rises 45°, the uplift pressure decreases. The use of an inclined cutoff raises the design factor of protection against uplift and piping, according to the findings.

(Al- Saadi and Al-fatlawy, 2007) investigated the seepage analysis through the soil foundation under dams. A two-dimensional model of quadrilateral isoperimetric element is used to predict the effect of degree of anisotropy and inclination of the hydraulic conductivity ellipse on exit gradient for the dam with and without sheet pile resting on anisotropic porous media. The problem has been analyzed in dam with upstream and downstream sheet piles and for various values of permeable layer thickness. A set of curves is obtained showing the exit gradient distribution under the dam. The effect of the length and location of the sheet piles are thoroughly investigated and different value of angle inclination for hydraulic conductivity ellipsoid from the major axis.

(Alsenousi and Mohamed, 2008) created a two-dimensional finite element analysis to simulate seepage flow beneath a dam with an inclined sheet pile within various flow conditions and soil properties (steady or unsteady flow conditions on homogeneous or non-homogeneous soil and isotropic or anisotropic seepage flow). By altering the slope angle of the sheet pile and varying the soil and flow conditions, the researchers discovered that measured exit gradient values, flow rates, and uplift pressure were affected. COMPAQ VISUAL FORTRAN 6 was used to make the software program.

(Al-Suhaili, 2009) looked into an empirical solution for the difference in exit gradient downstream of an inclined sheet pile. The exit gradient at downstream of an angled sheet pile is generally reduced as the angle of inclination calculated to the horizontal axis to the right of the sheet pile increases, according to the findings. In addition, the length of protection that is necessary to attain the factor of safety toward piping reduces with the angle of inclination in general. Beyond the angle of $2\pi/6$, this behavior can be noticed. However, the necessary length varies only slightly for each of the three kinds of soils downstream of the inclined

sheet pile. Moreover, when the angle of inclination is larger than 90° degrees, the distance of safety reduces as the angle of inclination increases.

(Obead, 2009) examined the influence of a Sheet Piles-downstream blanket of any length and position beneath a concrete dam of negligible thickness and restricted length on uplift heads, seepage discharges, and exit gradients beneath dam. The finite element approach is used to solve the problem numerically. The results are presented in a form of design charts, with dimensionless parameters; maximum uplift head, relative length of downstream blanket, relative depth of upstream and downstream cutoff walls, relative exit gradient, and relative seepage quantities. These geometry charts could be used to calculate highest uplift head, seepage discharge, and exit gradient below dams. The charts' applicability and validity were evaluated and compared to relevant existing approximations in the literature, and they performed well.

(Al-Saadi et al., 2011) studied the impacts of cut-off inclination on exit gradient and uplift pressure beneath hydraulic structure, determining the optimum position and angle of inclination of cutoff, and solving it using the finite element method in (ANSYS 11.0) software. The authors concluded that using a cut-off in downstream with angle less than 120 degrees inclined towards the downstream side is effective in rising the protection factor against the piping phenomenon.

(Singh, 2011) investigated the optimal solution for hydraulic structure using the genetic algorithm method, which finds the barrage dimension that gives the safe exit gradient and supplies optimal structure dimensions and contributes to reducing unit cost of construction works.

(Azizi et al., 2012) had investigated the effect of weep holes position and different depths of the dam cutoff walls on uplift pressure and exit hydraulic gradient.

According to the study, an upstream cutoff of 8 meters reduces uplift force by 63 percent and the exit gradient by 79 percent as compared to the case without the cutoff. Installing a weep hole in the downstream stilling basin reduces uplift force by 8% and the exit gradient by 74% compared to not having one. Based on this study, the construction of diversion dams can be carried out thus minimizing concrete costs, making it a cost-effective design.

(Ahmed, 2012) investigated the confined flow below hydraulic structures. Larger anisotropic heterogeneity ratios (the ratio of horizontal to vertical scale of fluctuation) significantly raised the seepage flow, according to the study. At greater coefficients of variance, the rise in flow was more pronounced. Uplift force followed a similar pattern, though it was fewer sensitive to anisotropic heterogeneity ratio increases. In addition, as the ratio of anisotropic heterogeneity increased, both the seepage flow and the uplift force appeared to deterministic values. The researchers have discovered that at some values of the anisotropic heterogeneity ratio, the exit hydraulic gradient reaches its maximum value.

(El-Jumaily et al., 2013) had studied the seepage analysis under hydraulic structures founded on isotropic, anisotropic, homogeneous, and non-homogeneous material. The research was performed using finite volume procedure with rectangular elements. This study compared the effects of heterogeneous and homogeneous foundations on uplift pressure and exit gradients. It also looked at the impact of cut-off location and inclination on uplift pressure and exit gradients. It also looked at how an impervious body within a structure or base affected uplift pressure and downstream gradients. A special code was written to program the finite volume method solutions, so the potential head and exit gradient can be obtained at any point within the flow domain.

(Al-Suhaili and Karim, 2014) had created a genetic algorithm model in conjunction with an artificial neural network to determine the optimal values of upstream, downstream cutoff lengths, floor length, and downstream protection length for a hydraulic structure. Different cases reaching 1200 were modeled and analyzed by the researcher using geo-studio modeling, with different values of input variables. Using the obtained ANN model, the researcher wrote a Matlab code to perform genetic algorithm optimization simulation. The optimal solution obtained for some selected cases was compared with the Geo-studio software to determine the length of protection required on the downstream and the volume required for the superstructure. The estimated values were found to be comparable to the Genetic algorithm.

(Mansuri et al., 2014) had checked into the impact of a cutoff wall on certain design parameters in the case of a hypothetical diversion dam cross-section by using Geo-Studio Software Seep/w. Various positions of cutoff wall with different angles of inclination were used in the dam base through using Geo-Studio software (GEO-SLOPE 2007). Results showed that:

- When the cutoff wall is placed upstream of the dam, the total uplift force is at the lowest value and the exit gradient is at the highest, and vice - versa.
- When the cutoff wall is placed on upstream or downstream of the dam, a significant reduction in seepage discharge happens. When the cutoff wall is close to the downstream, the proportion of overall uplift force decrement decreases as the angle is increased. It indicates that the overall uplift force near the downstream would be greater than in the other positions. Within the angle increment, this decrement percentage of overall uplift force have a larger increment near the upstream of the dam and a smaller increment near the downstream of the dam (total uplift force decrement percentage).

(Moharrami et al., 2015) used finite element method with the Geo-Studio program Seep/w (GEO-SLOPE 2007) to evaluate the influence of cutoff walls toward uplift pressure and piping phenomenon. The parametric analysis looked at how cutoff wall parameters like the inclination angle of one cutoff wall, their length in the upstream side, their spacing, and the number of cutoff walls underneath the hydraulic structure varied. The findings stated that using an inclined upstream cutoff wall with an angle of 70 or 90 degrees raised the hydraulic structure's safety toward piping phenomena and uplift force, respectively.

(Rasool, 2018) researched the seepage process (uplift pressure, flow rate, and exit gradient of hydraulic structures) and found that this process is one of the main causes of hydraulic structure failure or collapse, so it was minimized by using sheet piles below the floor of structures. Using the finite element software ANSYS, the effect of mutual interference piles on seepage phenomenon was investigated. The findings were compared to the (EL-Sayed et al., 2002) realistic results, which showed a strong correlation. By using of a pile in the upstream, the uplift pressures were decreased by 8.36%, while a pile in the downstream raised uplift pressure by 11.66%. Also, the flow rate was decreased by 66.8%, and the hydraulic structures' exit gradient was reduced by 28.28%.

(Saleh 2018) determined the rate of seepage and exit gradient below a dam with two sheet piles by using the SEEP/W model. The head variance, soil permeability factor, sheet pile spacing, heights, and inclined angles were all independent variables. This model was run for three various of independent variable values. The findings of the SEEP/W model were then used to construct two neural artificial network (ANN) models (A and B) with the rate of seepage (model A)

and exit gradient (model B) as output variables. For both models, the most suitable structure was with lowest relative errors. The coefficient of soil permeability, which had the greatest impact on seepage rate, the variation in the head (8 %), the length between piles (5.5 %), the depth of downstream pile (5 %), the depth of upstream pile (4 %), and the downstream and upstream inclined angles of the sheet piles, with ratios of about 1 % and 0.5 %, were found to be the most significant variables in the ANN models. The gap between piles had the greatest influence on the exit gradient, accounting for 35%, followed by the downstream inclination angle, length of downstream pile, head difference, length of upstream pile, inclined angle of upstream pile, and soil permeability, which accounted for 23%, 19%, 14 %, 7.5 %, 1%, and 0.5 %, respectively. These findings are consistent with a SEEP/W model study.

(Hassan, 2019) developed a genetic algorithm technique that was combined with a numerical model (finite element method) to determine the best cutoff position and inclination angle for barrages built on homogeneous anisotropic soil foundations. In the problem formulation, the exit gradient function is minimized as an objective function. Constraints involved uplift pressure and safety coefficient based on a minimum concrete floor thickness. Different degrees of anisotropy and values of a range of heads have been investigated. The SEEP2D GMS software was used to simulate over 1400 different scenarios. The pressure head and exit gradient for anisotropic soil foundations were predicted by using statistical nonlinear regression models at different degrees of inclination, relative position (b_1/b), and relative depth (d/b). Matlab was used to build a genetic algorithm optimization model. According to the findings, the computed optimum cut-off locations and relative inclination angles were influenced by changing the anisotropic ratio and relative cut-off depth. A cut-off wall with relative position ratios of b_1/b close to 0.8 and 30° , 123° , and 145° inclinations towards the D/S side can be regarded the

model's optimum solution. This improved protection factor towards piping by providing a minimum exit gradient for relative depth (d/b) of 0.1, 0.2, and 0.3, respectively. It also necessitates a floor thickness that is less than the same hydraulic structure's thickness without cut-off. When the cut-off was placed on the upstream side with angles ranging from 59° to 68° , the best solutions for other proportional depth values were discovered.

(Ghobadian et al., 2019) studied the effect of the slope of the impervious layer (downhill/uphill foundation slopes) on uplift pressure, seepage flow, and exit gradient beneath hydraulic structures. The researchers created a computational model in which a general equation of fluid flow is solved using the finite volume method, and demonstrated that as the downhill foundation slope (DFS) is increased, the uplift force decreases while the exit gradient and seepage discharge increase. Additionally, raising the uphill foundation slope (UFS) raises the uplift pressure while decreasing the exit gradient and seepage discharge. The exit gradient rises 19.75 % and 14.4 % for 1 m and 6 m cut-off distances, respectively, when the downhill foundation slope (DFS) is increased from zero to 15%. For the same cut-off depths, UFS has a lower exit gradient than DFS. Furthermore, raising the cut-off depths lowers the exit gradient. However, it has a greater impact on lowering the exit gradient in DFS than it does in UFS.

(Al-Juboori et al., 2019) focused on improving accurate surrogate models using the Artificial Neural Network (ANN) approach, that is trained using numerical simulated data sets produced by (SEEPW/Geo-Studio) software. The improved surrogate models are connected with the Genetic Algorithm (GA) optimization solution to optimize the hydraulic design of Concrete gravity dams whereas keeping design safety factors and construction costs. The evaluation

results show that the methodology has the potential to be used for efficient, safe, and cost-effective hydraulic design CGD on permeable soils.

2.3 Summary

The seepage problem under hydraulic structures is an important condition. It needs to be studied and treated, if seepage continues without being treated, the structure may eventually collapse, resulting in the loss of life and economic.

For hydraulic structures which built on penetrable soil foundations, the water flows through the soil and results uplifting force and contributing in erosion (piping phenomenon). some of control structures that minimize up lift pressure and seepage forces, such as cutoffs, sheet piles and concrete curtains at the base of the hydraulic structures that lengthen the seepage path. Thousands of studies have been performed to study and model seepage flow behavior and the effect of conditions and parametric on exit gradient and up lift pressure. All these models which include make high number of runs in GeoStudio software to create model and enter this results in genetic algorithm or artificial neural network to find the optimum solution, but in this study don't need to make high runs. This study is to develop an optimization model to design a safe hydraulic structure against seepage which adopt a coupled simulation-optimization techniques by using finite difference model represented as subroutine and Genetic Algorithm with finite difference programming GA-FDP techniques on MATLAB (2020a) platform, this model is economic and needs low time and efforts.

Chapter Three

Theoretical Aspect

3.1 Theory of seepage

Soil is made up of solid particles of varying sizes with interconnected void spaces. Water can flow from a point of high energy to a point of low energy through the continuous void spaces in soil. Permeability is the property of a soil that allows fluids to seep through its interconnected void spaces (DAS, 2008). All soils are permeable, allowing water to pass freely through the interconnected pores between the solid particles. As pore water flows through the soil, it exerts pressure, which is referred to as seepage pressure (Terzaghi et al., 1996).

3.1.1 Darcy's Law

Darcy (1856) published a precise relationship between discharge velocity and hydraulic gradient. Darcy's Law is true for laminar flow. Numerous investigations have been performed to determine the range over which Darcy's Law is accurate.

Many studies have been conducted to determine the scope of Darcy's law's applicability. The Reynolds number can be used as a criterion for investigating the scope, which is given by the following relation:

$$Re = \frac{vD\rho}{\mu} \quad (3.1)$$

Where:

- v = velocity in cm/s
- D = average diameter of the soil particle, cm
- ρ = water density g/cm^3
- μ = coefficient of viscosity $g/(cm.s)$

The experimental findings for laminar flow in soils indicate that

$$Re = \frac{vD\rho}{\mu} \leq 1 \quad (3.2)$$

Water beneath the water table may be static or seep through the soil according to a hydraulic gradient (Terzaghi et al., 1996). Bernoulli's principle can be used to measure the total head of flow at any point in the soil.

$$\text{Total head} = \text{pressure head} + \text{velocity head} + \text{elevation head} \quad (3.3)$$

Since the velocity head for flow in soil is so low, it can be ignored. (DAS, 2008),

$$\text{total head} = \text{elevation head} + \text{pressure head}$$

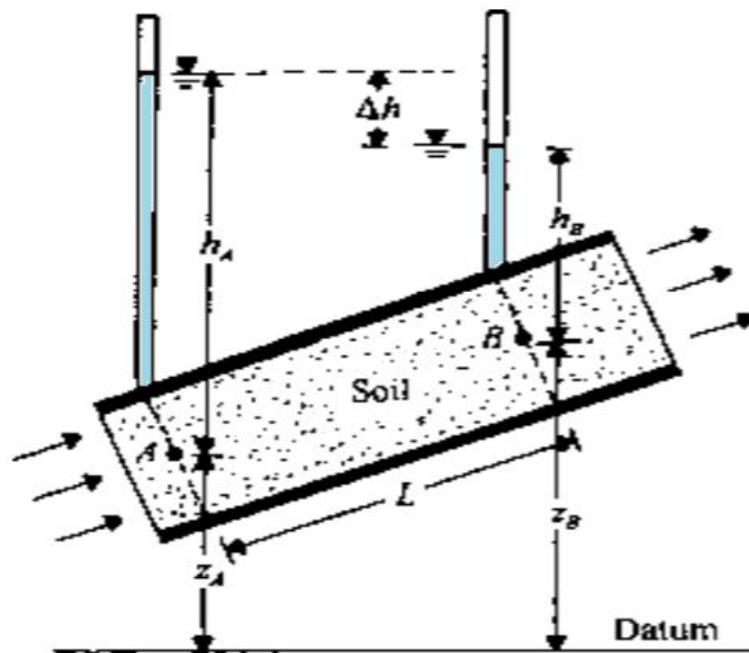


Figure (3-1) Development of Darcy's law.(DAS, 2008)

As result head can be calculated as follows:

$$h = \frac{P}{\gamma_w} + z \quad (3.4)$$

Where:

- h = total head
- γ_w = unit weight of water
- P = pore water pressure

- z = depth below a datum

Water will not flow in saturated soil where an impermeable boundary beneath the soil resists vertical flow under normal conditions, but when pressure heads are different (h), water will flow in the direction of the reduced head.

As seen in Figure 3.1, the head loss between A and B is calculated in the following way:

$$\Delta h = (Z_A + h_A) - (Z_B + h_B) \quad (3.5)$$

Darcy (1856) discovered an empirical model by analyzing the amount of water flow across granular soil, this known as Darcy's Law, it states that velocity is proportional to the hydraulic gradient. He devised the following equation based on his findings:

$$\frac{q}{A} = v = ki \quad (3.6)$$

The following equation is the hydraulic gradient (i) over a specific length (L) is calculated as follows:

$$i = \frac{\Delta h}{L} \quad (3.7)$$

Where, q is the flow (L^3/T), k is the coefficient of permeability for soil (L/T), A is the cross section area of a soil sample perpendicular to the flow path (L^2).

The force that causes water to flow is the hydraulic gradient, the amount of seepage, q with the s.i unit's m^3/s can be computed using the above equation as follows:

$$q = kiA \quad (3.8)$$

3.1.2 Coefficient of Permeability

The coefficient of permeability (hydraulic conductivity), k is measured in units of velocity and is an indicator of the soil's resistance to water flow. The coefficient of permeability is affected by a number of factors, the most of which are listed below.

1. The soil particle's shape and size.
2. Coefficient of Permeability increases as the void ratio increases.
3. Coefficient of Permeability increases as saturation level rises.
4. Composition of soil particles. Coefficient of permeability decreases as the thickness of the double layer increases.
5. Viscosity of the fluid. This factor is affected by the temperature. As the temperature increases, the viscosity decreases and thus the coefficient of permeability increases.

$$K \text{ can be displayed as: } K(\text{cm/s}) = \frac{K \rho g}{\mu} \quad (3.9)$$

Where:

- K = intrinsic permeability,
- ρ = mass density of the water, Kg/m^3 .
- g = acceleration of gravity, m/sec^2 .
- μ = dynamic viscosity for water, $\text{pa}\cdot\text{sec}$.

Table (1) shows several typical values for the coefficient of permeability. The coefficient of permeability for soils is generally reported at a temperature of 20 C.

Table (3-1): coefficient values of permeability for different soils (DAS, 2008)

Material	Coefficient of permeability (mm/s)
Coarse	10^{-1} – 10^3
Fine gravel, coarse, and medium sand	10^{-2} – 10
Fine sand, loose silt	10^{-4} – 10^{-2}
Dense silt, clayey silt	10^{-5} – 10^{-4}
Silty clay, clay	10^{-8} – 10^{-5}

3.2 Seepage Flow

Water travels through soil along paths known as flow channels. which are defined by flow lines, (Terzaghi, 1943) suggested that if the flow lines are straight and parallel, the flow is referred to as linear or one-dimensional flow. Also, suggested that flow is two-dimensional if water particles move along curves in parallel planes; all other kinds of flow are three-dimensional. In relation to the current study, the model dam was treated as a two-dimensional section for analysis.

3.2.1 Flow Nets

Forchheimer devised a graphical solution to estimate seepage under hydraulic structures and Casagrande (1940) documents this method. This is a graphical method used for the solution of simple, symmetrical problems. A flow net is a graphical representation of a flow region (Laplace equation solution) that comprises of a family of flow and equipotential lines. The path of water flow is indicated by the flow lines, and the distribution of potential energy is indicated by the equipotential lines or headlines (lines that represent the constant head). Flow nets are typically created through a trial-and-error process using sketches (Sachpazis, 2014).

The following rules should be followed when drawing flow nets, according to (Casagrande, 1940):

- In a homogeneous isotropic system, flow lines and equipotential lines should be perpendicular to each other.
- Flow lines must always be parallel to an impermeable boundary, while equipotential lines must always be perpendicular to it.
- Flow lines must always be perpendicular to a constant head boundary, while Equipotential lines must always be parallel to it.

Flow is typically calculated using graphs known as flow nets. The flow net concept is founded on Laplace's equation of continuity, which explains the steady flow condition for a specific point in the soil mass.

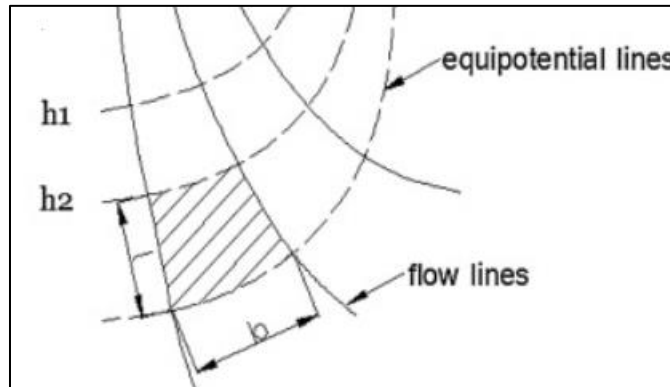


Figure (3-2): Flow net through a single flow element (Powrie, 2018)

To calculate the quantity of flow through a homogeneous isotropic soil under a hydraulic structure per unit length using a flow net Darcy's law is used ($q = Aki$). From observing the illustration in Figure 2-1 of how water flows through a single element in flow nets, the derivation of the formula used to calculate the quantity of pore water seepage using flow nets can be explained as follows. The head difference between the two equipotential drops is represented as Δh , from which the hydraulic gradient (i) is obtained by dividing Δh by the distance (l) between the two equipotential lines. As the distance between the flow lines (b) is curvilinear, therefore, Darcy's law becomes $q = k\Delta h$ where k is the coefficient of permeability. As all the squares are the same in a flow net, the quantity of flow through each one is the same with the same head loss through each curvilinear square. The result of this can be put into Darcy's equation as the hydraulic gradient for the whole system can be calculated by counting the number of flow channels (NF) and the number of equipotential drops (NH) and dividing the number of flow channels by equipotential drops. To calculate the rate of pore

water seepage per unit length using the method of flow nets, Darcy's equation can be rewritten as follows:

For one element, $q = Aki$

$$= b(1)k \frac{\Delta h}{l}$$

$$= k\Delta h \text{ if } b=1$$

For N_F flowtubes, N_H equipotential drops and an overall head drop of H (meters)

$$q_T = N_F * q$$

$$\text{and } \Delta h = \frac{H}{N_H}$$

$$\text{So } q_T = kHN_F / N_H \text{ m}^3 / \text{sec per meter length} \quad (3.10)$$

3.2.2 Seepage Failure Mechanisms

There are four failure mechanisms induced by seepage and seepage pressure as defined by Eurocode 7, they are as follows (NASI, 2005): “1.Failure due to uplift 2. Failure due to heave 3. Failure due to internal erosion 4.Failure due to piping”.

The important failures will be discussed now, but the others can be neglected because the low effects. Uplift occurs when the pore water pressure beneath a structure becomes greater than the vertical pressure resisting it and causes a buoyancy effect which pushes the structure upwards.

One of the critical pore water seepage problems that arise when don't dealt with adequately is the erosion of soils with the continued flow of the water through the soil leading to “piping failure” This occurs when the water starts removing soil from the exit point of the flow channel and forms a sand boil, it then erodes the soil backwards causing a hole to form. The flow channel will

increase rapidly as the water will rush through until the structure collapses due to the foundation eventually washing away. Piping phenomena typically occurs when the exit gradient exceeds the critical value, causing fines to be gradually leached from the soil and the hydraulic conductivity to increase (Powers et al., 2007).

Terzaghi (1929) provided an exit gradient approach to seepage study in his work on dam failure due to seepage and developed from the investigation of the vertical seepage forces generated by the upward flow of water, which tries to lift the soil grains. If the vertical gradient h/L reaches the critical value, the effective stress between soil grains becomes zero, and the soil becomes quick (Quicksand Condition), losing all of its strength. The quick condition causes a piping in hydraulic structures. Terzaghi calculated that for typical soils, the critical gradient is approximately 1.0. For the most common G_s and e combinations found in soils, i_{cr} ranges between 0.85 and 1.1.

Where

G_s : the specific gravity of soil

e : void ratio

3.2.3 Methods of preventing seepage failure

To prevent seepage failure under hydraulic structures, the following two critical points must be taken into account:

3.2.3.1 safety against piping

Harza (1935) analyzed the safety of hydraulic structures with piping and indicates that the factor of safety against piping FS , ranges from 3 to 4 can be defined as

$$FS_{\text{piping}} = \frac{i_{\text{critical}}}{i_{\text{exit}}} \quad (3.11)$$

where:

i_{critical} : Critical hydraulic gradient

i_{exit} : Exit gradient

- the exit hydraulic gradient represents the maximum hydraulic gradient on the exit face of the flow net, and it is given by:

$$i_{\text{exit}} = \frac{\Delta h}{L} \quad (3.12)$$

Where:

Δh : the difference in head between the final two equipotential lines

L: length of flow element

- Harza also provided charts for the maximum exit gradient of dams built over deep homogeneous deposits, as shown in Figure (2-2). The highest exit gradient can be expressed as follows:

$$i_{\text{exit}} = C \frac{h}{B} \quad (3.13)$$

Where :

C: constant.

h: the head difference between U/S and D/S.

B: the length of floor.

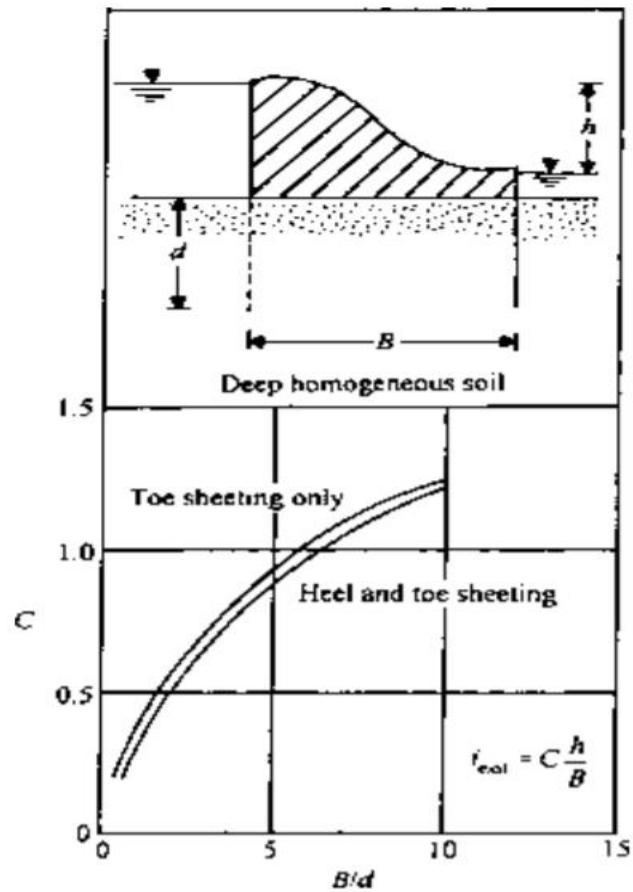


Figure (3-3): Critical exit gradient (DAS, 2008)

- Harr (1962) presented a theoretical answer for determining the highest exit gradient for a single pattern of sheet pile structures:

$$i_{\text{exit}} = \frac{1}{\pi} \left(\frac{\text{maximum hydraulic head}}{\text{depth of penetration of sheet pile}} \right) \quad (3.14)$$

- Lane (1935) researched dam safety against piping and proposed an empirical solution to the problem and created the name "weighted creep distance," which is calculated using the lowest flow path:

$$l_w = \frac{\sum l_h}{3} + \sum l_v \quad (3.15)$$

Where:

l_w = weighted creep distance

$l_h = L_{h1} + L_{h2} + \dots =$ the sum of the horizontal distances along the shortest flow path

$l_v = L_{v1} + L_{v2} + \dots =$ the sum of the vertical distances along the shortest flow path

After calculating the weighted creep length, the weighted creep ratio can be calculated as follows:

$$\text{Weighted creep ratio} = \frac{l_w}{H_1 - H_2} \tag{3.16}$$

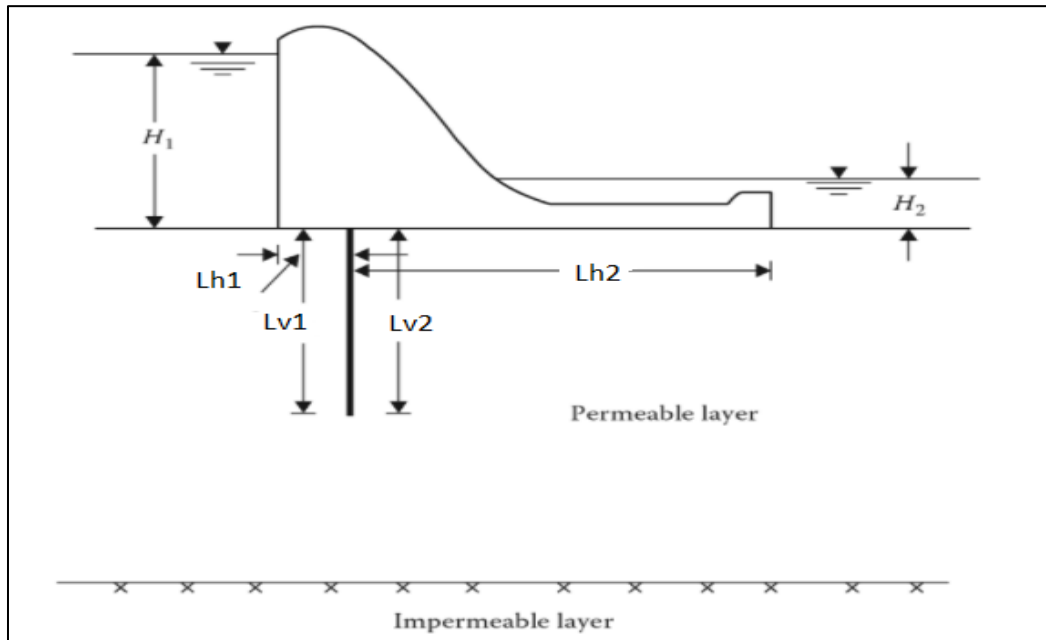


Figure (3-4): Calculation of weighted creep distance (DAS, 2008)

Lane (1935) proposed that for a structure to be safe toward piping, the weighted creep ratio be equal to or higher than the safe values as seen in Table 3-2.

Table (3-2): Values of the weighted creep ratio (DAS, 2008)

Material	Safe weighted creep ratio
Very fine sand or silt	8.5
Fine sand	7.0

Medium sand	6.0
Coarse sand	5.0
Fine gravel	4.0
Coarse gravel	3.0
Soft to medium clay	2.0–3.0
Hard clay	1.8
Hard pan	1.6

In the case of sheet piles Terzaghi (1922) found that hydraulic heave occurs at a point at a distance of half the depth of embedment of the sheet pile. Terzaghi concluded that the stability of this type of sheet pile structure can be determined by taking into account a soil prism on the downstream side of unit thickness and section($D \times D/2$). This is depicted in Figure 2-4.

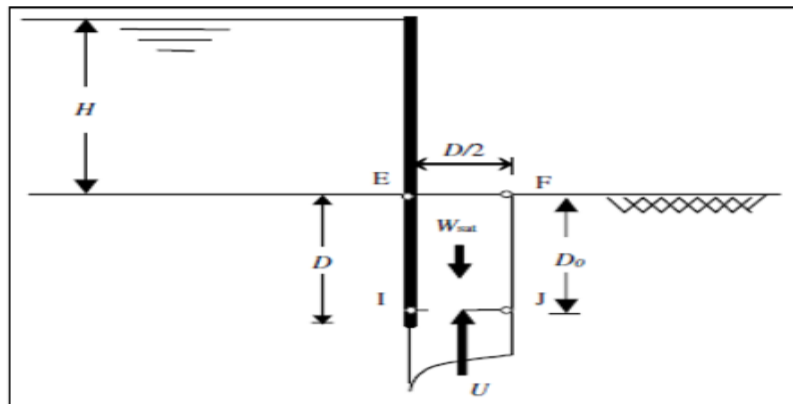


Figure (3-5): Hydraulic Heave in Sheet Pile Dam (Benmebarek et al., 2005)

3.2.3.2 Safety against uplift pressure

Uplift pressure, which occurs as a result of water seepage beneath the hydraulic structure, generates an uplift pressure on the hydraulic structure's floor. If the weight of the structure is inadequate to withstand the up lift pressure, the floor may burst and the effective length of impervious floor is thereby reduced. This may be prevented by:

- * providing impervious floor of sufficient length,

- * providing impervious floor of appropriate thickness at various points, and
- * providing pile at the U/S end so that the uplift pressure under the floor is reduced.

According to Bligh, a floor is safe against up lift pressure if downward weight of the floor is equal to or more than uplift force. The floor's thickness is calculated as follows:

Upward Water pressure = Downward structure pressure

$$\gamma_w (h + t) = \gamma_w \cdot Gs \cdot t \text{ or } t = \frac{h}{Gs - 1} \tag{3.16}$$

where:

- t : the floor's thickness under the gate (L);
- h : the residual head or the residual uplift pressure (L);
- γ_w : the unit weight of water (F/L³);
- GS : the specific gravity of floor's concrete = 2.4.

The gate is usually located at the third of the horizontal floor(b), so the uplift pressure under the gate is calculated as follows:

$$h = H(1 - \frac{b}{3}) \dots \dots \dots h = \frac{2}{3}H \tag{3.17}$$

where:

- H : the seepage head.
- b: the length of floor.

3.3 Equation of continuity (Laplace equation)

To obtain the equation of continuity of flow, The flows entering the soil prism in the x, y, and z directions at point A for the hydraulic structure shown in Figure (3-2) can be given from Darcy’s law as follows:

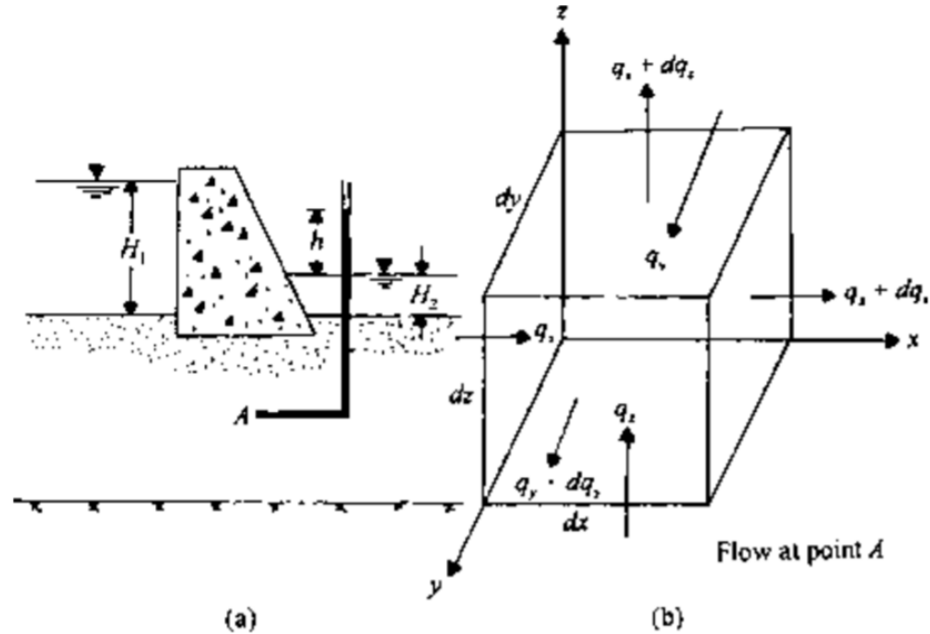


Figure (3-6): Derivation of continuity equation.(DAS, 2008)

$$q_x = k_x i_x A_x = k_x \frac{\partial h}{\partial x} dy dz \quad (3.18)$$

$$q_y = k_y i_y A_y = k_y \frac{\partial h}{\partial y} dx dz \quad (3.19)$$

$$q_z = k_z i_z A_z = k_z \frac{\partial h}{\partial z} dx dy \quad (3.20)$$

Where:

q_x, q_y, q_z = Flow enters in the x, y, and z directions, respectively..

k_x, k_y, k_z = permeability coefficients in the x, y, and z directions, respectively.

h = hydraulic head at point A

The flows exiting the soil prism in the x, y, and z directions are

$$q_x + dq_x = k_x (i_x + di_x) A_x \quad (3.21)$$

$$q_x + dq_x = k_x \left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2} dx \right) dy dz \quad (3.22)$$

$$q_y + dq_y = k_y \left(\frac{\partial h}{\partial y} + \frac{\partial^2 h}{\partial y^2} dy \right) dx dz \quad (3.23)$$

$$q_z + dq_z = k_z \left(\frac{\partial h}{\partial z} + \frac{\partial^2 h}{\partial z^2} dz \right) dx dy \quad (3.24)$$

The flow entering the soil prism is the same to the flow exiting from it in the steady flow across an incompressible medium. So,

$$q_x + q_y + q_z = (q_x + dq_x) + (q_y + dq_y) + (q_z + dq_z) \quad (3.25)$$

Combining Eqs. (3.18-3.25), we obtain

For three-dimensional steady flow across incompressible homogeneous anisotropic media the following formula can be used:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.26)$$

For two-dimensional flow, Eq. (3.26) is turned into the following formula:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \quad (3.27)$$

where:

h : is the piezometric head (L),

k_x : the permeability in horizontal direction (L/T),

k_y : the permeability in vertical direction (L/T).

If the soil is isotropic in terms to permeability, $k_x = k_z = k_y$, the following equation is simplified to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad (3.28)$$

Laplace's equation is the name given to this equation.

The Laplace equation is a partial differential equation used to explain the attitude of fluid potentials.

Using Darcy's Law, the equation can be put in terms of gradients and permeability's, as long as the following assumptions are made:

1. The law of Darcy holds true. (Laminar flow)
2. The soil is saturated. (The saturation level is 100 %)
3. The soil is homogeneous. (Permeability coefficients are constant in the soil medium.)
4. The soil is isotropic. (Permeability coefficients are same in all directions)
5. The amount of soil and water keeps constant through flow. (There is no expansion or contraction)
6. Water and soil are incompressible. (There is no difference in volume) (Sachpazis, 2014).

3.4 Boundary condition

The saturated soil which is considered for analysis must be defined by boundaries, permeability of the soil, and heads imposed upon the water. Normally, simplifying assumptions are required in order to establish boundaries which will make analysis feasible. The boundary conditions can be defined as follows (Hassan, 2019):

- ***Impervious boundary (no flow)***: As water cannot seep through these limits, On the surface, the component of vertical seepage flow, q_n , should be equal to zero, as shown below:

$$q_n = k_x \left(\frac{\partial h}{\partial x} \right) I_x + k_y \left(\frac{\partial h}{\partial y} \right) I_y = 0 \quad (3.29)$$

In which I_x and I_y indicate the direction cosine of a vertical vector on the surface with x and y directions, respectively. These boundaries perform a constant stream function, streamline.

- ***Boundaries of the reservoirs (Constant Head)***: Hydrostatic pressure is defined as the distribution of pressure on the steady head's boundaries. As a result, along the boundaries under the depth of water (h°), the pressure value (P) is as follows:

The piezometric head's value along these boundaries is steady:

$$h = \frac{p}{\gamma_w} + z \quad (3.30)$$

As such, all reservoir boundaries represent equipotential lines.

3.5 Isotropy and anisotropy

If a 'xyz' coordinate system is constructed in such a way that the coordinate orientation agrees with the anisotropy direction. k_x , k_y , and k_z are the hydraulic conductivity values of x , y , and z directions, respectively. Isotropic formations will be $k_x = k_y = k_z$ at any point with coordinates (x, y, z) , while anisotropic formations will be $k_x \neq k_y \neq k_z$.

The soil is said to be an isotropic flow medium if hydraulic conductivity is unaffected by velocity direction. The soil is said to be homogeneous and isotropic if its coefficient of hydraulic conductivity has the same value at all points within the flow region. The soil is said to be homogeneous and anisotropic if the coefficient of hydraulic conductivity is dependent on the direction of the velocity, and this coefficient is the same at all points of the flow field. The coefficient of hydraulic conductivity in homogeneous anisotropic soils is determined by the velocity direction but not by space coordination.

3.6 Seepage equation for homogenous anisotropic medium

The soil would be considered to be anisotropic in terms of permeability, despite being homogeneous. The coefficient of permeability in most natural soil is anisotropic, with a maximum value in the orientation of stratification and a minimum value in the orientation vertical to stratification; these directions are defined as x and z , respectively. (Al-Suhaili, 2014)

$$K_x = K_{\max} \quad \text{and} \quad K_z = K_{\min}$$

Now, for anisotropic soil $\rightarrow k_x \neq k_z$ first, the two soils will combined into a single isotropic one. ($k_{eq.x} = k_{eq.z} = k_{eq}$).

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.31)$$

Second, rewrite Laplace's equation to be in the necessary form to draw true flow net because the lines of flow path and equipotential are not perpendicular to each other.

The equation above divided by k_z

$$\frac{\partial^2 h}{\left(\frac{k_z}{k_x}\right) \partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{3.32}$$

Now let

$$x_t = x \sqrt{\frac{k_z}{k_x}} \quad \dots\dots\dots \text{(scale factor)} \tag{3.33}$$

The equation of continuity becomes:

$$\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{3.34}$$

That is the necessary form for drawing a true flow net and is the continuity equation in an $x_t z$ plan.

As a result, equation (3.33) states a scale factor that can be applied in the x direction to convert an anisotropic flow field into a hypothetical isotropic flow field in which the Laplace equation holds true. the flow net (representing the solution of the Laplace equation) for the transformed section has been drawn. The flow net for the actual section can be achieved by applying the inverse of the scaling factor

3.7 Escape and Critical Gradients

The escape or exit gradient, ie , is the rate of dissipation of head per unit of length in the area where seepage is exiting the porous media. For confined flow, the area of concern is usually along the uppermost flow line near the flow exit, e.g., at the downstream edge of a concrete or other impermeable structure.

Escape gradients for flow through embankments may also be studied by choosing squares from the area of interest in the flow net (usually at or near the exit face and downstream toe) and calculating gradients. If the gradient is too great where seepage is exiting, soil particles may be removed from this area. This

phenomenon, called flotation, can cause piping (the removal of soil particles by moving water) which can lead to undermining and loss of the structure. The gradient at which flotation of particles begins is termed the critical gradient, i_{cr} . Critical gradient is determined by the in-place unit weight of the soil and is the gradient at which upward drag forces on the soil particles equal the submerged weight of the soil particles.

The critical gradient is dependent on the specific gravity and density of the soil particles and can be defined in terms of specific gravity of solids, G_s , void ratio, e , and porosity, n . (Al-Suhaili, 2014)

$$i_{cr} = \frac{\gamma' m}{\gamma_w} = \frac{G_s(1-n)\gamma_w + n\gamma_w - \gamma_w}{\gamma_w} \quad (3.35)$$

$$i_{cr} = G_s(1 - n) + n - 1$$

$$i_{cr} = G_s(1 - n) - (1 - n)$$

$$i_{cr} = (G_s - 1)(1 - n)$$

$$\text{Or, since } e = \frac{n}{1-n} \quad \text{and} \quad n = \frac{e}{1+e} \quad (3.36)$$

$$i_{cr} = (G_s - 1)\left(\frac{n}{e}\right)$$

$$i_{cr} = (G_s - 1)\left(\frac{e}{1+e}\right)$$

$$i_{cr} = \frac{(G_s-1)}{(1+e)} \quad (3.37)$$

If typical values of G_s , e , and n equals to 2.65, 0.65, 0.39 respectively, are used in the above equations, i_{cr} will be approximately 1. Investigators have recommended ranges for factor of safety for escape gradient, $FS_{piping} = \frac{i_{cr}}{i_e}$ in the range of 4-5 (Harr, 1962) or 2.5-3 (Cedergren Harry et al., 1977) have been proposed depending on knowledge of soil and possible seepage conditions.

3.8 Finite difference method

3.8.1 Introduction

The finite difference method (FDM) is a well-known method that is used to approximate the solution of partial differential equations. It was already known by L. Euler (1707-1783) in one dimension of space and was probably extended to two dimensions by C. Runge (1856-1927). This method is effective when the domain of the problem has boundaries with regular shapes.

Physical and engineering problems such as equilibrium problems and steady state phenomena (independent of time) can be described as elliptic partial differential equations (elliptic PDEs). These equations express the behavior of such problems. Most of these problems are very hard to solve analytically, instead, they can be solved numerically using computational methods. Second order linear partial differential equations are mainly considered as:(Al-Rob, 2016)

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + c \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + F u = G(x, y) + D \quad (3.38)$$

Where A, B, C, D, E, F, and the free term G are the coefficients of Eq.(3. 31) which can be constants or functions of two independent variables x and y, and u is the dependent variable and is an unknown function of two independent variables x and y.

Eq. (3.38) is classified into three types depending on the discriminant $(B^2 - 4AC)$ as follows:

1. Hyperbolic if the discriminant is positive $(B^2 - 4AC > 0)$
2. Parabolic if the discriminant is zero $(B^2 - 4AC = 0)$
3. Elliptic if the discriminant is negative $(B^2 - 4AC < 0)$

3.8.2 The principle of finite difference method

The idea of FDM is to replace the partial derivatives of dependent variable (unknown function) with partial differential equation using finite difference approximations with $O(h^n)$ errors. This procedure converts the region (where the independent variables in PDE are defined on) to a mesh grid of points where the dependent variables are approximated. The replacement of partial derivatives with

difference approximation formulas depends on Taylor's theorem (Al-Rob, 2016). So, Taylor's theorem is introduced.

$$u(x_0 + h) = u(x_0) + h \frac{u_x(x_0)}{1!} + h^2 \frac{u_{xx}(x_0)}{2!} + h^3 \frac{u_{xxx}(x_0)}{3!} + \dots + h^{n-1} \frac{u_{n-1}(x_0)}{(n-1)!} + o(h^n) \quad (3.39)$$

3.8.3 Derivation the formula of finite difference approximation for Laplace equation

we use finite difference method for solving elliptic PDEs, A system of linear equations will be generated and should be solved to compute heads at nodes using several iterative schemes such as Jacobi, Gauss-seidel, successive over relaxation (SOR) and conjugate gradient methods (Al-Rob, 2016).

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = g(x, y) \quad (3.40)$$

This is poisson's equation or Laplace's equation (when $g(x,y) = 0$) which may be used to model the steady state temperature distribution in a plate or incompressible potential flow (Causon and Mingham, 2010)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (3.41)$$

The concept of the flow through a porous media is based on Laplace's equation of continuity, which represents the steady flow condition for a given point in the soil mass. To derive the formula of finite difference approximation for Laplace equation, We consider three points $i+1, i$, and $i-1$ which are located on X-axis with equal distance h between them as shown in Figure (3-7).



Figure (3-7): Schematic presentation of finite difference on the X-axis.

let the value of the function $u(x,y)$ at the points $(i-1,j)$, (i,j) , and $(i+1,j)$ be $u_{i-1,j}$, $u_{i,j}$ and $u_{i+1,j}$, respectively. Now, use Taylor series to express $u_{i+1,j}$ and $u_{i-1,j}$ in the form of Taylor expansions about the point I as follows:

$$u_{i+1,j} = u_{i,j} + \frac{h}{1!} \frac{\partial u}{\partial x}]_i + \frac{h^2}{2!} \frac{\partial^2 u}{\partial x^2}]_i + \frac{h^3}{3!} \frac{\partial^3 u}{\partial x^3}]_i + \frac{h^4}{4!} \frac{\partial^4 u}{\partial x^4}]_i + o(h^5) \quad (3.42)$$

$$u_{i-1,j} = u_{i,j} - \frac{h}{1!} \frac{\partial u}{\partial x}]_i + \frac{h^2}{2!} \frac{\partial^2 u}{\partial x^2}]_i - \frac{h^3}{3!} \frac{\partial^3 u}{\partial x^3}]_i + \frac{h^4}{4!} \frac{\partial^4 u}{\partial x^4}]_i + o(h^5) \quad (3.43)$$

By adding eq.(3.33)and eq.(3. 34), the following equation is obtained:

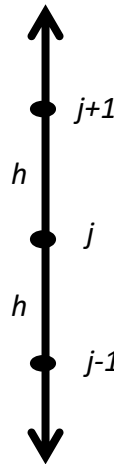
$$u_{i+1,j} + u_{i-1,j} = 2u_{i,j} + h^2 \frac{\partial^2 u}{\partial x^2}]_i + \frac{h^4}{12} \frac{\partial^4 u}{\partial x^4}]_i + o(h^5) \quad (3.44)$$

By rearranging the above equation, we get:

$$\frac{\partial^2 u}{\partial x^2}]_i = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + o(h^2) \quad (3.45)$$

Eq.(3.36) is a finite difference approximation formula with error term $o(h^2)$ of second order for $\frac{\partial^2 u}{\partial x^2}]_i$

Similarly, consider three points $j+1$, and $j-1$ which are located on the y-axis with equal distance h between them as shown in Figure (3-8).



Figure(3-8): Schematic presentation of finite difference on the Y-axis.

let the value of the function $u(x,y)$ at the points $(i,j+1)$, (i,j) , and $(i,j-1)$ be $u_{i,j+1}$, $u_{i,j}$, and $u_{i,j-1}$, respectively. Using Taylor series to express $u_{i,j-1}$ and $u_{i,j+1}$ in the form of Taylor expansions at the point j , the finite

difference approximation formulas with error term $o(h^2)$ of second order for $\frac{\partial^2 u}{\partial x^2}]_i$ and for first order for $\frac{\partial u}{\partial x}]_i$ are, respectively:

$$\frac{\partial^2 u}{\partial y^2} j = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} + o(h^2) \quad (3.46)$$

And

$$\frac{\partial u}{\partial y} j = \frac{u_{i,j+1} - u_{i,j-1}}{2k} + o(h^2)$$

Now, by combining Figure (4-7) and Figure (3-8) Together, the star shape(or 5-points stencil) region about the point (i,j) is obtained as shown in Figure (3-9) (Causon and Mingham, 2010)

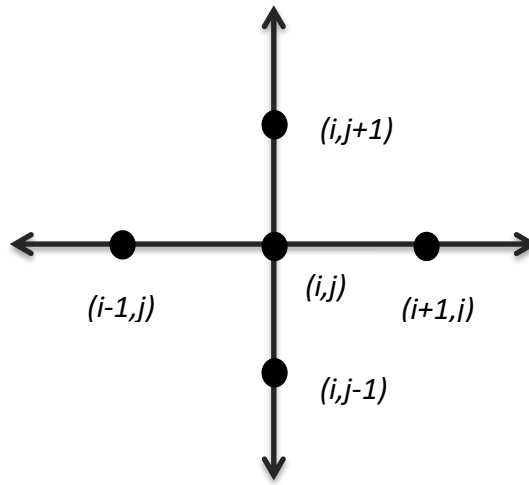


Figure (3-9): The star shape for point (i,j) in the finite difference method

Inserting Eq. (3.32) and eq. (3.33) into eq. (3.35) yields:

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)]_{(i,j)} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0 \quad (3.47)$$

$H=k$, By rearranging the above equation , the following equation is obtained:

$$(u_{i+1,j} + u_{i,j+1}) - 4u_{i,j} + (u_{i-1,j} + u_{i,j-1})=0$$

So,

$$u_{i,j} = \frac{1}{4} [u_{i+1,j} + u_{i,j+1} + u_{i-1,j} + u_{i,j-1}] \quad (3.48)$$

In general, u at any point in the domain, is the average of the values of u at the four surrounding points in the 5-points stencil as shown in Figure (3-9).

3.9 Genetic Algorithms Modeling (GAM)

3.9.1 introduction

In the early 1970s, John Holland introduced the concept of genetic algorithms. Genetic algorithms (or GA) are a type of optimization algorithm, meaning they are used to find the optimal solutions to a given computational problem that maximizes or minimizes a particular function and has been one of the most active research fields in artificial intelligence. Genetic algorithms are based on the genetic processes of biological organisms. Over many generations, natural populations evolve according to the principles of natural selection and “survival of the fittest” first clearly stated by Charles Darwin in The origin of species (Beasley et al., 1993)

In Genetic algorithm, some of key elements are as follows:

Genes: Genes are the basic units’ “instructions” for building the genetic algorithms. Genes may represent a possible solution to a problem without actually being a solution.

Individuals (chromosomes): An individual is a single solution involved currently in the search process. Each individual is named a chromosome or string. A chromosome is a sequence of genes, compared to chromosomes in natural systems. A chromosome is a main key element, which the genetic algorithm is dealing with.

Populations: A population is a group of individuals currently involved in research

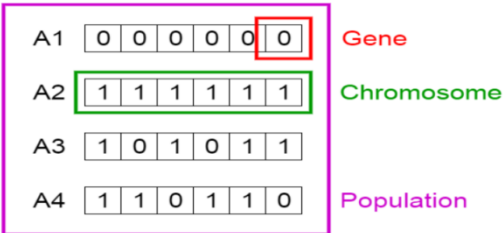


Figure (3-10) Elements in genetic algorithm

3.9.2 Outline of the basic genetic algorithm

Step 1: Represent the problem variable domain as a chromosome of a fixed length, choose the size of a chromosome population N , the crossover probability p_c and the mutation probability p_m .

Step 2: Define a fitness function to measure the performance, or fitness, of an individual chromosome in the problem domain. The fitness function establishes the basis for selecting chromosomes that will be mated during reproduction.

Step 3: Randomly generate an initial population of chromosomes of size N : x_1, x_2, \dots, x_N

Step 4: Calculate the fitness of each individual chromosome: $f(x_1), f(x_2), \dots, f(x_N)$

Step 5: Select a pair of chromosomes for mating from the current population. Parent chromosomes are selected with a probability related to their fitness.

Step 6: Create a pair of offspring chromosomes by applying the genetic operators - crossover and mutation.

Step 7: Place the created offspring chromosomes in the new population.

Step 8: Repeat Step 5 until the size of the new chromosome population becomes equal to the size of the initial population, N .

Step 9: Replace the initial (parent) chromosome population with the new (offspring) population.

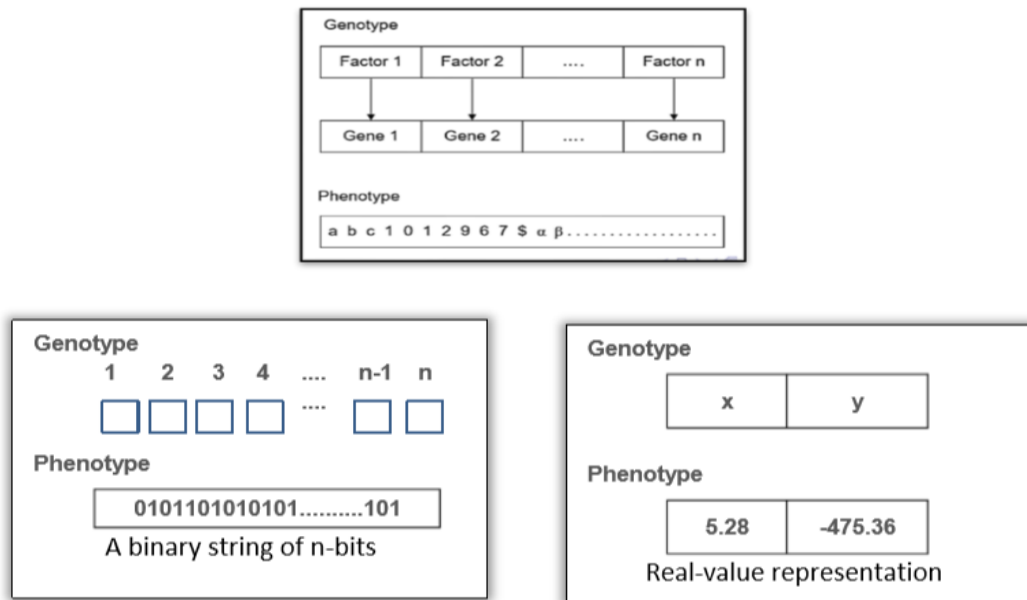
Step 10: Go to Step 4, and repeat the process until the termination criterion is satisfied.

3.9.3 Basic Principles of Genetic Algorithm

Before a GA can be run, a suitable coding or representation, for the problem must be devised. There is also require to a fitness function, which assigns a figure of merit to each coded solution. During the run, parents must be selected for reproduction and recombined to create offspring. These aspects are described below:

3.9.3.1 Coding or representation

The key issue when using GAs is the encoding of a solution to the problem. The issue has been investigated in many aspects, such as mapping characters from genotype space to phenotype space when individuals are decoded into solutions and metamorphosis properties when individuals are manipulated by genetic operators (Gen and Cheng, 1999). An individual is a single solution while a population is a set of individuals at an instant of the searching process. An individual is defined by a chromosome. A chromosome stores genetic information (called phenotype) for an individual as shown in Figure (3-11).



Figure(3-11): Chromosome

According to what kind of symbol is used for gene, the encoding methods can be classified follows:

- **Binary encoding:** Representing a gene in terms of bits (0 and 1) strings.

Chromosome A	101100101100101011100101
Chromosome B	111111100000110000011111

Figure (3-12): Binary encoding

- **Real-number encoding:** Representing a gene in terms of values or symbols or string. It can be anything connected to the problem.

Chromosome A	1.2324 5.3243 0.4556 2.3293 2.4545
Chromosome B	ABDJEIFJDHDIERJFDLDFLFEGT
Chromosome C	(back), (back), (right), (forward), (left)

Figure (3-13): Real-number encoding

- **Integer or literal permutation encoding (order encoding):** Representing a sequence of elements. It can be used in ordering problems, such as the travelling salesman problem or task ordering problem.

Chromosome A	1 5 3 2 6 4 7 9 8
Chromosome B	8 5 6 7 2 3 1 4 9

Figure (3-14): Order encoding

- **General data structure encoding (tree encoding):** Representing in the form of a tree of objects. It is used mainly for evolving programs or expressions of genetic programming.

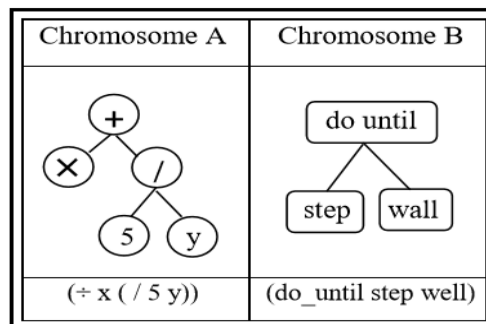


Figure (3-15): Tree encoding

Real-number encoding is best used for function optimization problems (Gen and Cheng, 1999)

3.9.3.2 Fitness function

Evaluate the fitness of each chromosome by knowing its fitness function. To calculate the fitness of the chromosome must first be encoded and then find the objective function of it. The fitness does not only indicate how good the solution is, but also corresponds to how close the chromosome is to the optimal one. For maximization problems, the fitness function can be considered to be the same as the objective function, for minimization problems, the number of such transformations are possible. The following fitness is often used:

$$F(i) = \frac{1}{\text{objective function}(i)} \quad (3.49)$$

The fitness function must be more sensitive than just detecting what is a ‘good’ chromosome versus a ‘bad’ chromosome: it needs to accurately score the chromosomes based on a range of fitness values, so that a somewhat complete solution can be distinguished from a more complete solution.

The highest fitness value is considered the best solution and the least value the poorest solution. If the problem is to minimize the cost, then high cost solutions will have low fitness, and low-cost solutions will have high fitness.

3.9.3.3 Selection

The process of choosing two individuals from the original population for reproduction in an evolutionary process is called selection. Proportional selection is one of the common forms for selection. The name indicates that this approach involves the establishment of a number of offspring in proportion with the fitness of the individual. This method was suggested and analyzed by Holland (1975). It has been widely used in many applications of evolutionary algorithms.

The operator's selection in exploiting the characteristics of the best candidate solutions is aimed at improving these solutions in all generations. The selection operator is the most important parameter that may affect the performance of the GA. The operator directs the genetic search to promising areas in the search space. In selection, the fitness values of individuals are taken into account only.

During the literature review many selection methods have been proposed in the proposed model and seven different selection techniques will be used in the genetic algorithm. These techniques are presented here, namely: the Roulette Wheel Selection (RWS), the Rank Roulette Wheel Selection (RRWS), the Linear Rank Selection (LRS), the Stochastic Universal Sampling (SUS), the Tournament Selection (TOS), the Truncation Selection (TRS) and the Random selection (RMS). A brief description of the Roulette Wheel Selection (RWS), as follows:

3.9.3.3.1 Roulette Wheel Selection (RWS)

The roulette wheel is the simplest and traditional random selection method proposed by the Holland. It is classified under a proportional selection as individuals select on the basis of a probability proportionate with their fitness. The roulette selection principle is a linear search through the roulette wheel with slots in the wheel weighted in proportion to the fitness values of the individual. All chromosomes (individuals) are placed in the population on the roulette wheel according to the value of their fitness (Goldberg, 1991). Assigned to each individual a part on the roulette wheel. The size of each part in the roulette wheel is proportional to the value of an individual's fitness, The higher the value, the greater the part. Then, the virtual roulette wheel is spin. Then selected the individual corresponding to the part on which roulette wheel stops, as shown in the Figure (3-16). The process is repeated until the desired number of individuals is selected. Individuals with higher fitness have more probability of selection. This may lead to biased selection towards high fitness individuals. It can also possibly miss the best individuals of a population. There is no guarantee that good individuals will find their way into the next generation. Roulette wheel selection uses exploitation technique in its approach.

The conspicuous characteristic of this selection method is in fact, every member (i) of the current population that is given the probability (pi) of being selected (Hancock and et al., 2000), proportional with its fitness (fi)

$$Pi = \frac{f_i}{\sum_{i=1}^n f_i} \quad (3.50)$$

Where n denotes to the population size in terms of the number of individuals.

It is very important to refer to the known disadvantage in this technique, which is the risk of rapid convergence premature of the GA to a local optimum, since the existence of a dominant individual who always wins the competition and selects as a parent. In roulette wheel selection, individuals are given a probability of being selected that is directly proportionate to their fitness. Two individuals are then chosen randomly based on these probabilities and produce offspring.

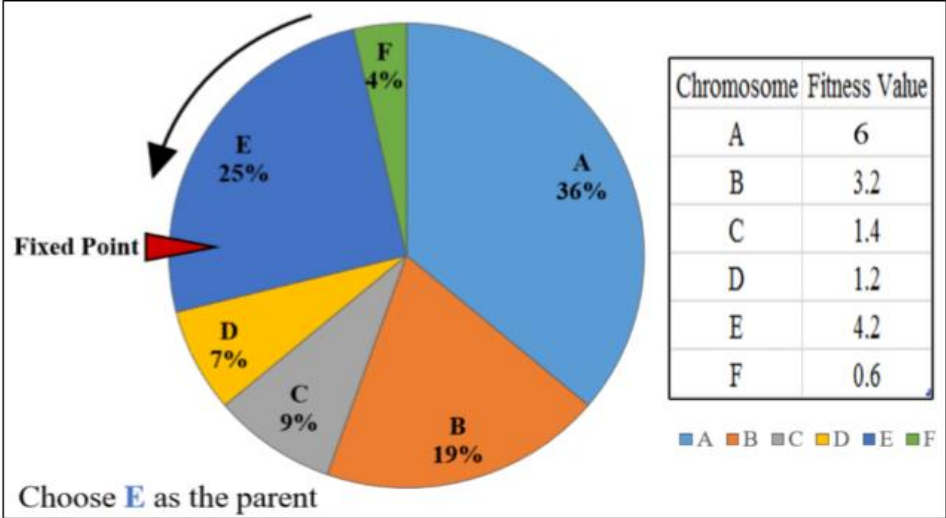
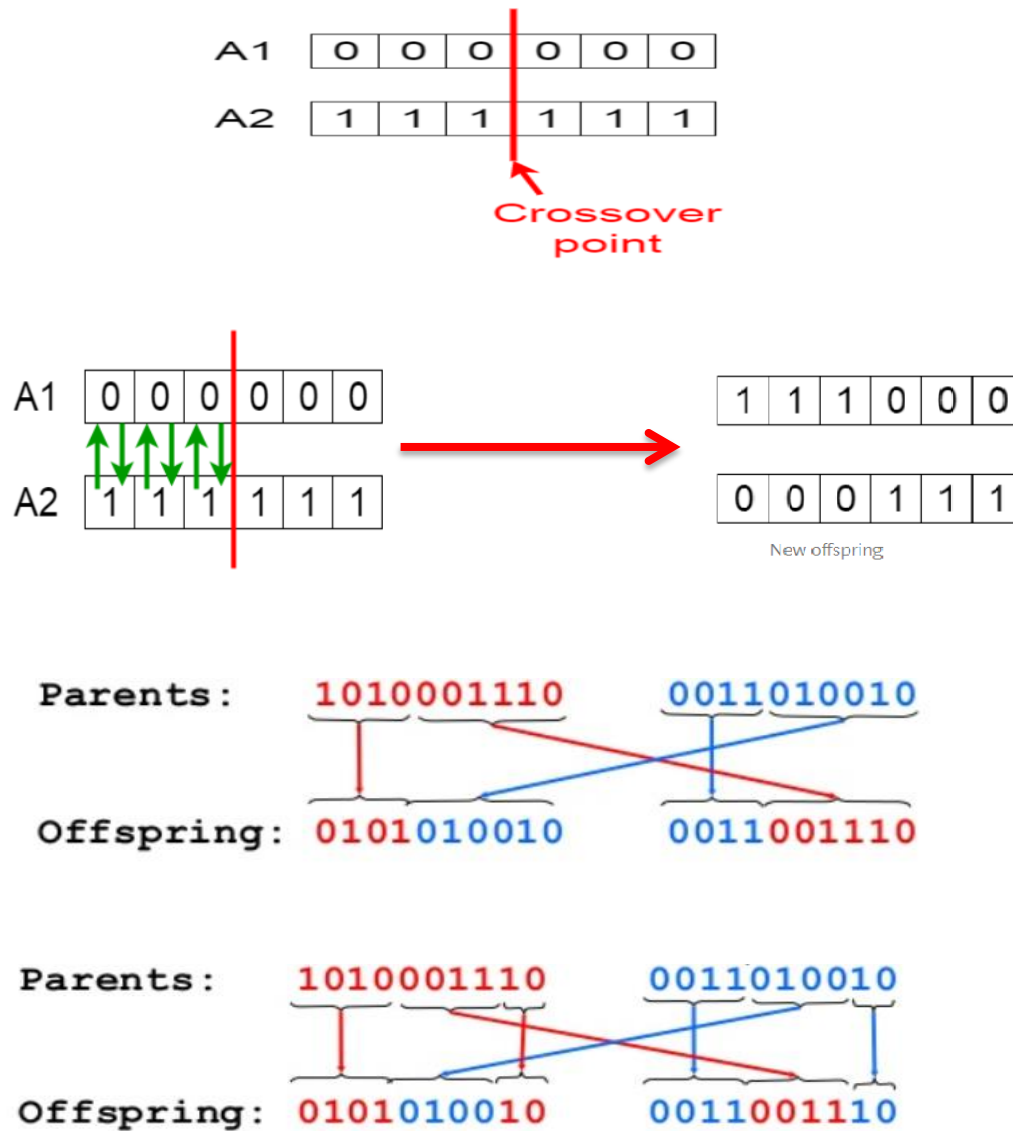


Figure (3-16): Roulette-wheel selection mechanism (Hancock and et al., 2000).

3.9.4 Crossover

Crossover, which also called recombination, is a genetic operator used to combine the genetic information of two parents to generate new offspring. Like its counterpart in nature, crossover produces new individuals that have some parts of both parent’s genetic material.

In most crossover operation, two strings are picked from the mating pool at random, and some portions of the strings are exchanged between the strings. A single-point crossover operator is performed by randomly choosing a crossing site along the string and by exchanging all bits on the right side of the crossing site as shown below:



Figure(3-17): Types of crossover.

If good strings are created by crossover, there will be more copies of them in the next mating pool generated by the reproduction operator. But if good string are not created by crossover, they will not survive too long, because reproduction will select against those strings in subsequent generations.

It is clear from this discussion that the effect of crossover may be detrimental or beneficial. Thus, in order to preserve some of good strings that are already present in the mating pool, not all strings in the mating pool are used in crossover. When a crossover probability of p_c is used, only $100p_c$ percent strings in the

population are used in the crossover operation and $100(1 - p_c)$ percent of the population remains as they are in the current population.

The most common type is single point crossover. In single point crossover, a locus at which remaining alleles are swap from one parent to the other is chosen. This is complex and it is best understood visually. It is clearly shown that the children take one section of the chromosome from each parent. The point at which the chromosome is broken depends on the randomly selected crossover point.

Crossover does not always occur, however, sometimes, based on a set probability, no crossover occurs and the parents are directly copied to the new population. The probability of crossover occurring is usually from 60% to 70%.

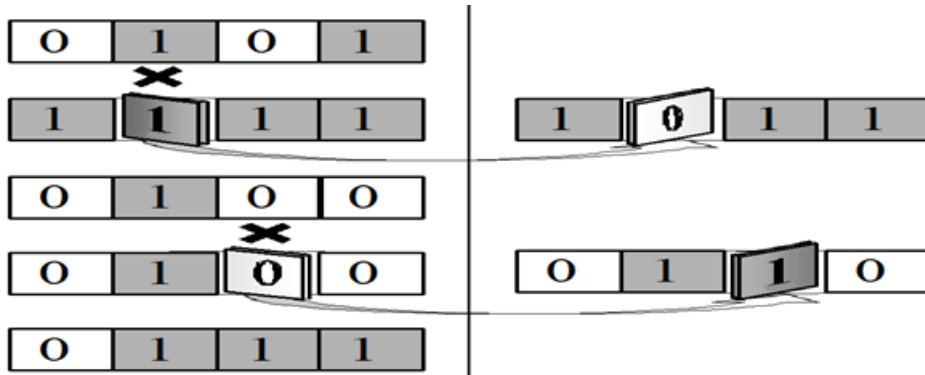
3.9.5 Mutation

Chromosomes are subjected to mutation after crossover. The main advantage of the mutation is to prevent the algorithm from being stuck at the local minimum. It provides new genetic structures in the population where the individual's units are randomly modified. There are many different forms of mutations for different types of representation. One of the most important parameters in the technique of mutation is mutation's probability (P_m). The probability of a mutation determines the number of times that parts of the chromosomes will mutate. If the mutation will not occur, the offspring will be copied directly after the crossover without any modification. If the mutation occurs, one or more of the chromosomes building units will be changed. The entire chromosome will change if the probability of the mutation is 100%, and nothing will change if it is 0%. The mutation should not occur too much, because the genetic algorithm will change to random search in fact.

The mutation operator alters a string locally to hopefully create a better string. It changes 1 to 0 and vice versa with a small mutation probability, p_m . The bit-wise mutation is performed bit by bit by flipping a coin with a probability p_m . If at any bit the outcome is true then the bit is altered; otherwise, the bit is kept unchanged. The need of mutation is to create a point in the neighborhood of the current point, thereby achieving a local search around the current solution. The mutation is also used to maintain diversity in the population. For example,

- Mutation represents a change in the gene.

- Mutation is a background operator. Its role is to provide a guarantee that the search algorithm is not trapped on a local optimum.
- The mutation operator flips a randomly selected gene in a chromosome.
- The mutation probability is quite small in nature, and is kept low for GAs, typically in the range between 0.001 and 0.01



Figure(3-18): Mutation operate

Chapter Four

MODELS DEVELOPMENT

4.1 Problem identification:

The primary purpose of a dam may be defined as to provide for the safe retention and storage of water. Gravity dams are solid concrete structures that maintain their stability against design loads from the geometric shape and the mass and strength of the concrete. That's mean it depend upon its own mass for stability.

As previously mentioned, the most critical design of a hydraulic structure is the foundation design. the dimensions of the length of floor(B), depth of upstream cutoff (d1), depth of downstream cutoff (d2) for a given head difference (H) and a given depth of impervious layer (D), Figure (4-1) shows these dimensions.

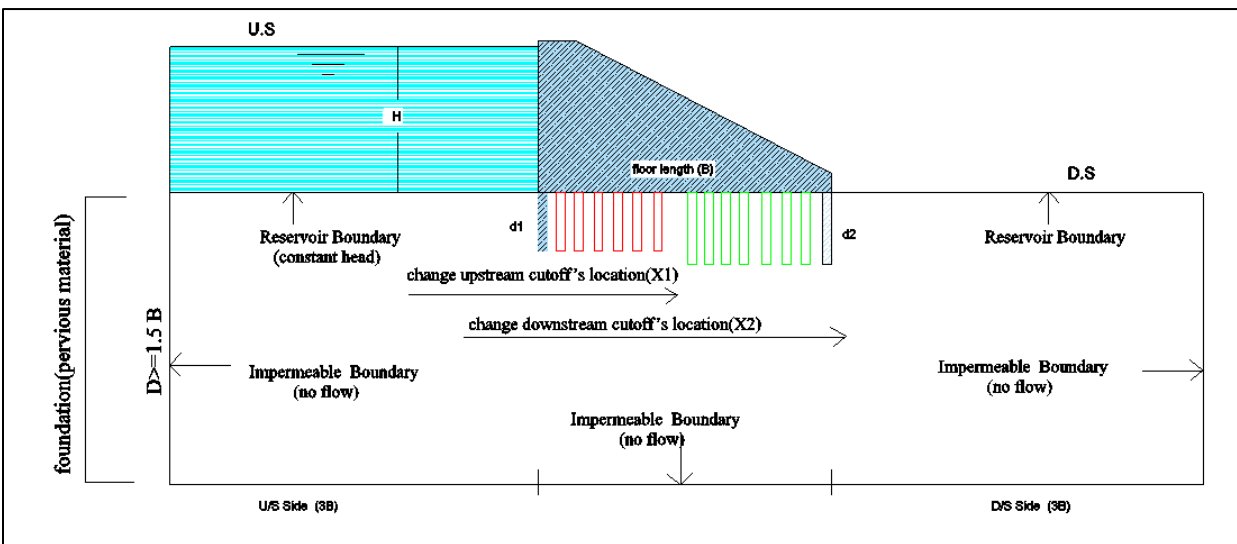


Figure (4-1): Sketch of the proposed model showing the boundary conditions and the foundation properties.

These values are affected by the maximum expected difference in head between the upstream and downstream sides of the hydraulic structure (H) and the soil strata properties (K_x and K_y). The most critical failure that may occur for such a structure are either due to uplift pressure or due to erosion of the

downstream side, when the hydraulic gradient exceeds the critical exit gradient. The designer can control these failures by providing the recommended factors of safety against both uplift pressure and exit gradient failures. The controlling process is done by selecting the dimensions of d_1, d_2, x_1, x_2, B for a given (H) , (D) and (K_x/K_y) . It is better to select optimum dimensions.

4.2 Finite difference Programming (FDP)

Elliptic partial differential equations appear frequently in various fields of science and engineering. These involves steady state distribution problem such as temperature distribution on a heated plate, seepage of water under dam, and the electric field near the point of a conductor. The most common examples of such equations are Laplace equation. This equation is classified as second order linear partial differential equations.

The concept of the flow through a porous media is based on Laplace's equation of continuity, which represents the steady flow condition for a given point in the soil mass.

The problem of seepage under concrete dam is performed in Matlab programming (finite difference). This code resolves the problem and find uplift force under dam, and exit gradient after entering the inputs data isotropic degree K_r , total head at upstream H_U . S and total head at downstream H_D . S , this code will be discussed with details in the section (4-3).

4.2.1 Solution Technique

The numerical solution of elliptic PDEs such as Laplace equation begins with replace the partial derivatives which appears in the equation by finite difference approximations (central differences) as shown in Figure (4-2).

When these differences are evaluated at each of the mesh points, the PDEs is transformed into an algebraic difference equation which can be solved either directly such as Gauss Elimination, or by iterative procedures such as Jacobi method, Gauss_sedi method.

4.2.1.1 The Laplacian Difference Equation

Each partial derivatives are replaced by Central differences which illustrated in below equations:

$$\frac{\partial^2 h}{\partial x^2} \Big|_i = \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{h^2} + o(h^2) \quad (4.1)$$

$$\frac{\partial^2 h}{\partial y^2} \Big|_j = \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{k^2} + o(k^2) \quad (4.2)$$

Which have errors of $o[\Delta(x)^2]$ and $o[\Delta(y)^2]$, respectively. substituting this expression into Eq. (3.31) gives

$$k_x \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{h^2} + k_y \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{k^2} = 0 \quad (4.3)$$

For the square element in grid as shown in the figure below, $h=0.5=k$, $k_x = k_y$ (isotropic) and by collection of terms, the equation becomes

$$h_{i+1,j} + h_{i-1,j} - 4h_{i,j} + h_{i,j+1} + h_{i,j-1} = 0 \quad (4.4)$$

This equation, which holds for all interior points in the domain and represent the dependent equation in the code, is referred to as the Laplacian difference equation as shown in Figure (4-2).

$$h_{i,j} = \frac{1}{4} [h_{i+1,j} + h_{i,j+1} + h_{i-1,j} + h_{i,j-1}] \quad (4.5)$$

Where , $k_x \neq k_y$ anisotropic, equation(4.3) divided by k_y , the equation becomes

$$\frac{k_x}{k_y} \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{h^2} + \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{k^2} = 0 \quad (4.6)$$

$$k_r \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{h^2} + \frac{h_{i,j+1} - 2h_{i,j} + h_{i,j-1}}{k^2} = 0 \quad (4.7)$$

$$k_r h_{i+1,j} - 2(k_r)h_{i,j} + k_r h_{i-1,j} + h_{i,j+1} - 2h_{i,j} + h_{i,j-1} = 0 \quad (4.8)$$

$$h_{i,j} = \frac{[k_r h_{i+1,j} + h_{i,j+1} + k_r h_{i-1,j} + h_{i,j-1}]}{(2*k_r)+2} \quad (4.9)$$

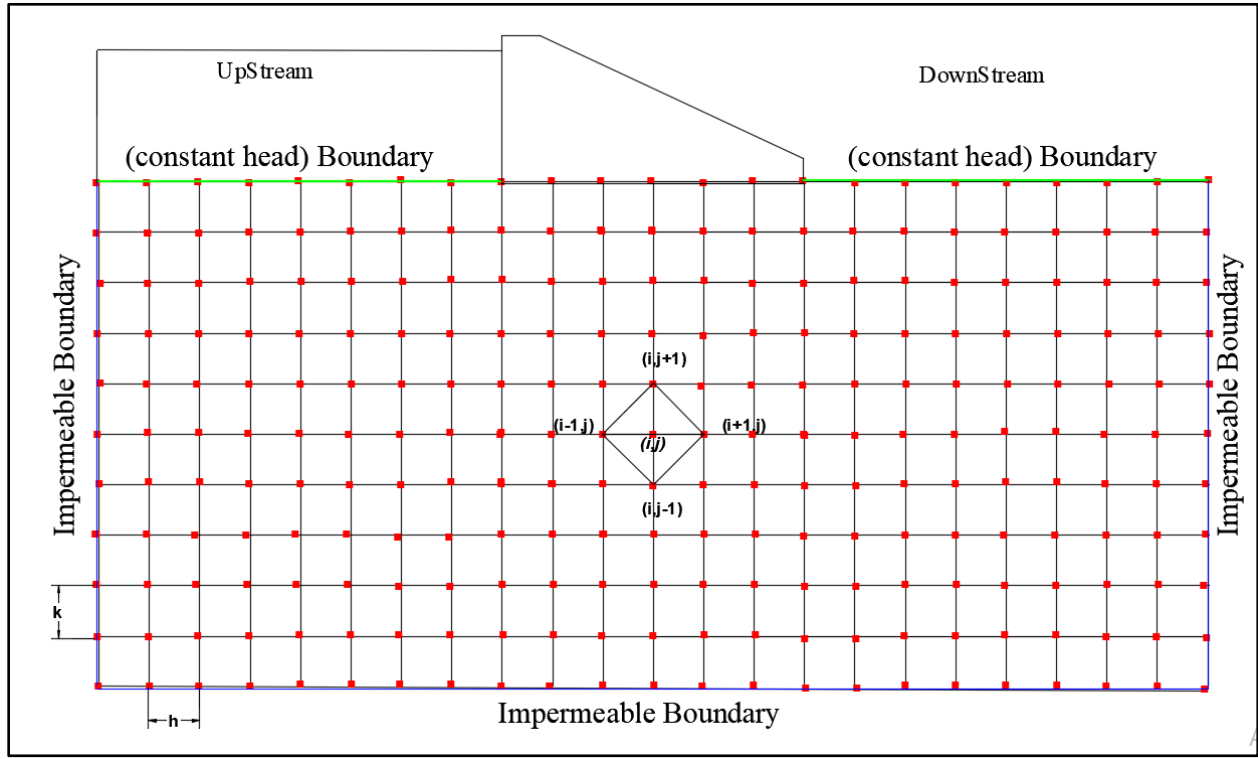


Figure (4-2) : The finite differences grid in X and Y directions of flow domain.

4.2.1.2 Boundary Conditions

The boundary conditions along the edge of the domain must be specified to obtain a unique solution. Two boundary conditions were used in this study, the first is the simplest case where the head at the boundary is a fixed value, This called a Dirichlet boundary condition (constant head) in upstream and downstream. The second is the condition where the value of the head derivative ($\partial H/\partial n = 0$), this called the Neumann boundary condition. These boundary conditions are illustrated in Figure (4-2).

4.3 Formulation of Optimization

The optimization model was formulated to find a safe and minimum cost seepage design related to a concrete gravity dam that impounds significant amount

of water. The main optimization components are summarized as follows:

4.3.1. Objective function

The objective function has represented the subject of finding the optimal design of dam. The objective function is considered a minimization function because it is a construction cost for a safe hydraulic structure represented by cost of cutoffs, excavations.

The objective function is defined as:

$$\begin{aligned} \text{Minimize: } f(x) &= c_1 v_1 + c_2 v_2 & (4.10) \\ f(x) &= c_1 * d_1 * t1 * 1 + c_2 * d_2 * t2 * 1 \end{aligned}$$

Where: $f(x)$ is the objective function, which represents the construction cost of seepage control elements of the concrete gravity dam. The objective function incorporates the decision variables and design parameters, Where v_1, v_2 are the volume of U/S and D/S cutoffs, respectively, where $t1, t2$ are the thickness of U/S and D/S cutoffs (assumed 0.5m), respectively.

c_1, c_2 are costs coefficients related to construction and excavation of U/S and D/S cutoffs per unit volume (m^3), respectively, which can be expressed by equation as a function of the cutoff depth which discussed in the next chapter.

Genetic algorithms are basically unconstrained optimization techniques. If used to solve constrained problems like the seepage problem in GA, the constrained problem has to be converted to an unconstrained one. A penalty method is usually used for this purpose, which includes constraints in the objective function via a penalty cost, leading to the following form of penalized objective function:

$$f(x) = (c_1 v_1 + c_2 v_2) + g * (FS_{uplift} + FS_{piping}) \quad (4.11)$$

Where:

g : The penalty rate,

FS_{uplift} : The factor of safety against uplift pressure,

FS_{piping} : The factor of safety against piping.

4.3.2 Constraints

Constraints are physical conditions or design requirements, and the optimal design must satisfy these conditions. The design of a concrete gravity dam has many requirements and conditions that are formulated as constraints in an optimization model, as shown by:(Al-Suhaili, 2014)

The standard stabilization requirements of hydraulic structures against uplift pressure was provided by the factor of safety against uplift force:

$$FS_{uplift} = \frac{\gamma_c V}{\text{uplift force}} \geq 2 \quad (4.12)$$

Where:

F.O.S is the factor of safety against uplift pressure,

V: volume of concrete of the superstructure, (L^3)

γ_c : concrete weight density equal to 2.4, (F/L^3)

Uplift force is calculated along the base of structure.

The other constraint is the factor of safety against piping:

$$FS_{piping} = \frac{i_{cr}}{i} \geq 3 \quad (4.13)$$

Where:

i_{cr} is the critical exit gradient $\cong 1$,

i is the computed exit gradient at the downstream side of the structure.

The Figures (4.4), (4.5) and (4.6) show the distribution of up lift force in different locations for cutoffs which result from the effect of seepage water under dam as illustrated in below equation.

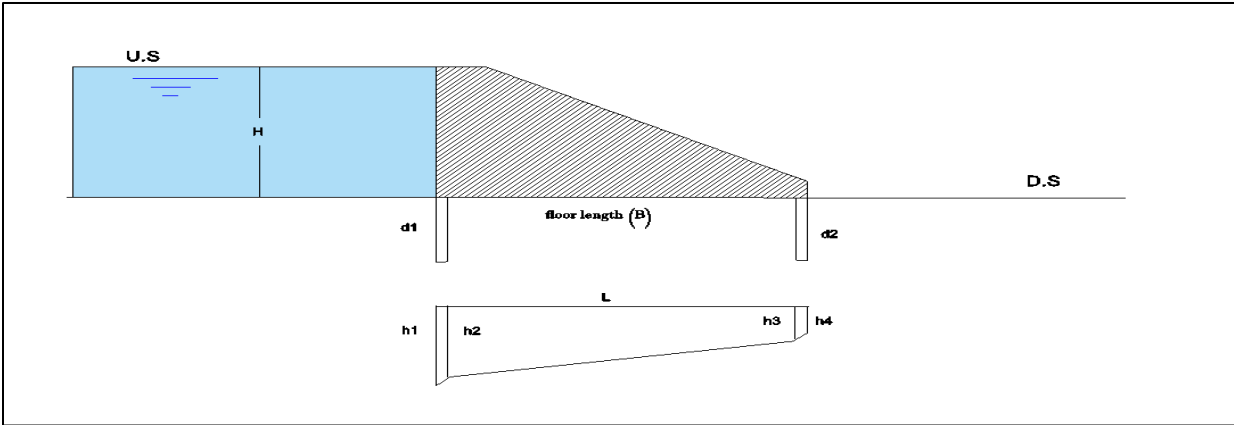


Figure (4-4): Uplift pressure distribution

According to Figure (4-4), uplift pressure is calculated as follows:

$$F_{\text{uplift}} = F1 + F2 + F3, \quad \gamma_w = 9.81 (\text{KN}/\text{m}^3), \quad t_{\text{cutoff}} = 0.5\text{m} \quad (4.14)$$

$$F1 = \frac{h1 + h2}{2} \times 0.5 \times 1 \times \gamma_w$$

$$F2 = \frac{h2 + h3}{2} \times L \times 1 \times \gamma_w$$

$$F3 = \frac{h3 + h4}{2} \times 0.5 \times 1 \times \gamma_w$$

*If there is a water on the downstream surface floor, $h4$ is not equal to zero.

*If there is no water on the downstream surface floor, $h4$ is equal to zero.

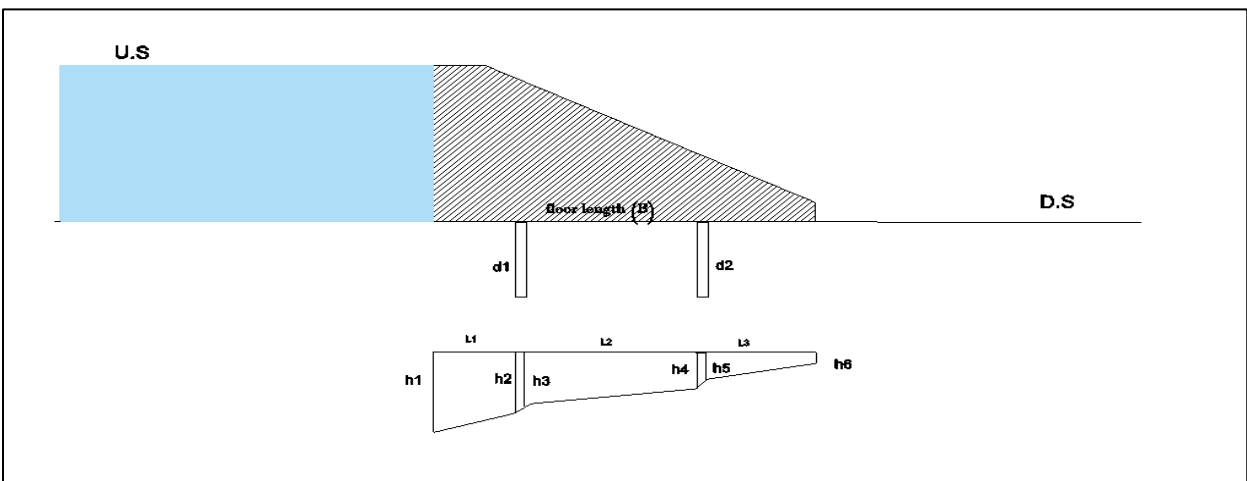


Figure (4-5): Uplift pressure distribution

$$F_{\text{uplift}} = F1 + F2 + F3 + F4 + F5 \quad (4.15)$$

$$F1 = \frac{h1 + h2}{2} \times L1 \times 1 \times \gamma_w$$

$$F2 = \frac{h2 + h3}{2} \times 0.5 \times 1 \times \gamma_w$$

$$F3 = \frac{h3 + h4}{2} \times L2 \times 1 \times \gamma_w$$

$$F4 = \frac{h4 + h5}{2} \times 0.5 \times 1 \times \gamma_w$$

$$F5 = \frac{h5 + h6}{2} \times L3 \times 1 \times \gamma_w$$

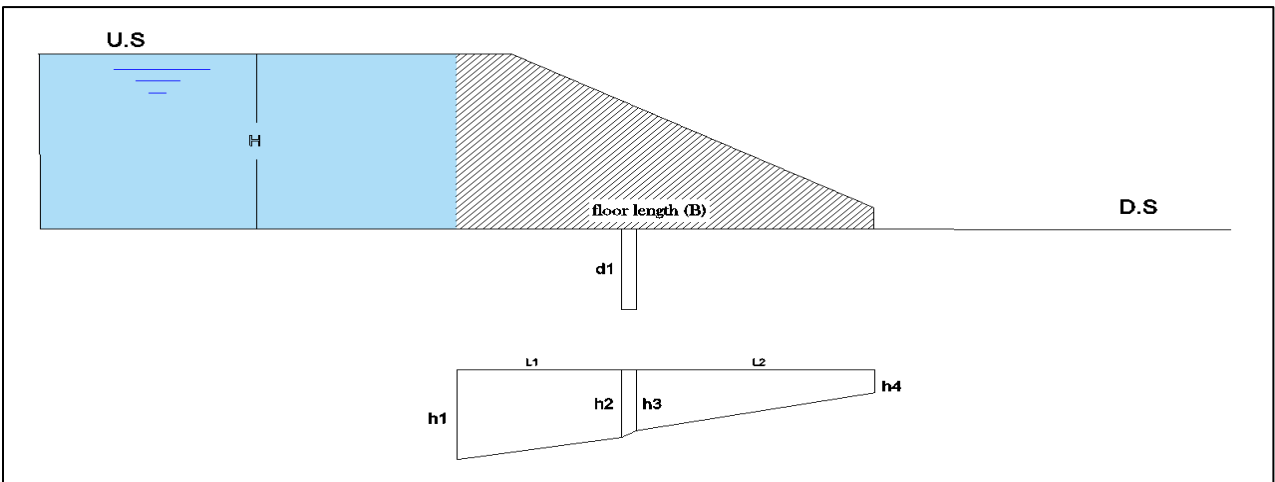


Figure (4-6): Uplift pressure distribution

$$F_{\text{uplift}} = F1 + F2 + F3 \quad (4.16)$$

$$F1 = \frac{h1 + h2}{2} \times L1 \times 1 \times \gamma_w$$

$$F2 = \frac{h2 + h3}{2} \times 0.5 \times 1 \times \gamma_w$$

$$F3 = \frac{h3 + h4}{2} \times L2 \times 1 \times \gamma_w$$

4.3.3 Formulation of the GA-FDP model

In order to solve the formulation problem presented in Section 4.2, a new approach is proposed in this study. It is a combination of Genetic Algorithm and Finite Difference programming called GA-FDP. The following steps are suggested in the proposed GA search for the safe hydraulic structure design:

- 1. Encoding the design variables:** the genetic algorithm requires that any trial solution of the design problem should be represented by a coded string of finite length, similar to the structure of the chromosome of a genetic code. Each chromosome from a population signifies one design, the depth and location of two cutoffs coded as genes. The length of a chromosome is equal to four gene, the depth and location for the first cutoff and depth and location for the second cutoff; any gene in a chromosome represents a depth and location for cutoff coded with integer coding. A selection of cutoff depth were considered and represented as integer coding, as shown in Table 4-1.

Table(4-1): Integer coding

integer coding	depth, distance (m)
1	0.5
2	1
3	1.5
4	2
.	.
.	.
.	.
N	d1,x1,d2,x2

- 2. Generation of initial population:** this step generates an initial random population of chromosomes, which represent trial solutions to hydraulic structure design problem. Hence, population sizes equal to 50 are used in the GA-FDP model.

3. Decoding the population: the population of the strings is decoded using the mapping defined in step 1, to produce a population of trial solutions to the corresponding seepage problem.

4. Calling sub-routine FD algorithm: for every chromosome located within the parent pool, the FD algorithm is used to determine the head at any point in the given domain, uplift force, exit gradient, quantity of seepage and the structure cost of each generated chromosome as follows:

a. Computation of hydraulic characteristics for the dam: this step calculates, the total head at any node in the domain of study, uplift force, exit gradient and quantity of seepage. All these characteristics are calculated using steady flow analysis.

- In this step, the **initial** values for the counter and the error of head (the difference) between two successive attempts are defined as $k=0$ and $error=1$, respectively.
- The head at each node had calculated by applying Eq. (4.5).
- This algorithm was run for a number of iterations and was stopped when the error (the summation of differences between head in two successive attempts for all nodes) less than the maximum error ($e_0=1 \times 10^{-9}$).
- depending on the head that was calculated in the last iteration, the total head, the up lift force under dam, the quantity of seepage and the exit gradient were found.

b. Computation of penalty costs: the GA assigns a penalty cost to each chromosome if a suggested solution does not satisfy one or more of the constraints.

c. Computation of total costs: the total cost is computed by the sum of the cutoff construction and penalty costs.

5. Computation of fitness: this is the fitness for each chromosome in the population. This step computes the fitness function, described as the inverse of the total cost.

6. Generation of a new population: this creates a new population by repeating the following steps until the new population is complete:

6.1. Selection: two parent chromosomes are selected from a population according to their fitness (the better the fitness, the bigger the chance of being selected). This work used Roulette Wheel Selection (RWS) method.

6.2. Crossover: regarding crossover probability, parent's crossover to form new offspring (children). If no crossover is performed, the offspring are an exact copy of the parents. Hence, crossover one method was considered where $P_c = 0.95$.

6.3. Mutation: a random mutation with some specified probability of mutation, P_m , is carried out for each of the strings that have undergone crossover. In an integer coded GA, a random mutation operator with $P_m = 0.05$ is employed by replace the depth of cutoff1 instead of the depth of cutoff2 in a chromosome.

7. Production of successive generations: the three operators described above, produce a new generation of hydraulic structure trial designs.

8. Convergence of the basic GA-FDP model: steps 3–7 are repeated until the convergence criteria for the basic GA search, set by the user, are met. Hence, the basic GA search is considered to be converged if the best solution from the search is not improved by number of generations.

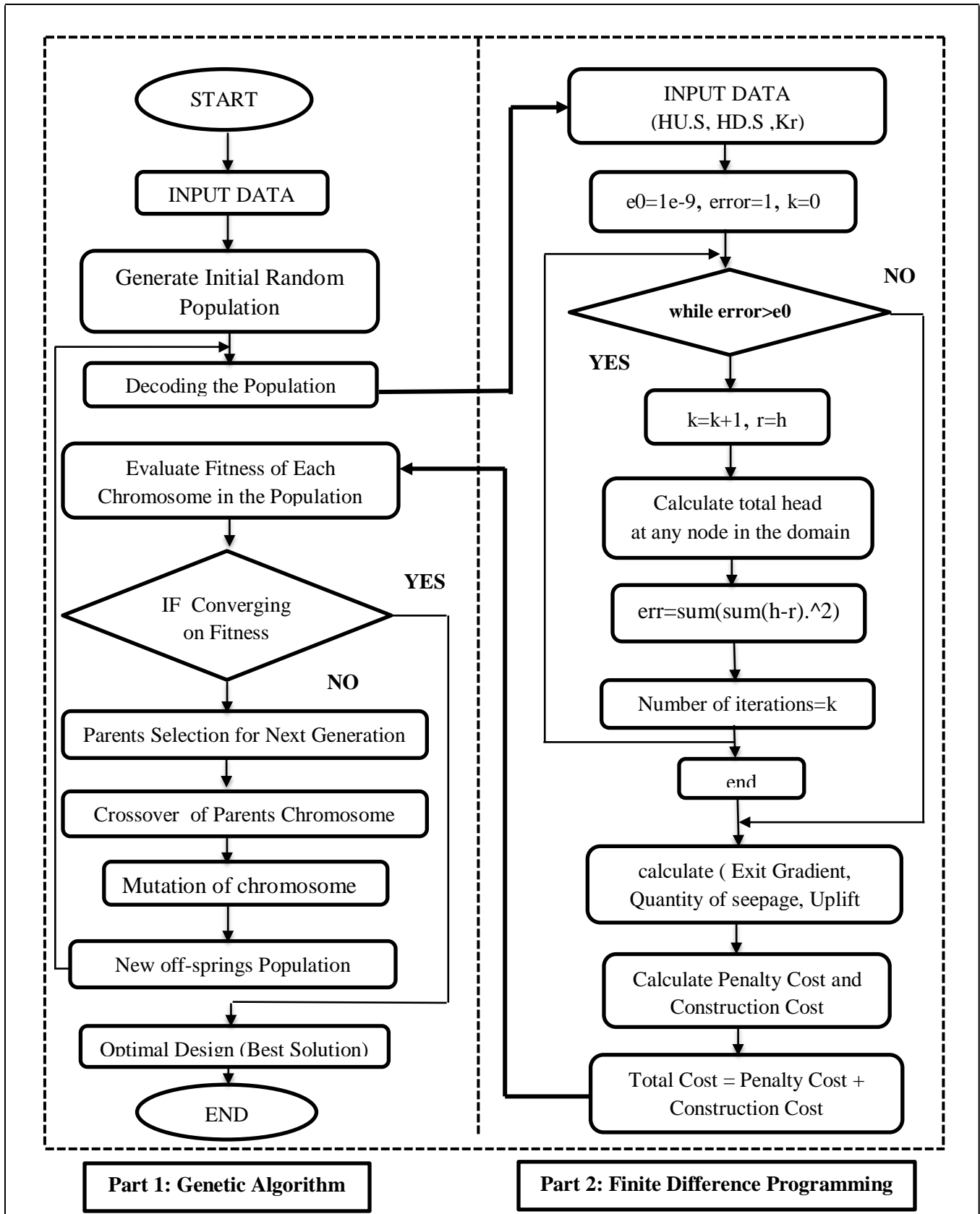


Figure (4-7): Flow chart of the proposed GA-FDP model.

Chapter Five

Analysis and Discussion of Results

5.1 Introduction

The results in this chapter were analyzed and discussed at three different parts. The first part includes the validation of the numerical simulation using the hand calculation for example in soil mechanics (Lambe and Whitman, 1991) with numerical simulation results of the GeoStudio (2018 R2) then comparing the numerical simulation results of finite difference code in Matlab (R2020a) with the results of finite difference code. The second part includes numerical simulation results by finite difference code at different locations and depths for cutoff with different differential head and anisotropic degree (K_r). The selected variables for numerical simulation were (differential head, depth of cutoff1, depth of cutoff2, location of cutoff1&2 on the floor of the dam and anisotropic degree). The third part presents the results of optimum position and depth of cutoff that give minimum cost, and min. exit gradient and uplift pressure by optimization using GA_FDP program that includes the safety of the structure by using different locations and depths for cutoff with different differential head.

5.2 Model Validation

As the problem of seepage under hydraulic structures complex and difficult to measure, it is primarily studied in numerical work. This validation were studied in two parts:

5.2.1 Verification of SEEP/W Numerical Software

This part include comparison the results of GeoStudio 2018 R2 SEEP/W with an example in soil mechanics (Lambe and Whitman, 1991) of two-dimensional flow under a concrete dam with homogenous and isotropic soil was

taken as shown in Figure (5-1) by comparing the seepage rate value, exit gradient and pressure heads at points A to H obtained by hand calculation with that obtained from the numerical program to establish confidence.

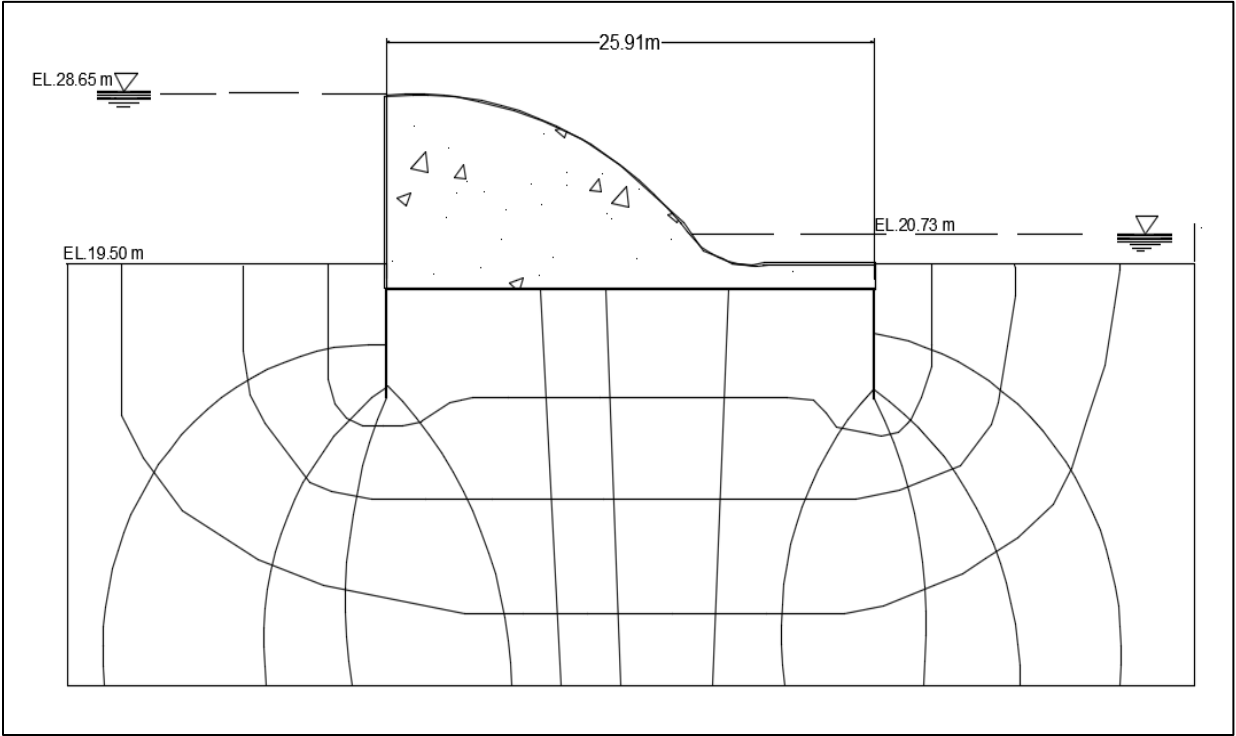


Figure (5-1): development the flow net under concrete dam (Lambe and Whitman, 1991).

5.2.1.1 Hand calculation

$$K = (0.1 \text{ ft/min}) * (0.3048/60) = 5.08 * 10^{-4} \text{ m/sec}$$

Number of flow channels $N_f = 4$ and equipotential lines $N_d = 12.6$

$$\frac{Q}{L} = K H \frac{N_f}{N_d} = 5.08 * 10^{-4} * (28.65 - 20.73) * \frac{4}{12.6} = 1.277 * 10^{-3} \text{ m}^3/\text{sec}/\text{m}.$$

Exit gradient = 0.19

Figure (5-2) shows the results of Pressure head at points A to H.

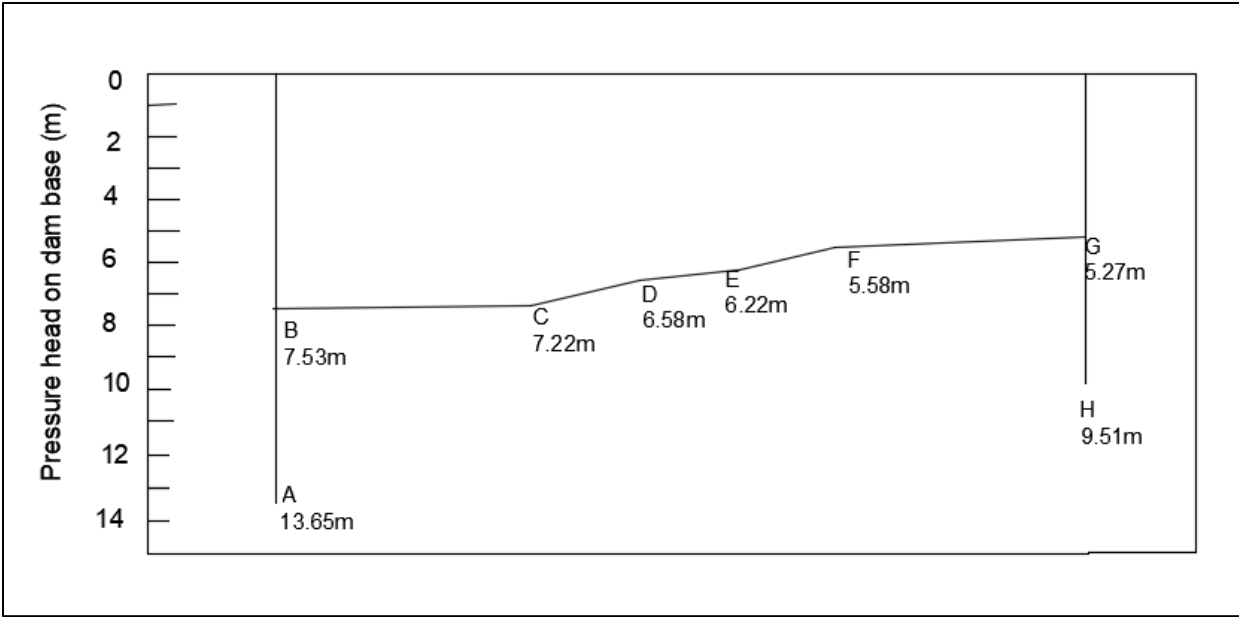
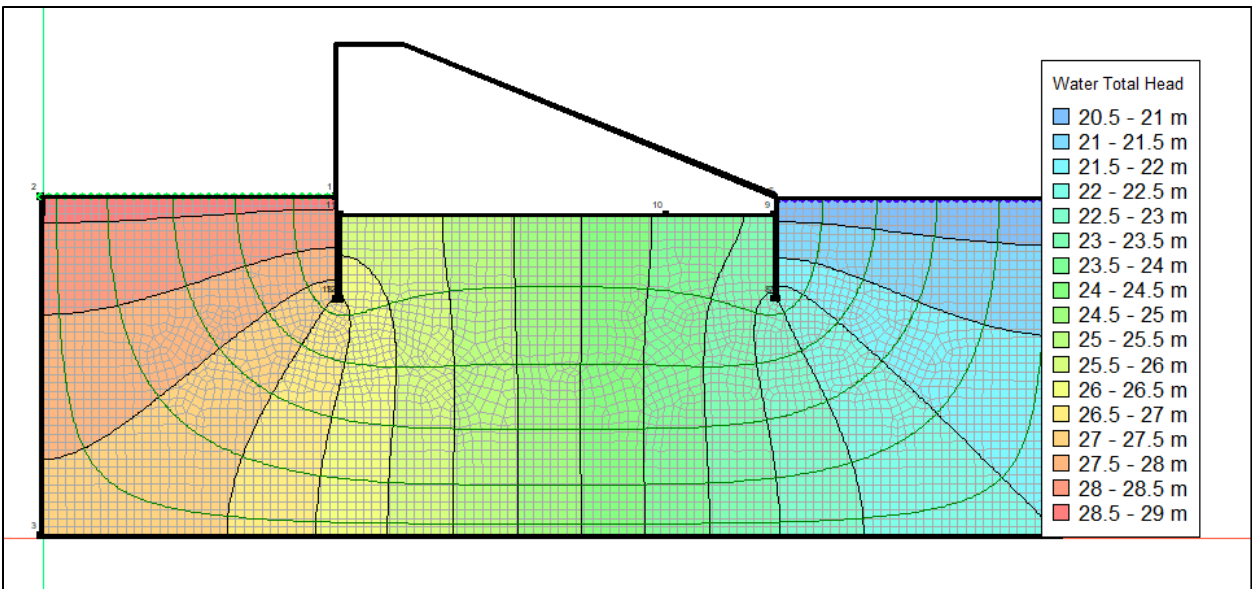


Figure (5-2): The pressure head on dam base.

5.2.1.2 SEEP/W model

To test the effectiveness of the numerical model, the numerical model as shown in figure (5-3) was simulated in similar conditions to the above example. The boundary conditions were applied to the numerical model.



Figure(5-3): The flow net under concrete dam (SEEP/W) model.

The result from the numerical model was $q = 1.254 \times 10^{-3} \text{ m}^3/\text{sec}/\text{m}$ with exit gradient $= 0.203$, Figure (5-4) presents the behavior of pressure head on dam base when using two cutoff. The results showed a good agreement with hand calculation for this example in soil mechanics (Manual, 2012).

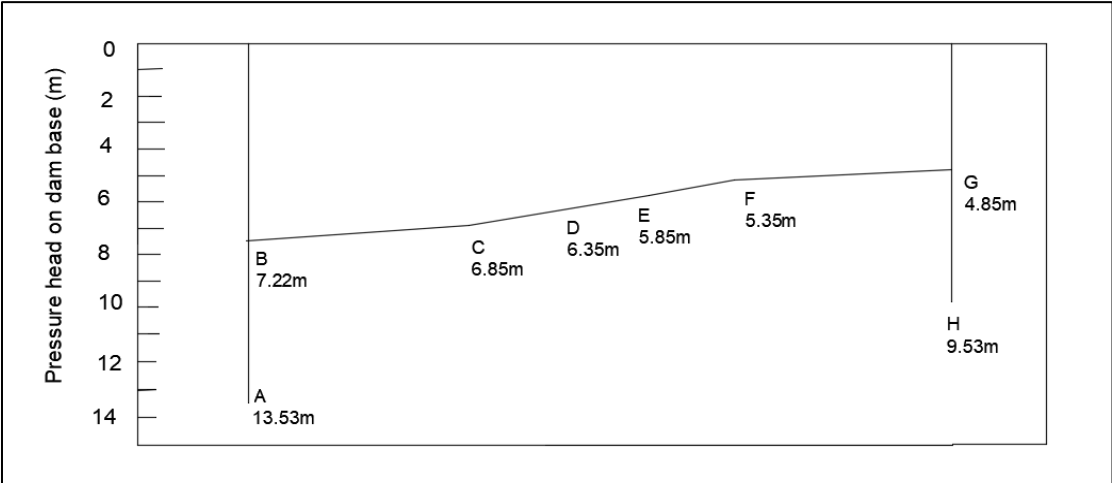
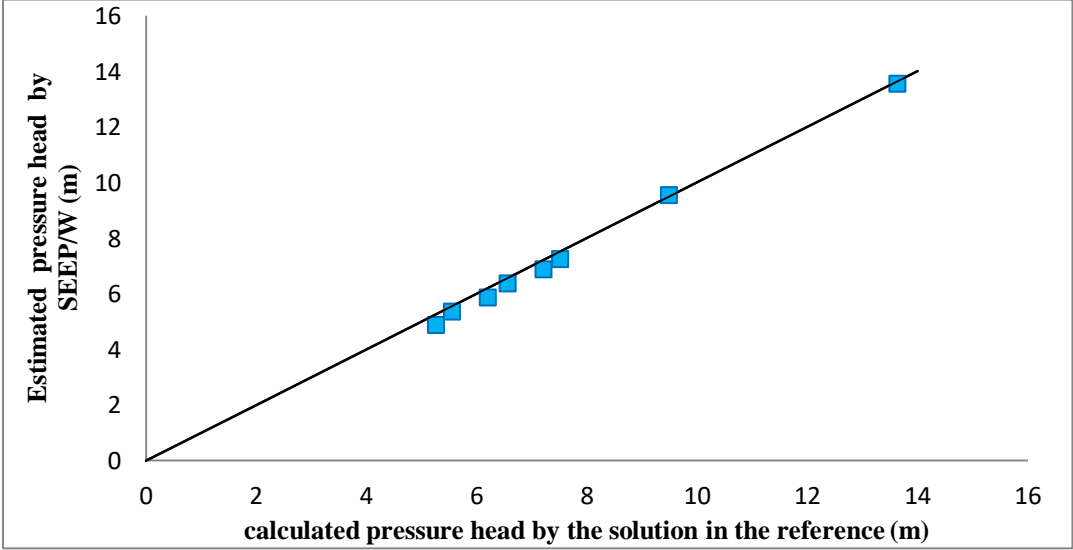


Figure (5-4): The pressure head on dam base (SEEP/W) model.

The below scatterplot of estimated values in GeoStudio software versus calculated values by solution in (Lambe and Whitman, 1991) is used to investigate the degree of similarity between the models, these models give a high value which indicating very little discrepancy between the pressure head of the two models..



Figure(5-5): The scatterplot of pressure head in SEEP/W versus the solution in(Lambe and Whitman, 1991).

5.2.2 Verification of the numerical simulation finite difference code

This part includes the comparison of the results of GeoStudio 2018 R2 SEEP/W with the numerical simulation results of finite difference code in Matlab (R2020a). This numerical model was simulated in similar conditions to the GeoStudio model. An example of flow under a concrete dam was taken in some cases which depending no water in downstream (critical condition) to verify the finite difference code by comparing the total water head at specific position (such as at 1/3 from the dam's floor), exit gradient value obtained by the computer program, Geo Studio, with that obtained from the finite difference code to establish confidence.

Case1: Without cutoff ($H=10\text{m}$, $D=30\text{m}$, $B=15\text{m}$, $Kr=1$). where H is the water head at upstream; D is the depth of domain; B is the length of floor. Figure (5-6) shows the flow net of finite difference code, while Figure (5-7) shows the results in GeoStudio software.

Case2: with one cutoff ($H=10\text{m}$, $D=30\text{m}$, $B=15\text{m}$, $X1=10.5\text{m}$, $d1/B=0.2$, $Kr=1$), where $X1$ the location of cutoff1 on the floor; $d1$ the depth of cutoff1. Figures(5-8),(5-9) show the flow net when using one cut off for finite difference code and GeoStudio software.

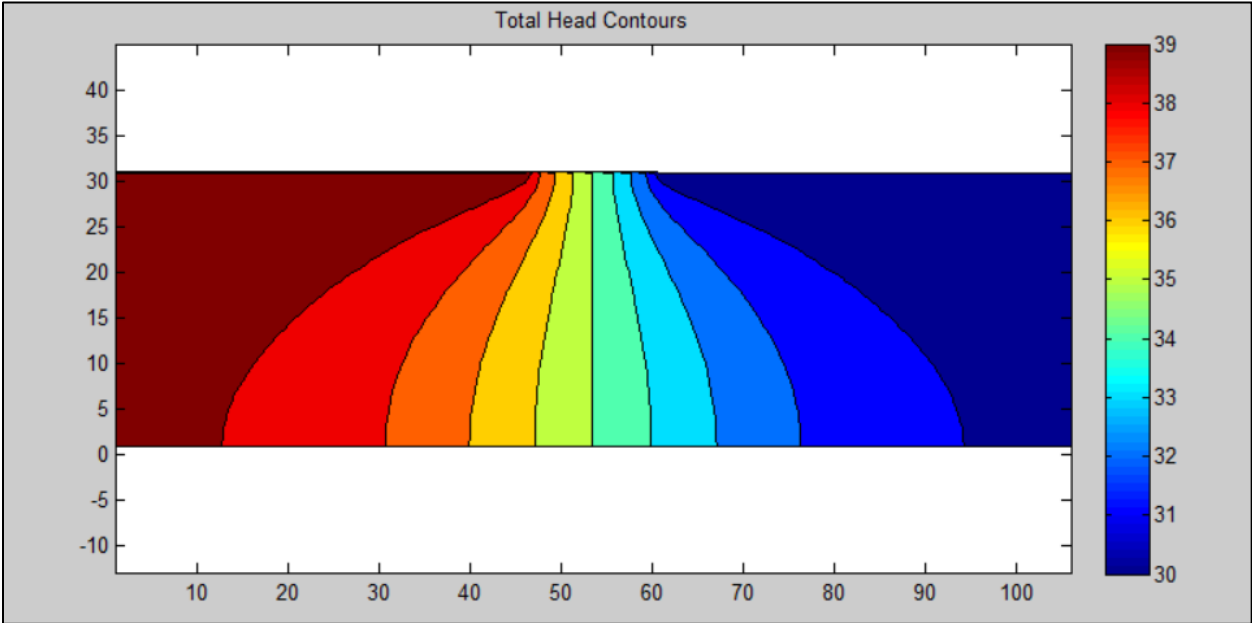
Case3: with two cutoffs ($H=10\text{m}$, $D=30\text{m}$, $B=15\text{m}$, $x1=0$, $x2=15\text{ m}$, $d1/B=0.2$, $d2/B= 0.2$), Figure (5-10),(5-11) show the flow net when using two cutoffs for finite difference code and GeoStudio software.

Case4: when using one cutoff at an isotropic degree ($K_r=2$) with one cutoff ($H=10\text{m}, d_1=3\text{m}, x_1=0, B=15\text{m}$). Figures (5-12),(5-13) shows the results in GeoStudio software and Matlab code.

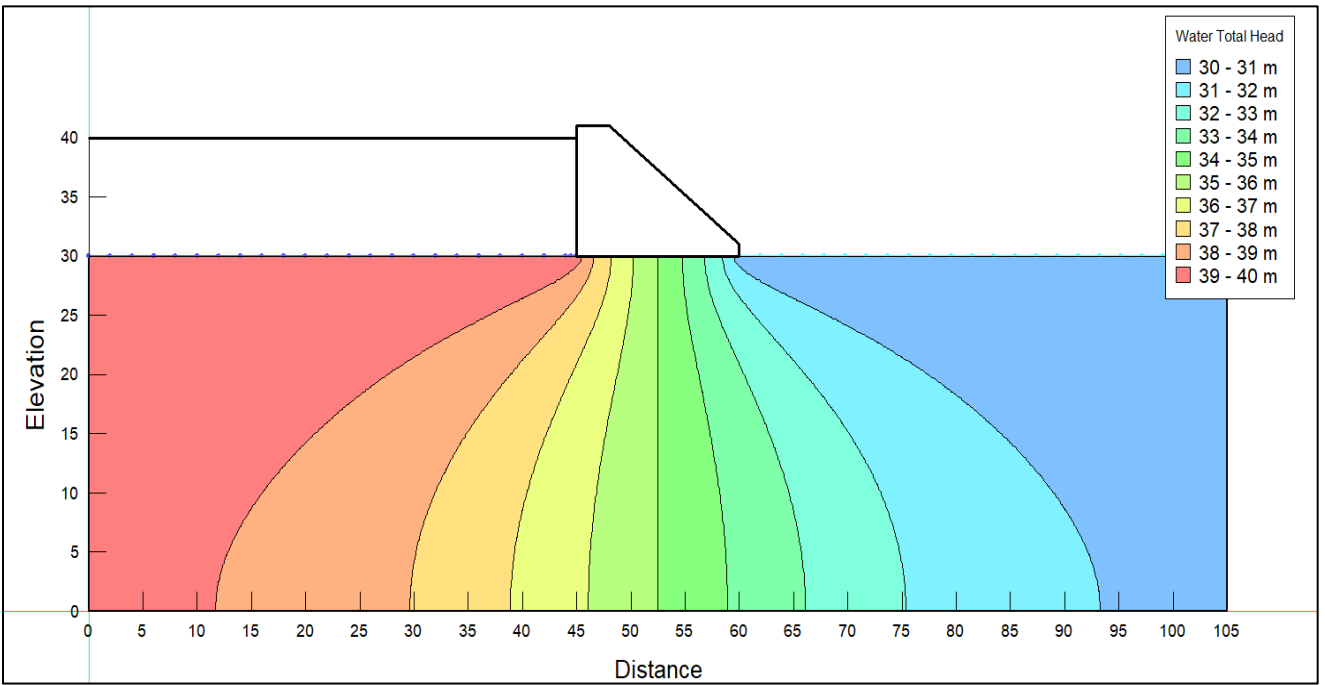
Case5: when use one cutoff at anisotropic degree ($K_r=4$) with the same dimensions of the previous case. Figures (5-14), (5-15) shows the results in GeoStudio software and in Matlab code.

Case6: when use anisotropic degree ($K_r=8$) with the same dimensions of the previous case. Figures (5-16),(5-17) shows the results in GeoStudio software and in Matlab code. All results of the this cases are summarized in Table (5-1).

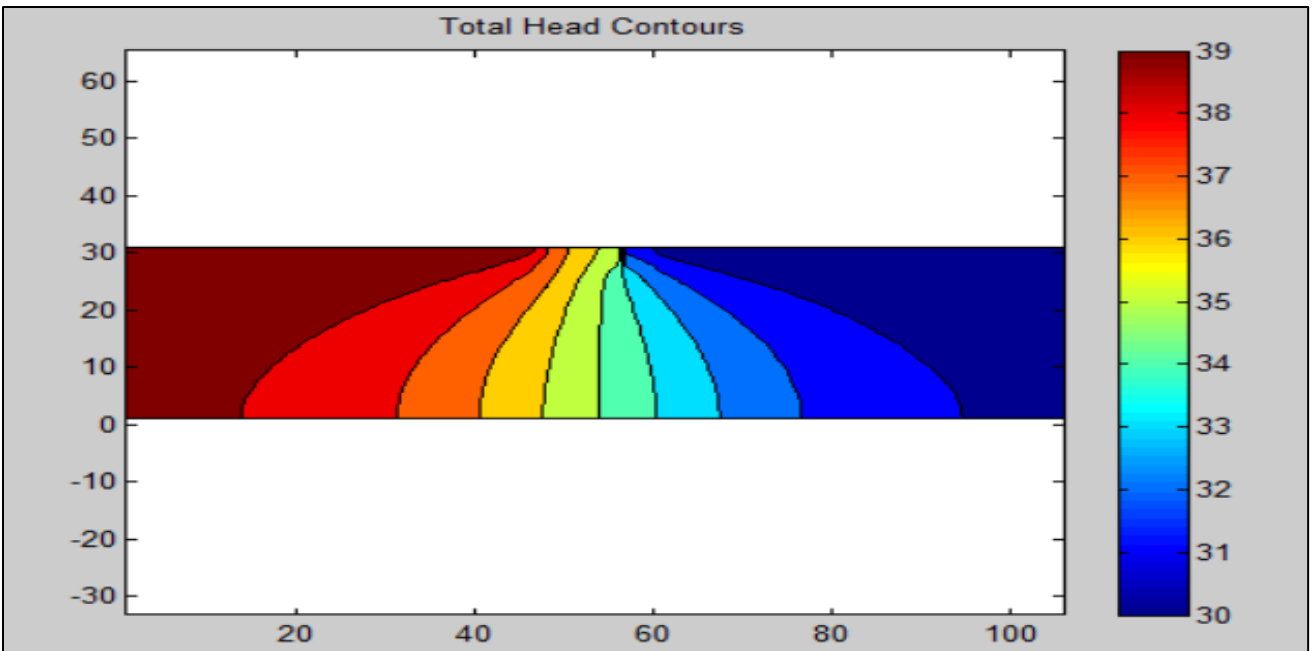
From all below figures, the results present good fitness between equipotential lines in Matlab and GeoStudio with a little variance between them.



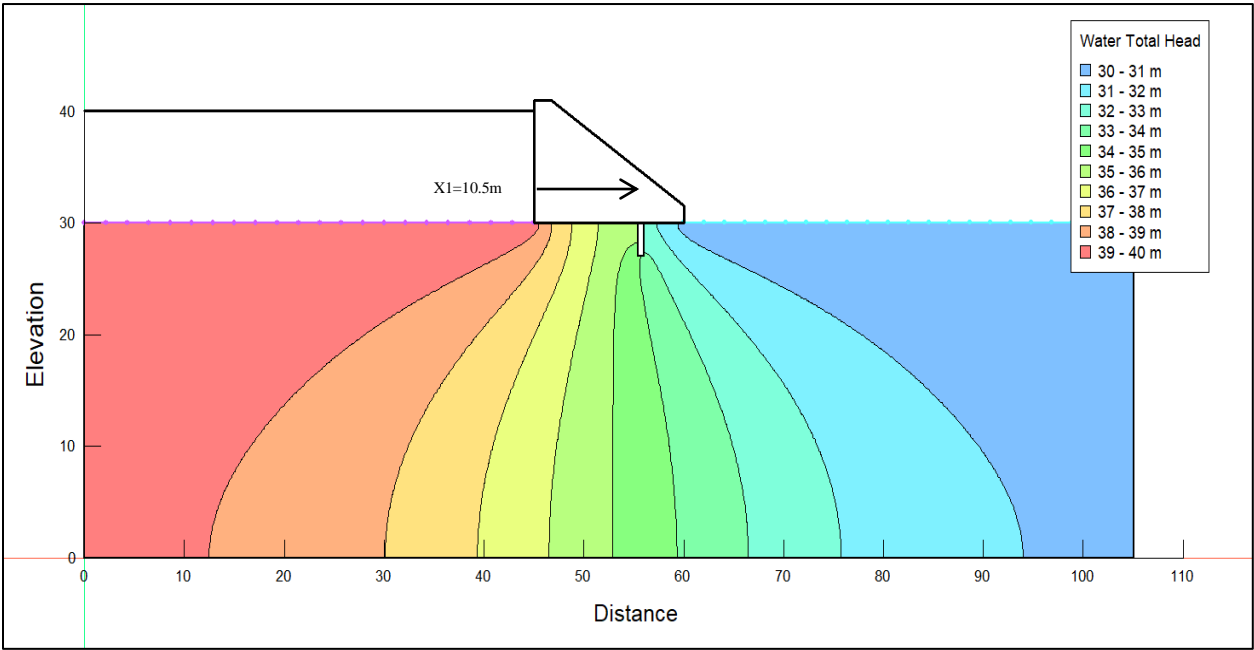
Figure(5-6): The equipotential lines under dam for case1 by Matlab (R2020a).



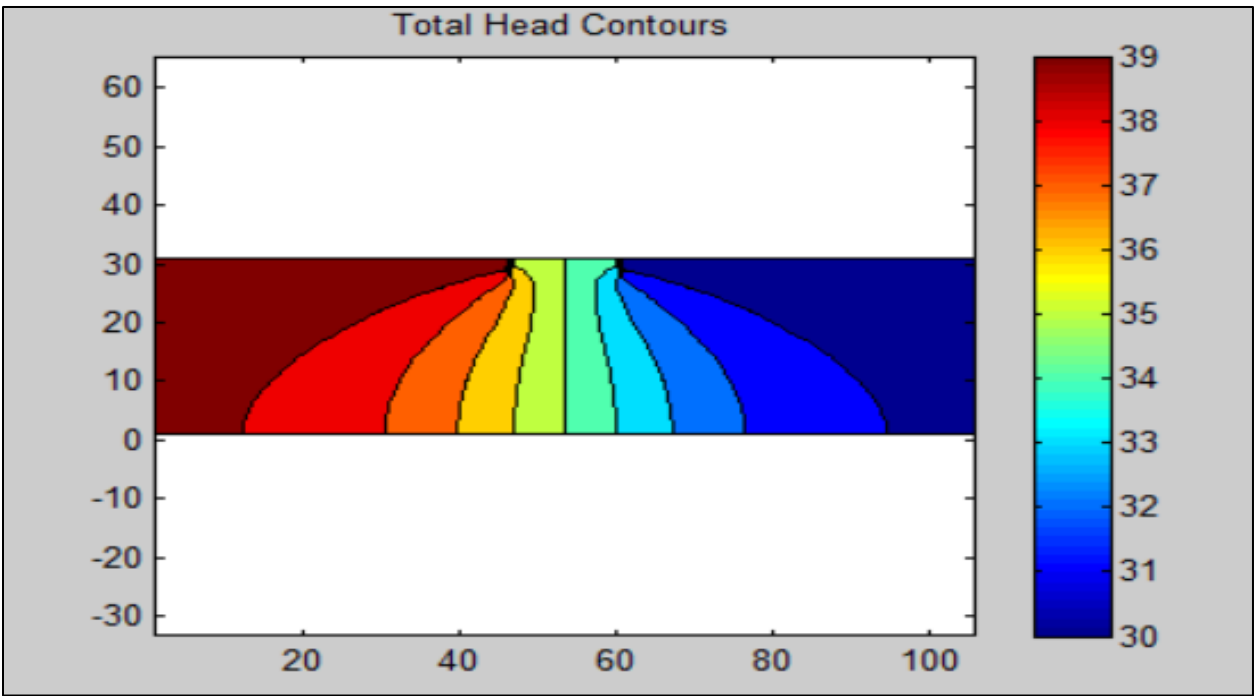
Figure(5-7): The equipotential lines under dam for case 1 by GeoStudio 2018R2.



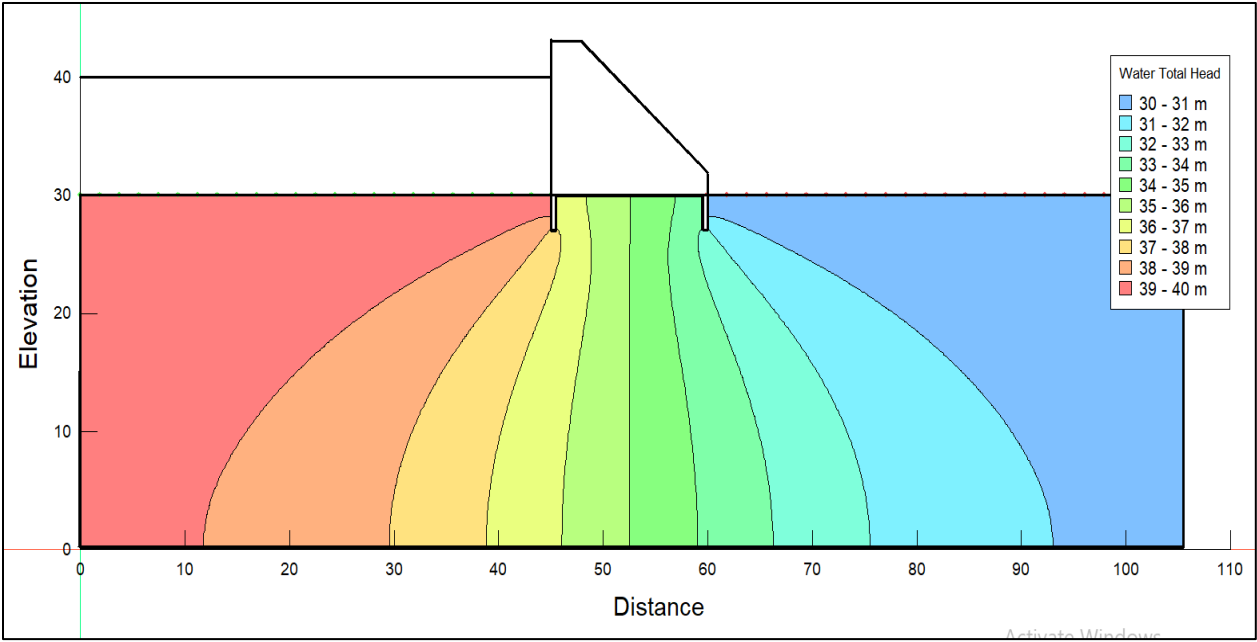
Figure(5-8): The equipotential lines under dam for case2 by Matlab.



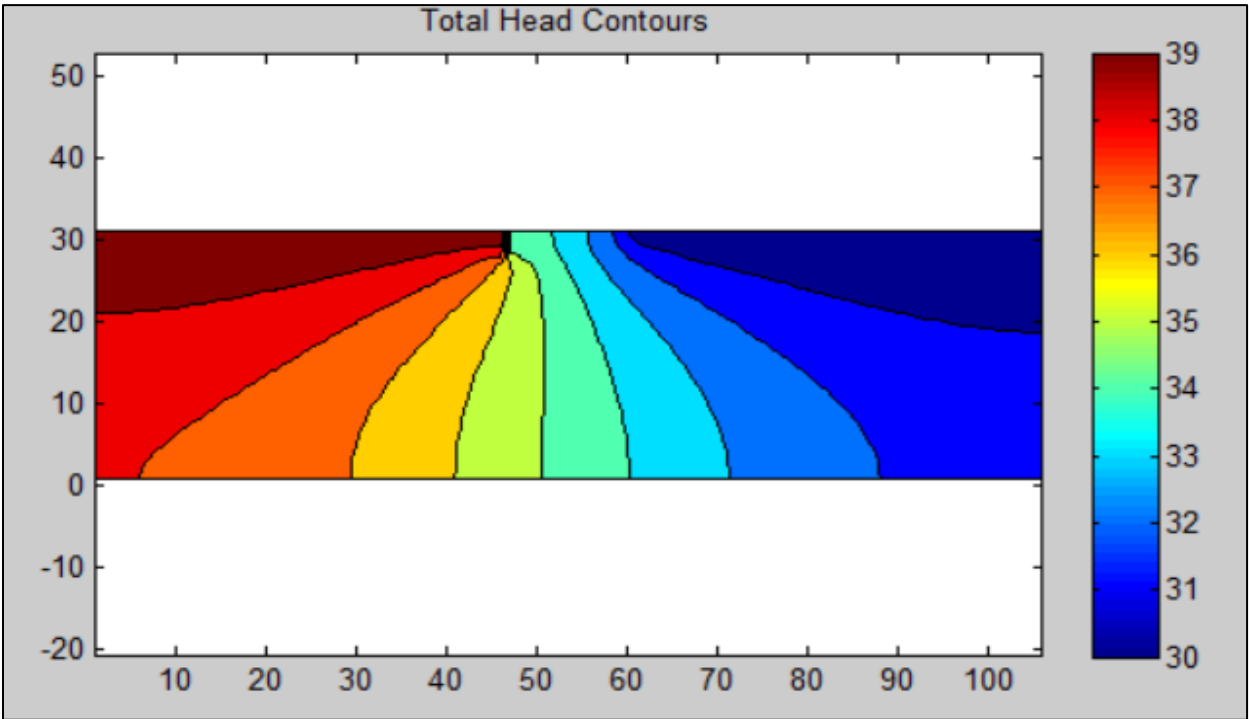
Figure(5-9): The equipotential lines under dam for case2 by GeoStudio.



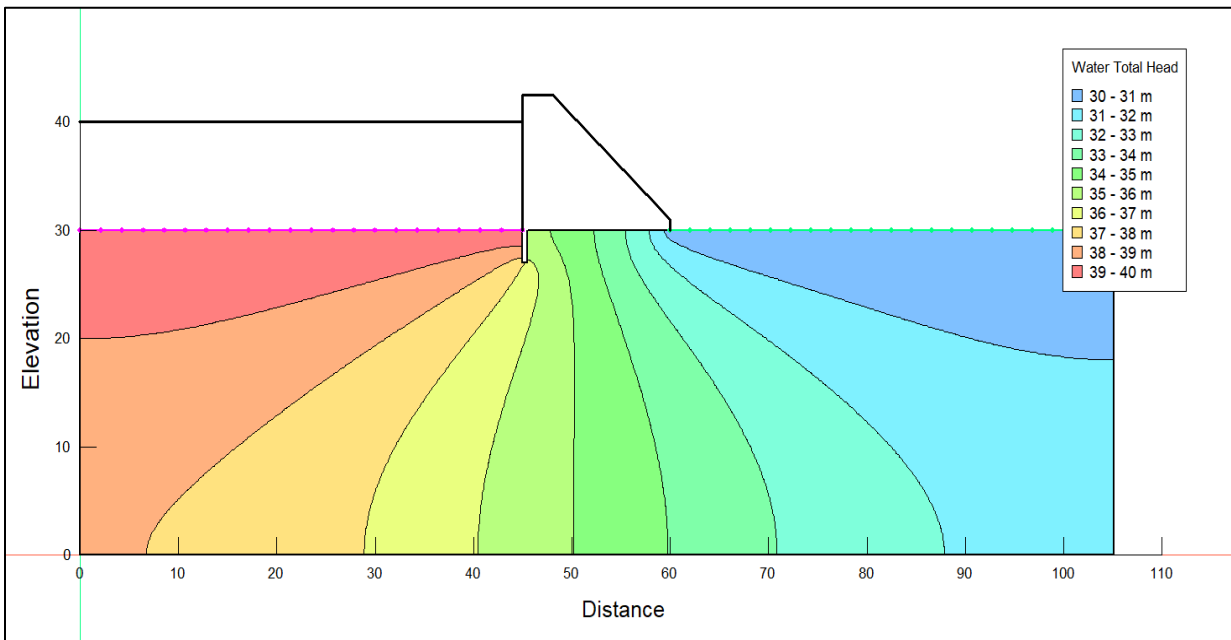
Figure(5-10): The equipotential lines under dam for case2 by Matlab.



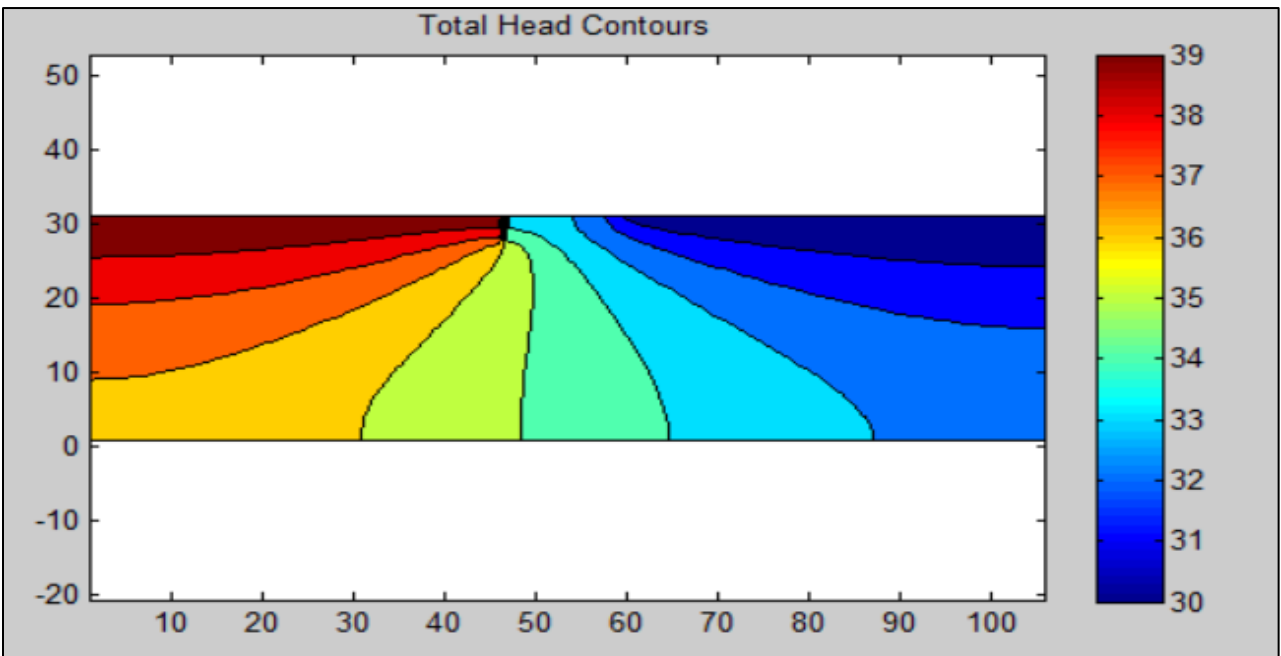
Figure(5-11): The equipotential lines under dam for case3 by GeoStudio.



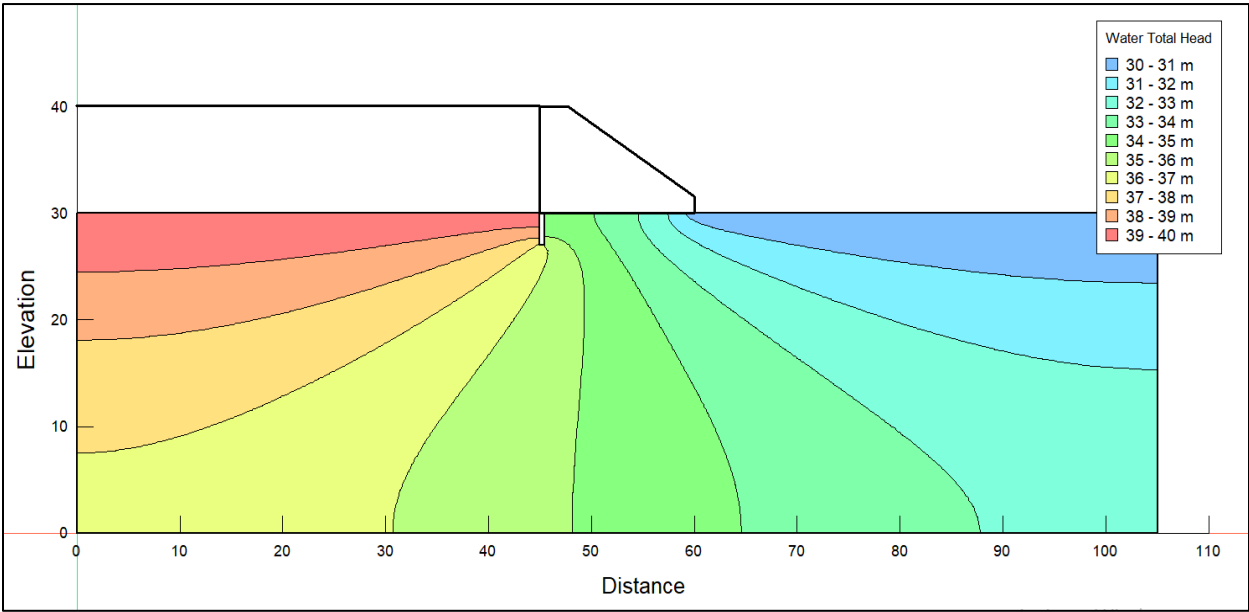
Figure(5-12): The equipotential lines under dam for case4 by Matlab.



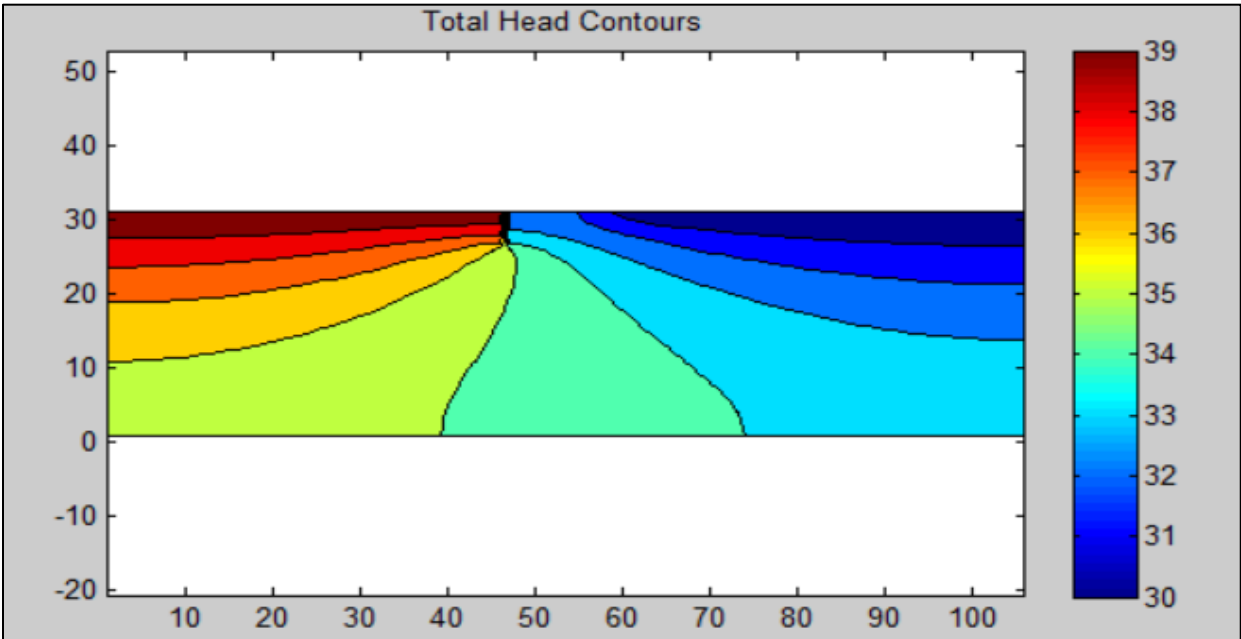
Figure(5-13): The equipotential lines under dam for case4 by GeoStudio software.



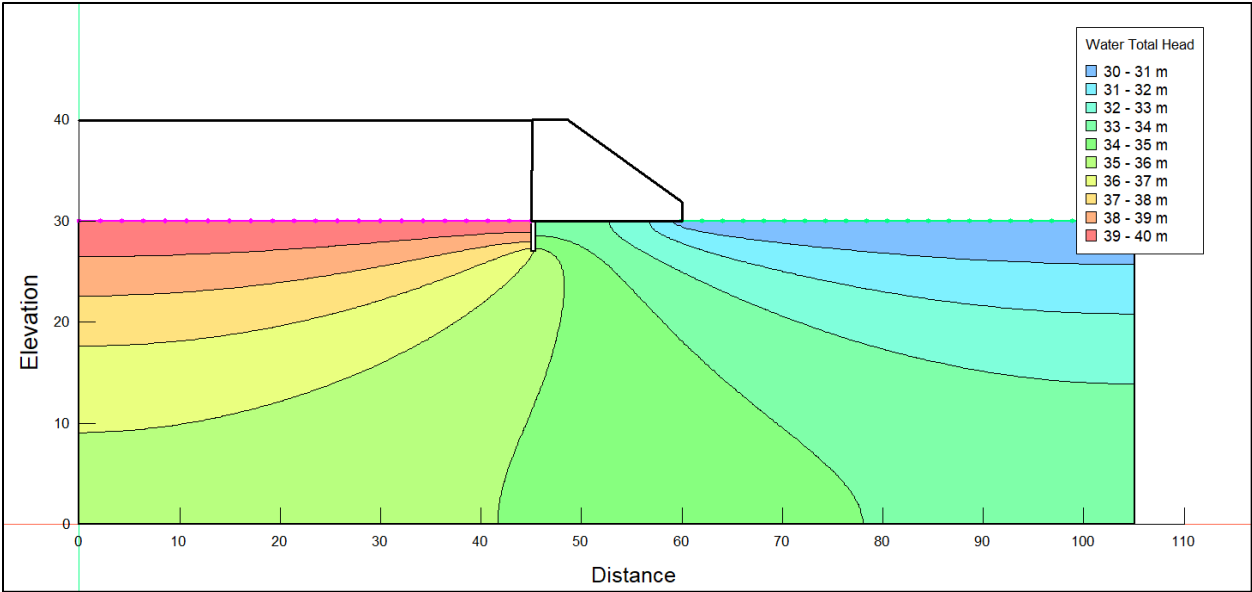
Figure(5-14): The equipotential lines under dam for case5 by Matlab.



Figure(5-15): The equipotential lines under dam for case5 by GeoStudio software.



Figure(5-16): The equipotential lines under dam for case6 by Matlab



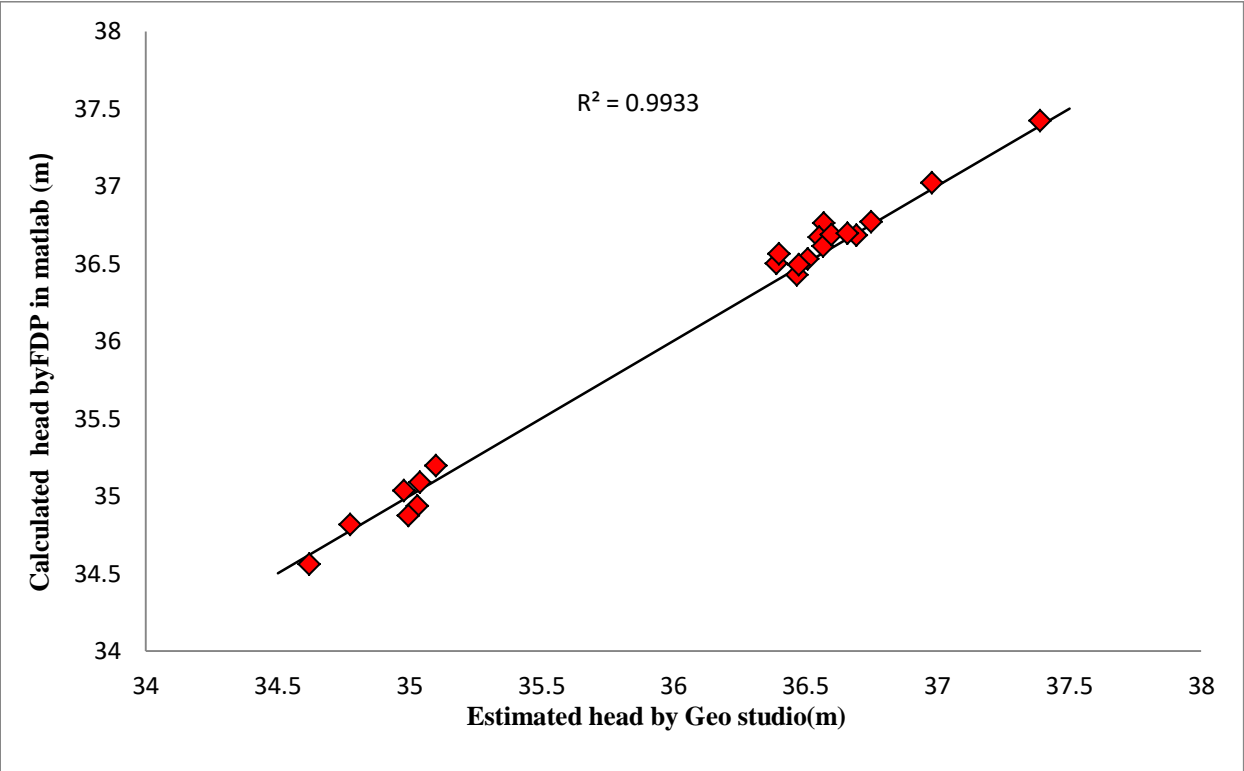
Figure(5-17): The equipotential lines under dam for case6 by GeoStudio software

Table(5-1):The values of exit gradient, total water head by GeoStudio and Matlab

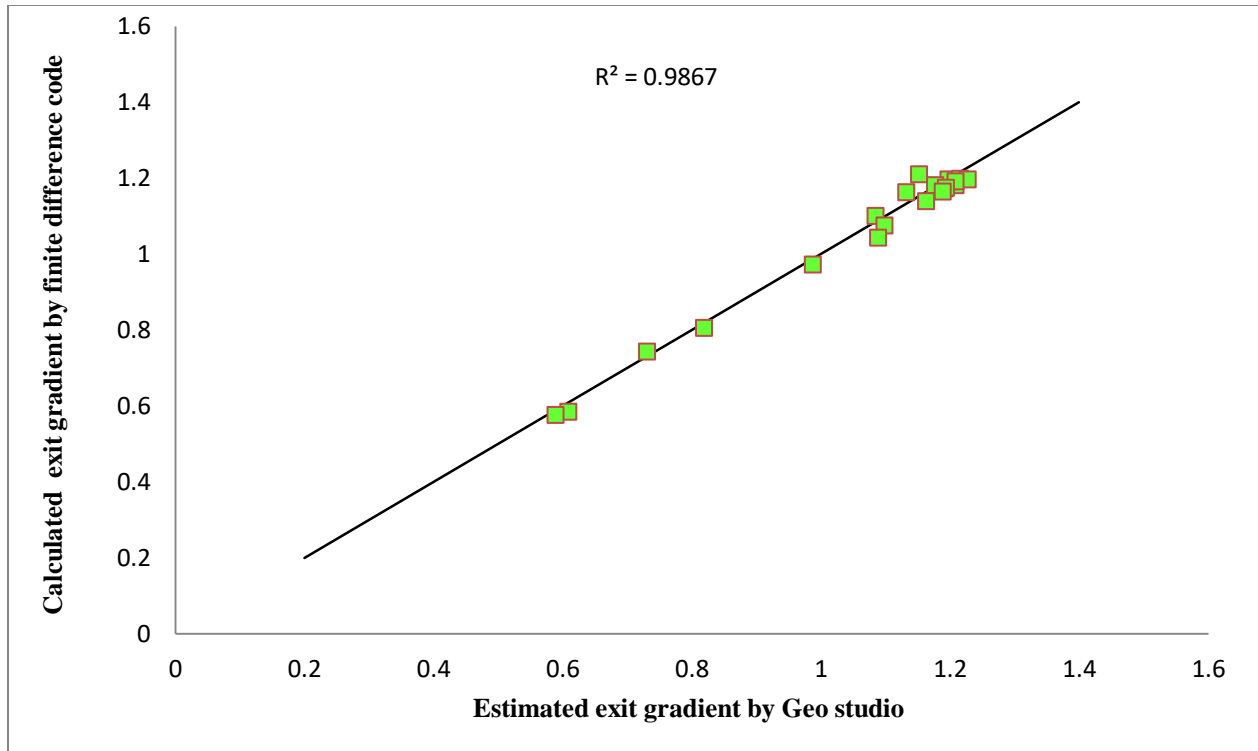
Case	Model	Total water head at (1/3)B	Exit Gradient
case 1	GeoStudio	36.107	1.312
	Matlab	36.1196	1.3108
case2	GeoStudio	36.5105	1.1003
	Matlab	36.5977	1.0731
case3	GeoStudio	35.6014	0.5325
	Matlab	36.5578	0.4787
case 4	GeoStudio	33.32	1.02
	Matlab	33.21	1.1
case5	GeoStudio	32.65	1.04
	Matlab	32.52	1.13
case6	GeoStudio	32.04	0.98
	Matlab	31.91	1.11

Now, the following input variables are used as example to state the verification between them, the flow under a concrete dam with one cutoff (H=10m, D=30m, B=15m, Kr=1, $x_1/B = 0$ to 1, $d_1/B=0.2$).

The scatterplot (5-18), (5-19) of calculated values by finite difference code versus estimated values in GeoStudio is used to investigate the degree of similarity between them, the result indicates that acceptable scatter can recognize between predicted and measured exit gradient and total head.



Figure(5-18): Scattered plot of total water head by Geostudio versus that calculated by code.



Figure(5-19): Scattered plot for exit gradient by Geostudio versus that estimated by code

5.3 Influence of main parameters on exit gradient and uplift pressure

There are several parameters that can control the exit gradient and the up lift pressure. In this section, the effects of the main controlling parameters clearly defined by viewing it's performance in charts. The effect of these parameters presented on series of runs have been conducted for the foundation of the structure at different conditions to study the variation of values of the exit gradient and uplift pressure. (3474) runs were carried out by finite difference code using the input variables for the case study to identify the effect of various depths and locations for cutoffs on the exit gradient and uplift pressure: differential head (H)= 5,10,15m, floor length(B)=20m, depth of impervious layer(D=30), and isotropic ratio ($k_x/k_y = 1,2,4,8$). Six various depths ratio ($d/B=0.1-0.6$), for each depth ratio, various cutoff locations, ($X/B=0-1$) were used, all results are represented in Appendix A.

5.3.1 Influence of cutoff location ratio on exit gradient and uplift pressure

The cutoff location has a direct influence on the exit gradient and uplift pressure. To show the impact of the cut off on them, the runs were made for the region under structure with one cutoff at different location ratio for cutoff starts from upstream and ends in downstream. Figures (5-20), (5-21) show the specific exit gradient and total uplift force applied on the structure versus cutoff location ratio for $d1/B = 0.1$, $Kr=1$.

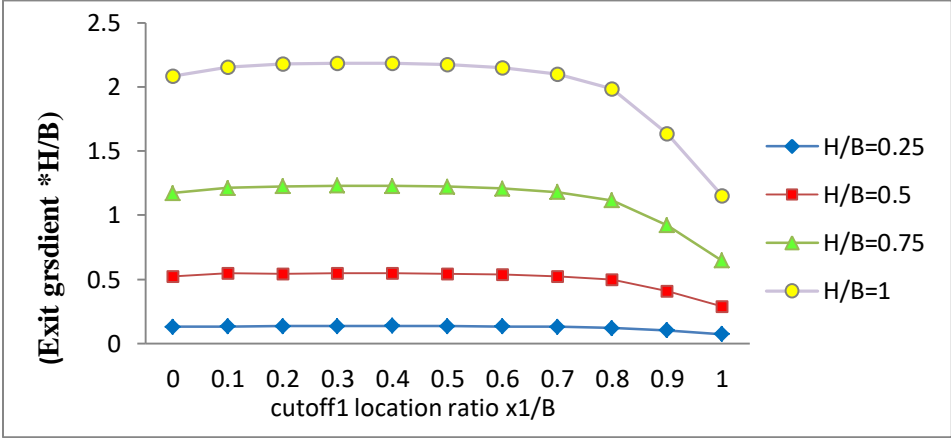


Figure (5-20): Exit gradient versus cutoff1 location ratio ($d1/B=0.1, Kr=1$).

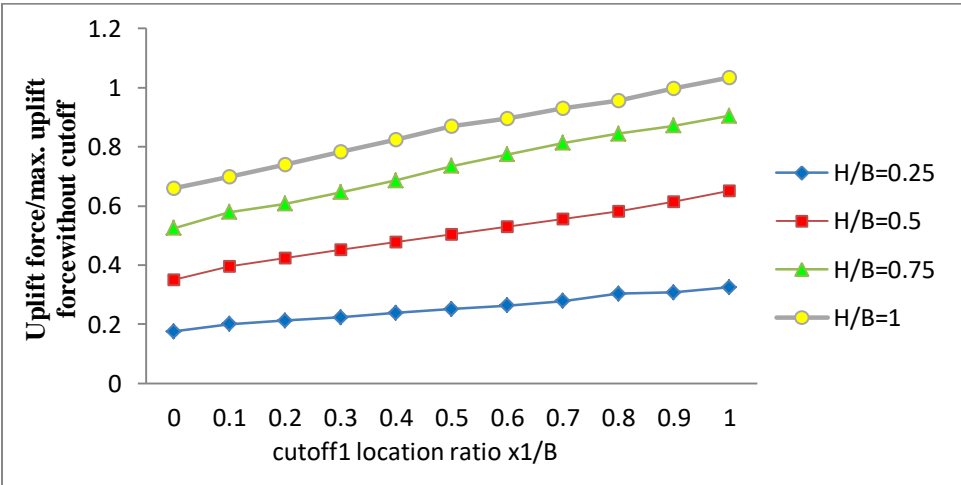


Figure (5-21): Uplift force versus cutoff1 location ratio ($d1/B=0.1, Kr=1$)

It is clear from these figures, that the maximum specific exit gradient occurs when the cutoff's location is at upstream and this maximum value decreases with cutoff location for all head differential until it reaches the minimum value at downstream, This is because the cut-off decreases the seeping velocity causing the exit gradient to decrease as well. While the minimum specific uplift force occurs when the cutoff's location is at upstream and this value increases with cutoff location for all head differential until it reaches the maximum value at downstream. So, using the cutoff at upstream is particularly effective in reducing the uplift pressure, while the cutoff in downstream is more effective in reducing the hydraulic gradient at the exit (Al-Suhaili and Karim, 2014).

Figures (5-22), (5-23) and (5-24) illustrate the effect of the location of cut-offs on uplift pressure with different depths (d) and ($H/B=0.75$) under the floor of the hydraulic structure. It can be seen from these figures that when there is a cut-off at the upstream, the uplift pressure is decreased along the base of floor. Also, the uplift distribution decreases with increasing the depth of cut-off, This will be discussed in the next subject. This is because the cut-off causes an increase in the length of creep, which increases the head loss. Whenever a cut-off is located a drop in the uplift pressure at that location is observed as expected (Al-Mussawi, 2006).

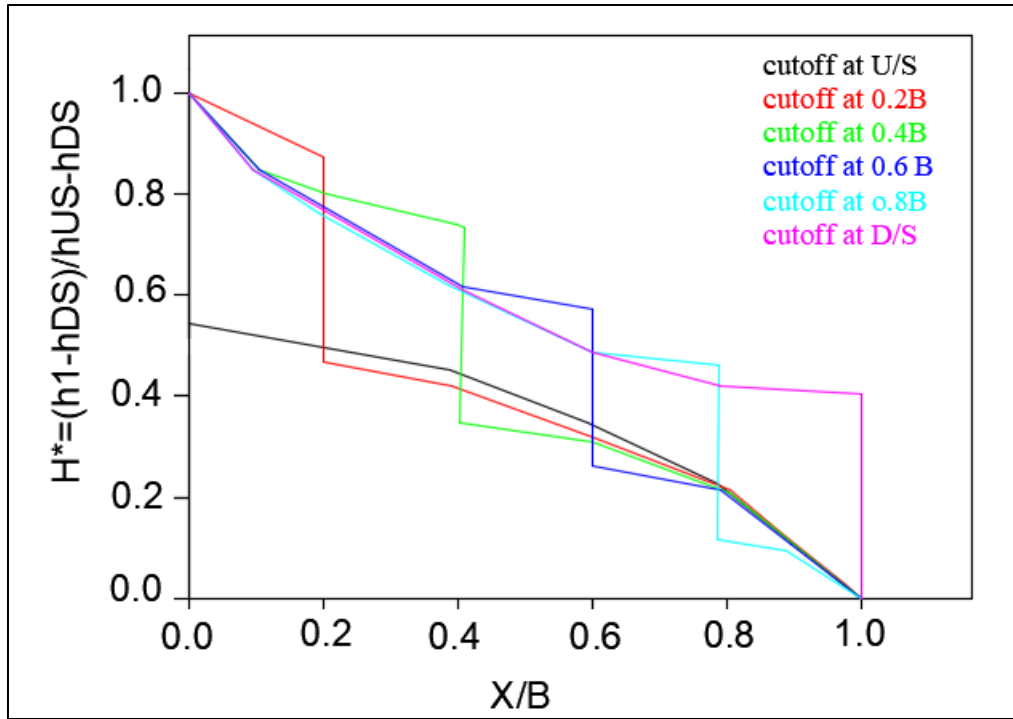


Figure (5-22): Uplift pressure distribution under a hydraulic structure with various locations of cut-off, ($d_1/B=0.2$, $K_r=1$).

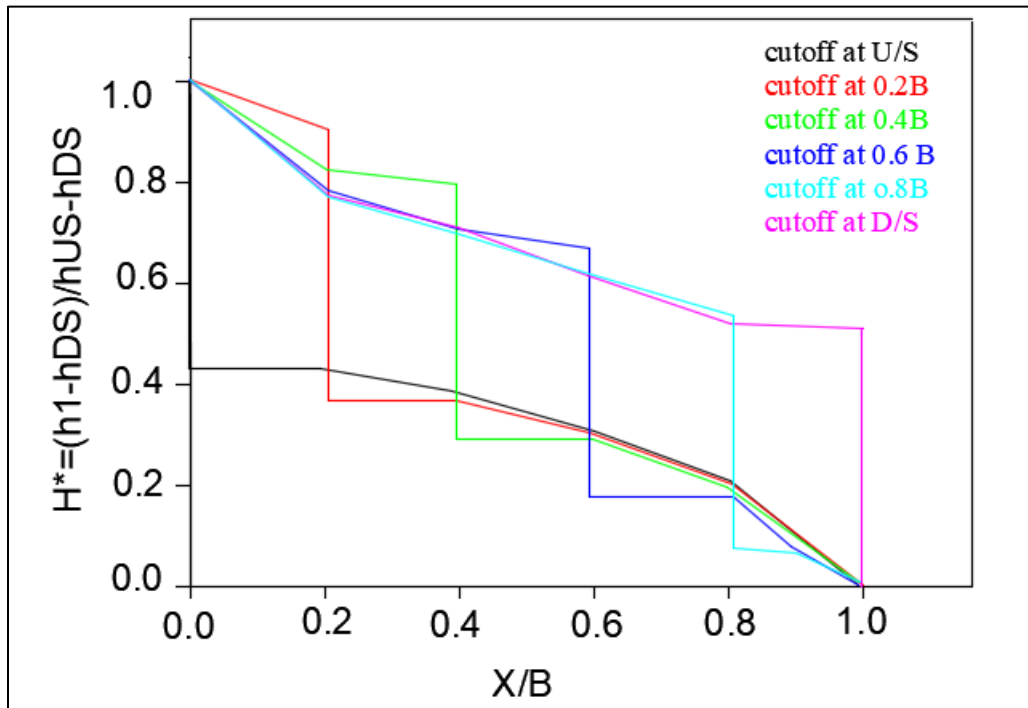
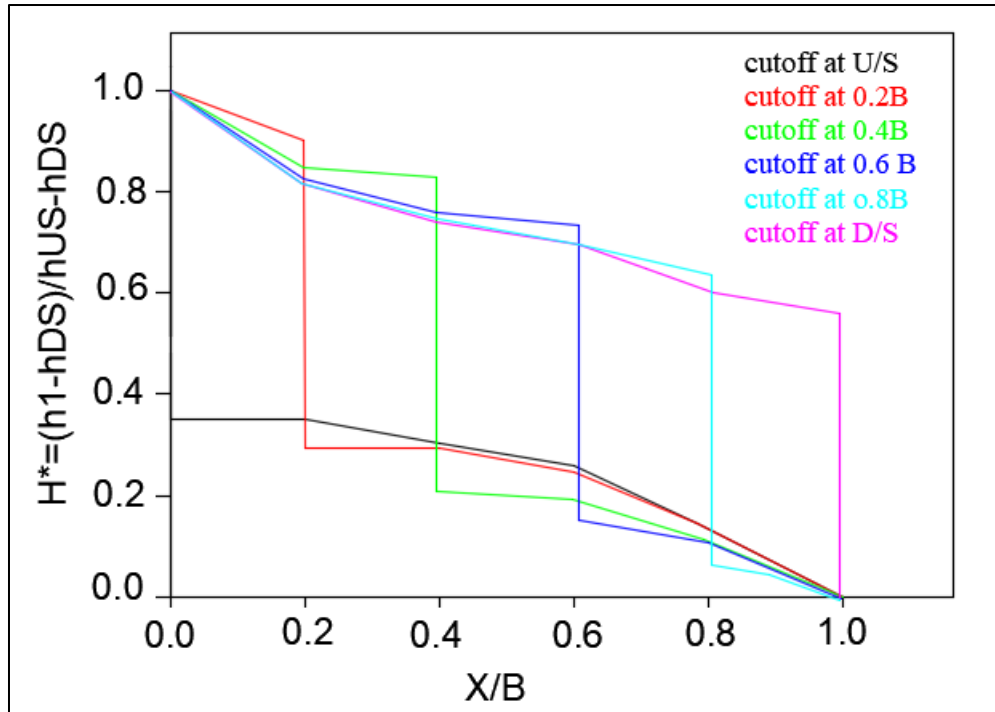


Figure (5-23): Uplift pressure distribution under a hydraulic structure with various locations of cut-off, ($d_1/B=0.4$, $K_r=1$).

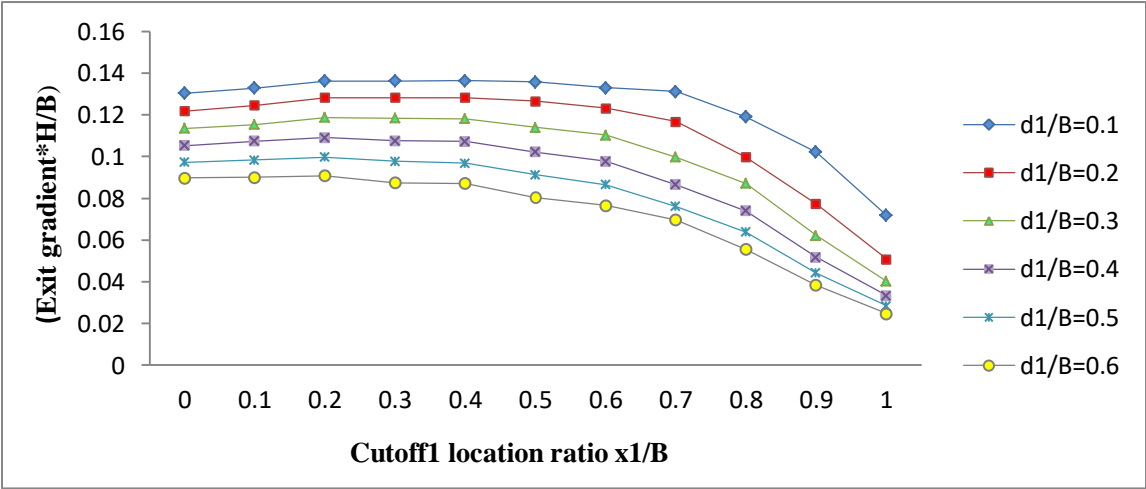


Figure(5-24):Uplift pressure distribution under a hydraulic structure with various locations of cut-off,($d1/B=0.6$, $Kr=1$).

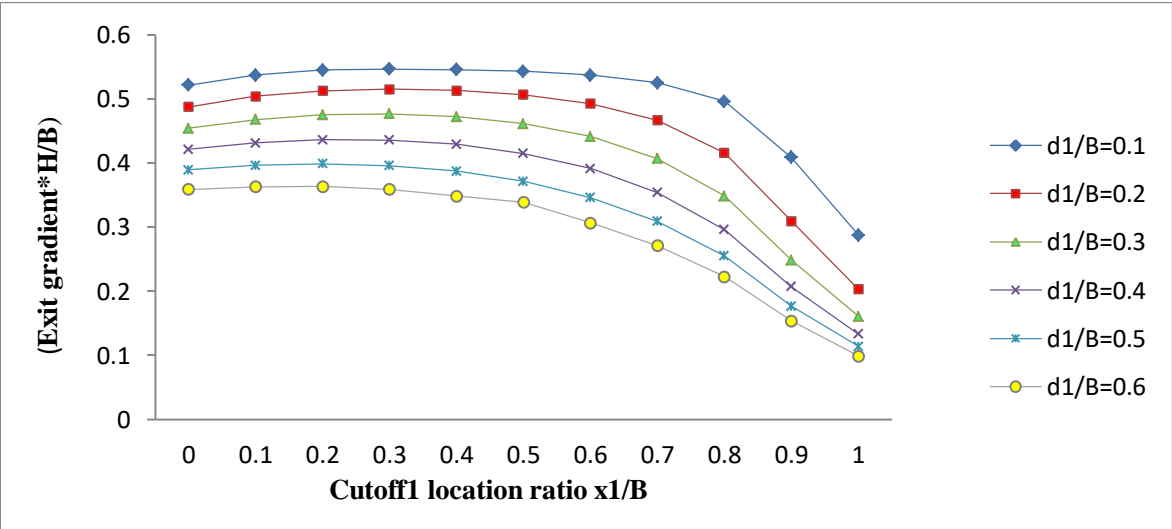
5.3.2 Influence of cutoff's depth ratio on exit gradient & uplift pressure

The main indicator of piping difficulties conditions is the exit gradient . Piping occurs if the exit hydraulic gradient at the downstream point approaches the critical hydraulic gradient (AL-Musawi, 2006). To prevent piping, it is necessary to reduce the velocity of the seeping water to a safe value. This can be accomplished by lengthening the seepage path. One of the methods of such lengthening is to introduce sheet piles or cutoff walls within the dam foundation (Alsenousi and Mohamed, 2008). the increasing in the depth of the cutoff will increase in the seepage path, wherefore the seepage process is inversely proportional to cutoff's depth.

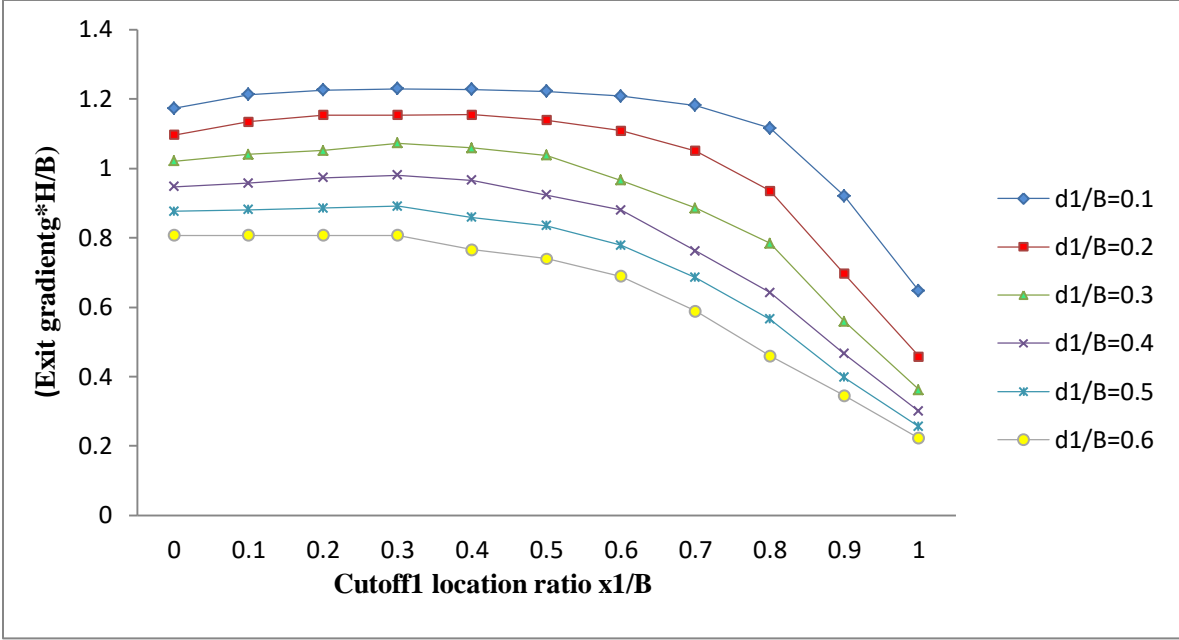
To represent the influence of cutoff depth on exit gradient and uplift pressure with one cutoff, Figures (5-25-5-30) were presented for (H/B) of 0.25, 0.5, 0.75 and 1 respectively. Each of these figures shows six curves for $d1/B = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6 at different value of cutoff location $x1/B = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and 1 to state the behavior of exit gradient and uplift pressure with the variation of cutoff depth.



Figure(5-25): Specific Exit gradient versus cutoff1 location ratio at different values of cut off depths $d2/B=0, x2/B=0, H/B=0.25, Kr=1$.



Figure(5-26): Specific Exit gradient versus cutoff1 location ratio at different values of cut off depths $d2/B=0, x2/B=0, H/B=0.5, Kr=1$.



Figure(5-27): Specific Exit gradient versus cutoff1 location ratio at different values of cut off depths $d_2/B=0$, $x_2/B=0$, $H/B=0.75$, $k_r=1$.

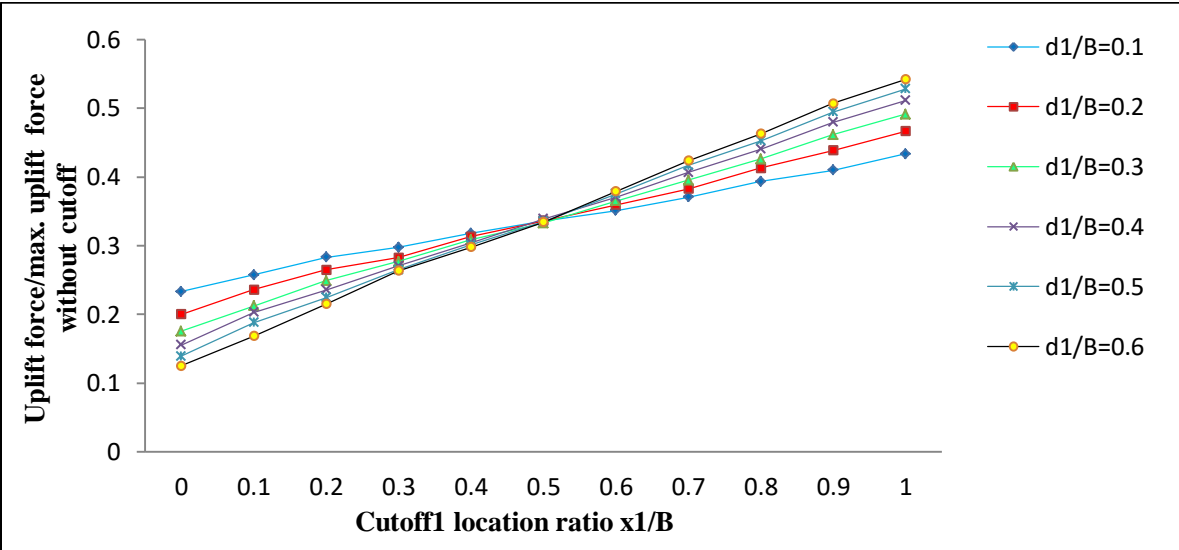


Figure (5-28): Specific Uplift force versus cutoff1 location ratio at different values of cut off depths for $d_2/B=0$, $x_2/B=0$, $H/B=0.25$, $K_r=1$.

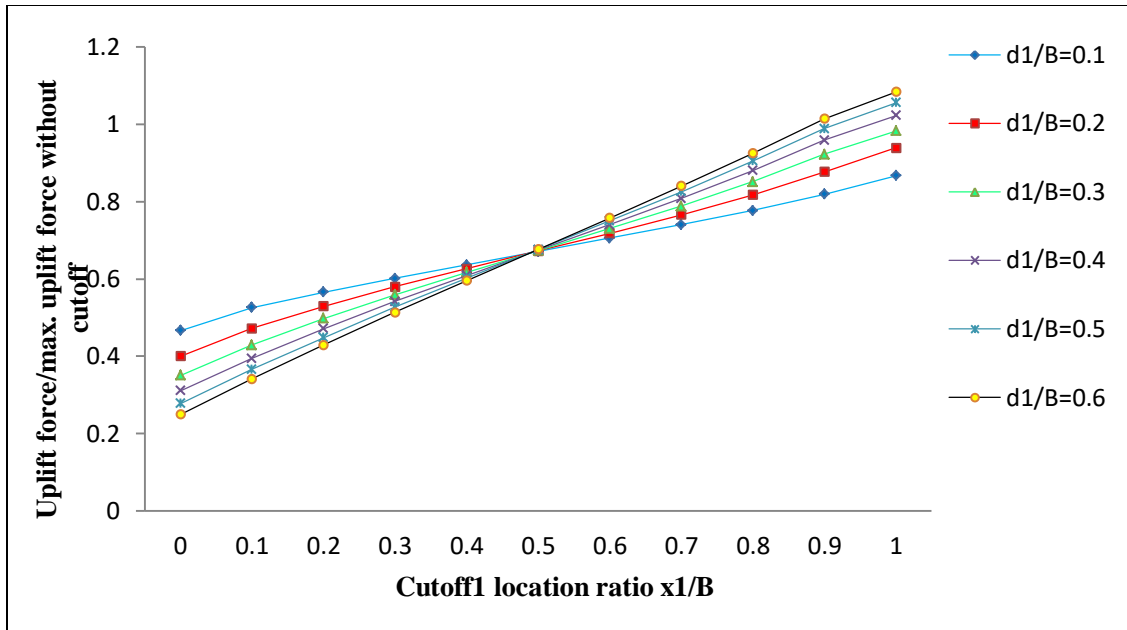


Figure (5-29): Specific Uplift force versus cutoff1 location ratio at different values of cut off depths for $d_2/B=0$, $x_2/B=0$, $H/B=0.5$, $Kr=1$.

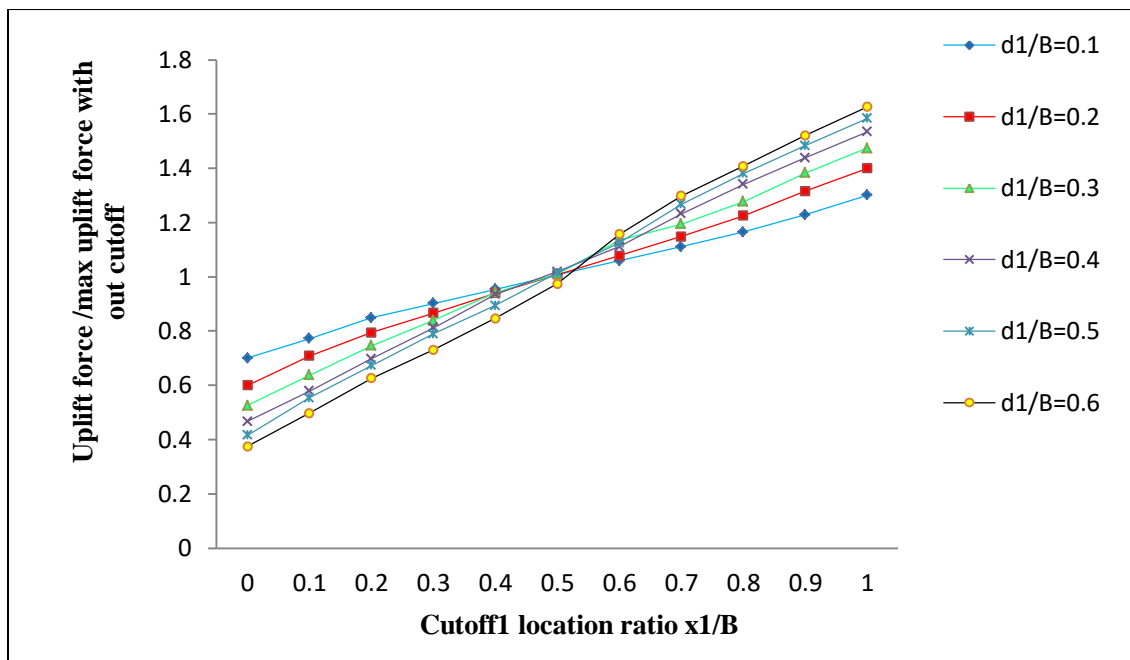


Figure (5-30): Specific Uplift force versus cutoff1 location ratio at different values of cut off depths for $d_2/B=0$, $x_2/B=0$, $H/B=0.75$, $Kr=1$.

Figure (5-27) indicates that exit gradient reach a minimum value at downstream with depth ratio($d_1/B=0.6$), while Figure (5-30) indicates that minimum uplift force occurs at upstream with depth ratio ($d_1/B=0.1$).

Accordingly, From these figures, with increasing cutoff depth, the exit gradient decreases while the uplift pressure decreases at the first half of floor’s length, but it increases at the last half of the floor’s length. This is because the cut-off causes an increase in the length of creep, which increases the head loss. All results are represented in Table A.1 in Appendix A.

5.3.3 Influence of two cut off on exit gradient and uplift pressure

To design safety hydraulic structures against piping and floating as a result to both exit gradient and uplift pressure, the hydraulic structures are supplied by cutoffs at U.S and D.S sides of the base. In general, upstream cutoffs lower the lifting pressure and exit gradient. However, they reduce the uplift pressure in a rate higher than that for exit gradient influence. To reduce the exit gradient, a downstream cutoff should be provided that has a significant impact on the exit gradient(Al-Suhaili and Karim, 2014).

Figures (5.31, 5.32 and 5.33) were presented for (H/B) of 0.25, 0.5 and 0.75 respectively. Each of these figures shows six curves for $d_2/B = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6 at different values of cutoff location x_2/B 0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9 and 1 at cutoff depth $d_1/B = 0.1$ and cutoff location $x_1/B = 0$ to state the behavior of exit gradient with using two cutoff.

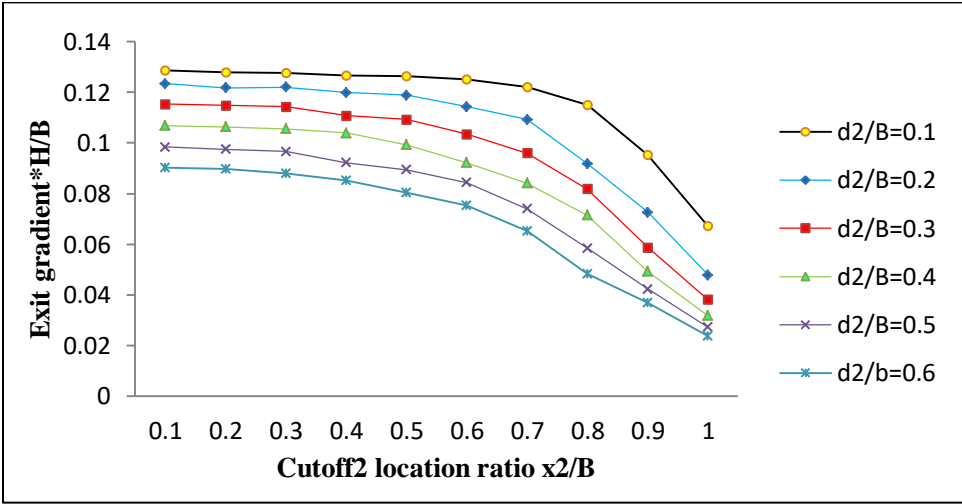


Figure (5-31): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1, x_1/B=0, H/B=0.25, Kr=1$.

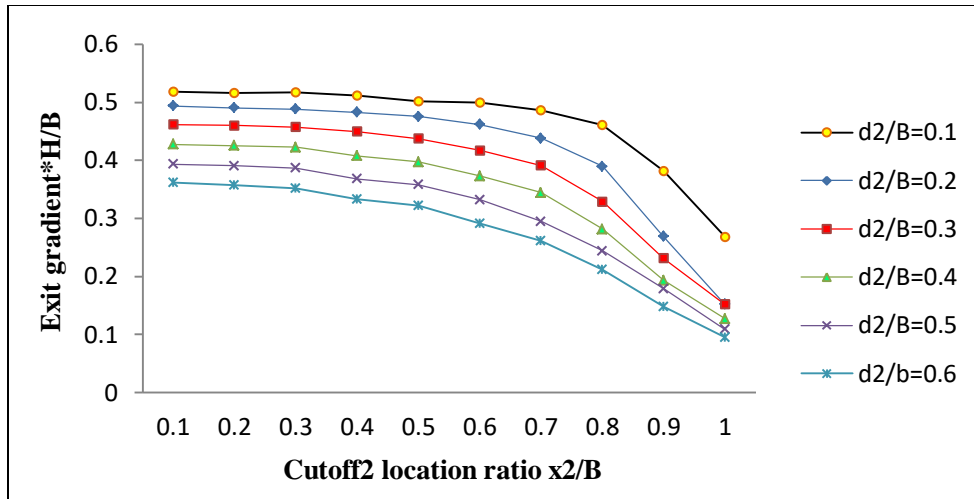


Figure (5-32): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.5$, $K_r=1$.

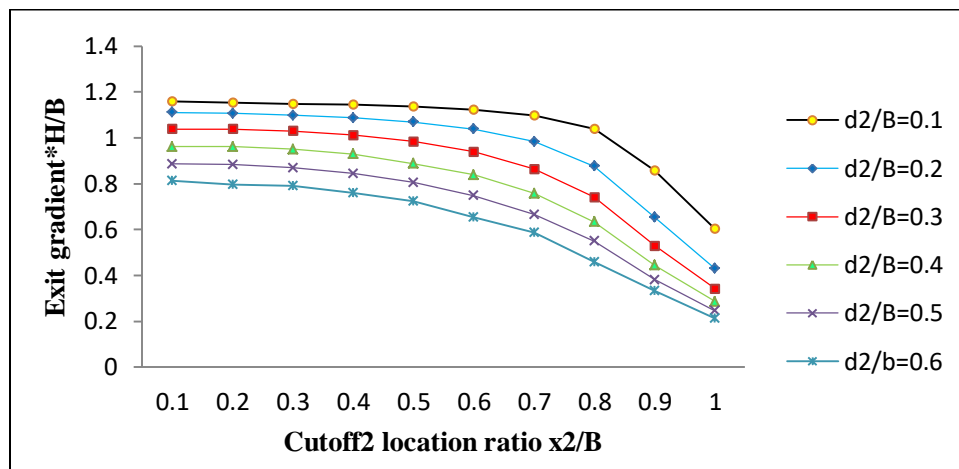


Figure (5-33): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$, $K_r=1$.

It is clear from above figures that the exit gradient decreases with the distance downstream of the hydraulic structure. Also, it is shown that the magnitude of the exit gradient decrease when the cut-off moves from upstream to downstream of the hydraulic structure. This is because the cut-off decreases the seeping velocity causing the exit gradient to decrease as well.

In most practical cases two cut-offs are used, one at each end of the floor for seepage control. Thus, Figure (5-34) illustrates the distribution of the uplift pressure for different lengths of cut-offs. The figure shows that the uplift pressure starts decreasing compared with the case of no cut-off is used; this behavior is

reversed beyond the point where $x = 0.5B$, because of using the two cut-offs at the ends.

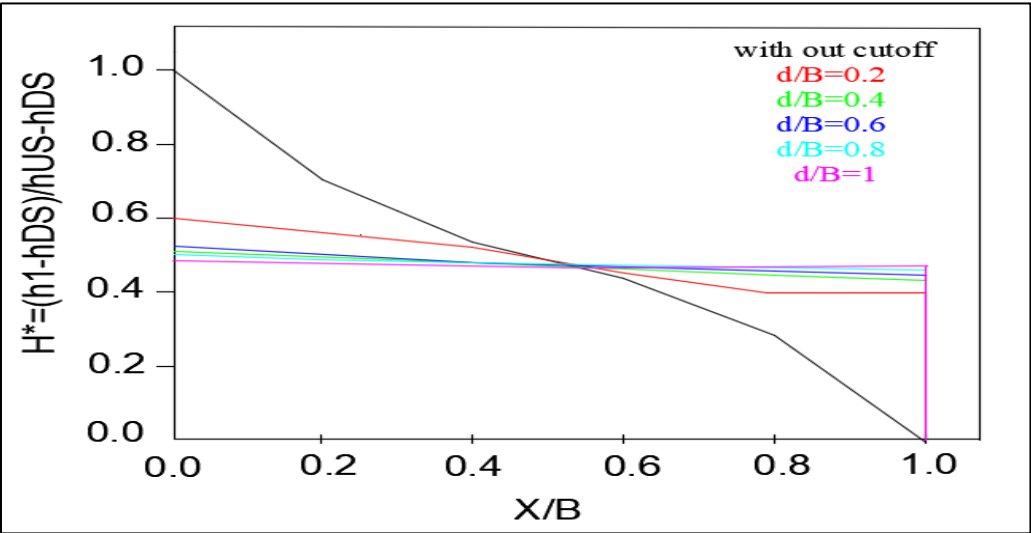


Figure (5-34): Uplift pressure distribution under a hydraulic structure with various depths of U/D cut-offs with $Kr=1$.

Figure (5-35) includes two cutoff, the first cutoff changes from upstream approaches to the downstream at different values of cutoff location for $(x1/B)$ 0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8 and 0.9, the second in downstream at constant value of cutoff location of $x2/B=1$. This figure shows five curves for the cutoffs depths of $(d1/B),(d2/B)$ 0.1, 0.2, 0.3, 0.4 and 0.5 to state the behavior of exit gradient when the cutoff1 approaches to the downstream with constant location for the cutoff2.

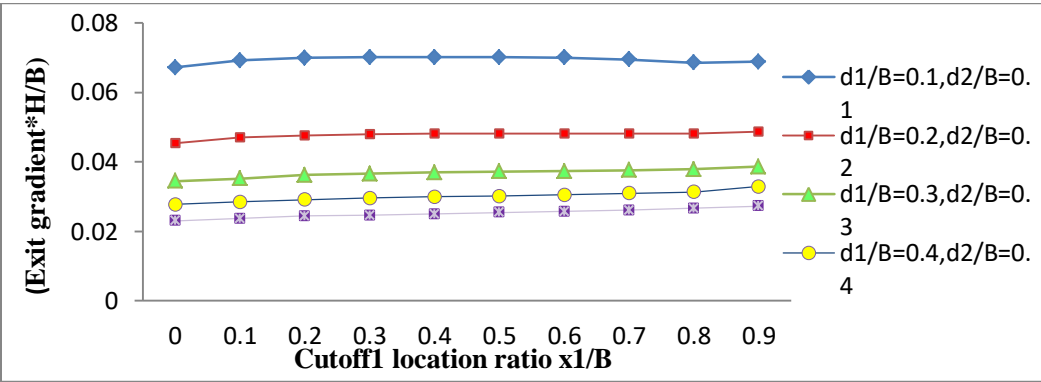


Figure (5-35): Exit gradient versus cutoff1 location ratio $x2/B=1$, $H/B=0.25$, $Kr=1$.

This figure showed the exit gradient will increase when the cutoff1 approached to the downstream and cutoff 2 in constant location (downstream).

5.3.4. Influence of (d1/d2) ratio on exit gradient

To study the effect of the variation of lengths for the cutoffs when using two cutoffs on exit gradient and uplift force at $x1/B=0$, $x2/B=1$, as illustrated in Figures (5-36), (5-37).

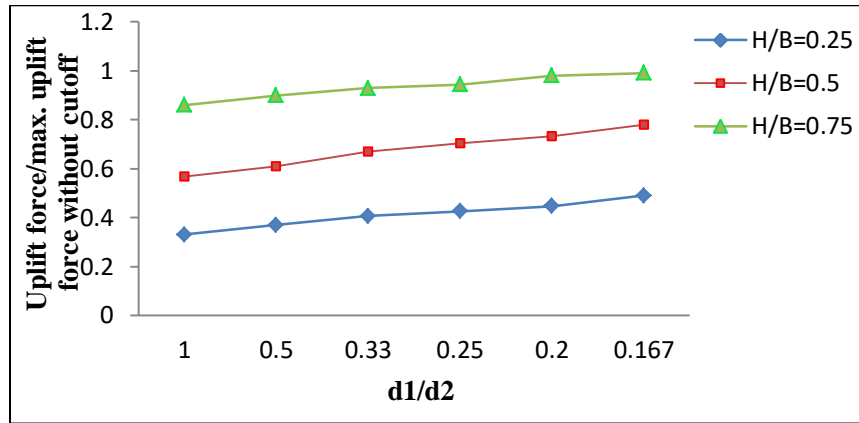


Figure (5-36): Specific uplift force versus (d1/d2) ratio.

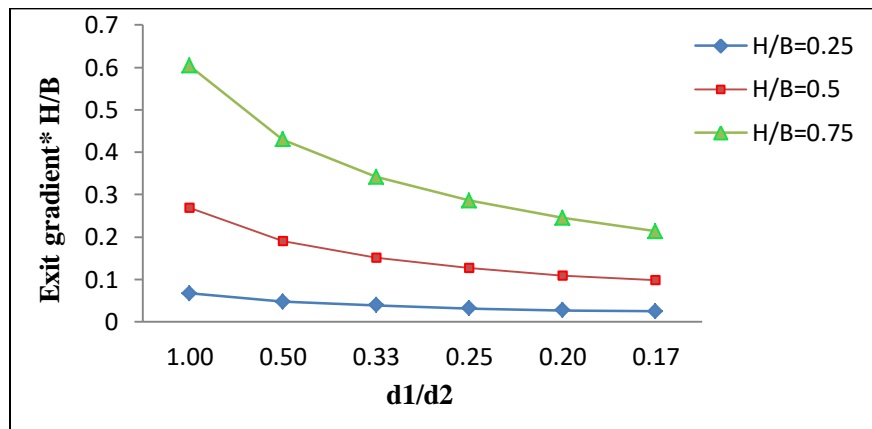


Figure (5-37): Specific exit gradient versus (d1/d2) ratio.

Figure(5-36) indicates that the minimum uplift pressure occurs when $(d1/d2 = 1)$. This minimum value will increase as the ratio of $(d1/d2)$ decreased. Figure(5-37) indicate that the maximum exit gradient occurs when the ratio of the depth of upstream cutoff to the depth of downstream cutoff is equal to one $(d1/d2 = 1)$. This maximum value of the exit gradient will decrease as the ratio of $(d1/d2)$ decreased (increasing the cutoff depth in downstream will significantly effect on

the value of exit gradient than cutoff depth in upstream) (Karim and Drainage, 1988).

5.3.5. Influence of differential head ratio, (H/B) ratio on exit gradient and uplift pressure

In order to investigate the effect of differential head ratio, (H/B) ratio on the exit gradient and the uplift pressure. For (H/B = 0.25, 0.5 and 0.75) each of above figures indicate that the value of H/B had a considerable effect on increasing the exit gradient and uplift pressure, especially when the value of the differential head (H/B =0.75).

5.3.6. Influence of an isotropic degree (Kx/Ky) ratio on exit gradient and uplift pressure

In order to investigate the effect of isotropic degree ratio (Kr) on the exit gradient, as illustrated in Figures 5-38,5-39,5-40,5-41 and the uplift pressure, as illustrated in Figures 5-42,5-43,5-44, 5-45. These runs are operated at different locations and depths of cutoffs for different soil properties (Kr= 1,2,4,8). The results indicate that the minimum exit gradient occurs when the ratio of the hydraulic conductivity in x direction to the hydraulic conductivity in y direction is equal to one (Kx/Ky =1). This min. value of the exit gradient will increase as the ratio of (Kx/Ky) increased until reach the value of 8.

Kr=1

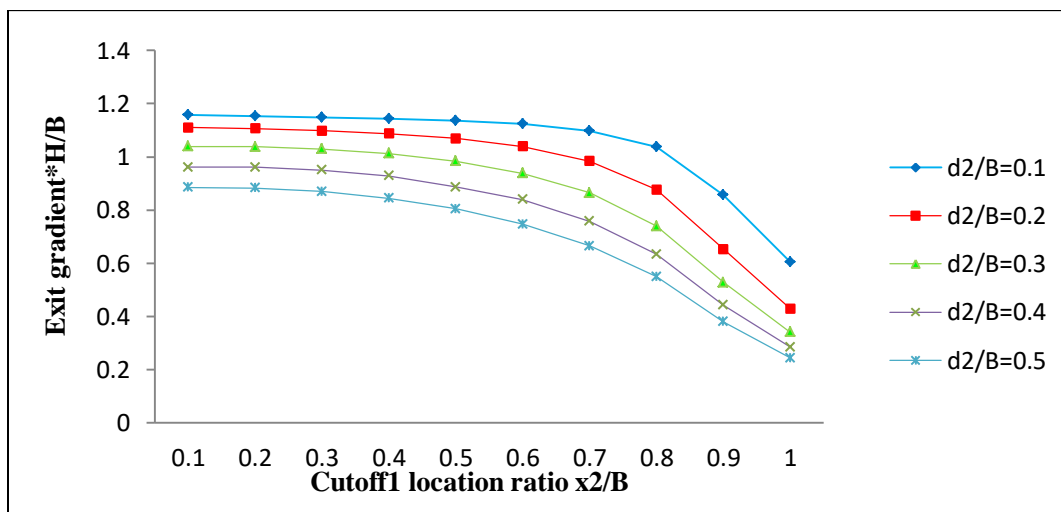


Figure (5-38): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d1/B=0.1$, $x1/B=0$, $H/B=0.75$.

Kr=2

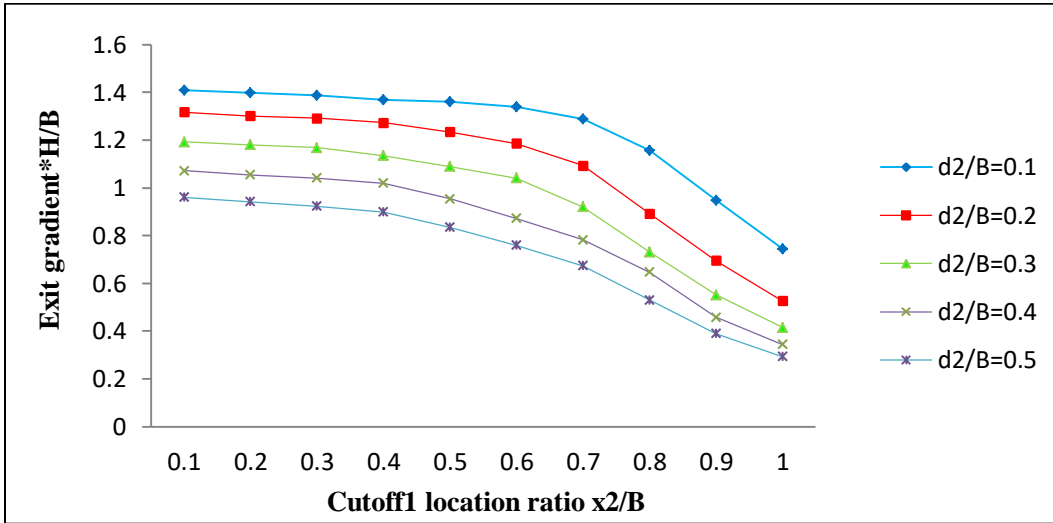


Figure (5-39): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d1/B=0.1$, $x1/B=0$, $H/B=0.75$.

Kr=4

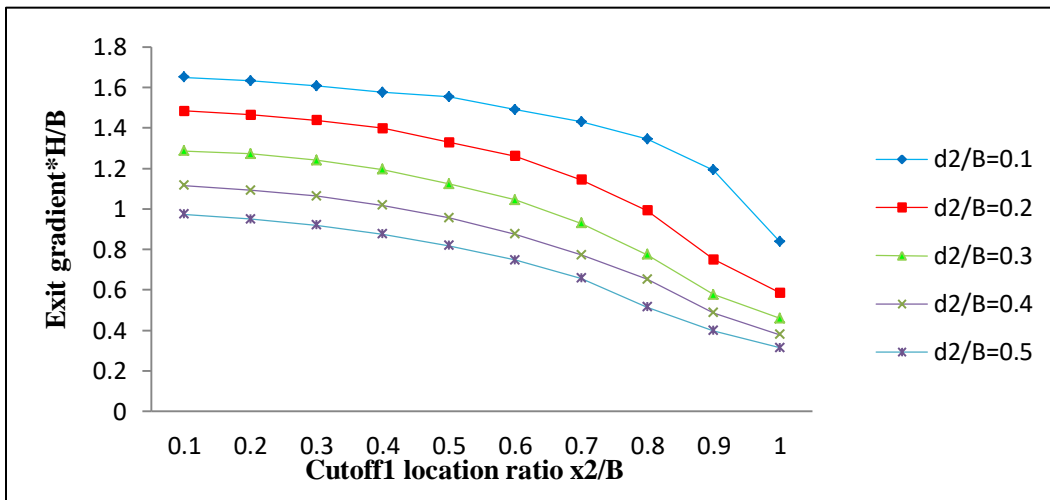


Figure (5-40): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d1/B=0.1$, $x1/B=0$, $H/B=0.75$.

Kr=8

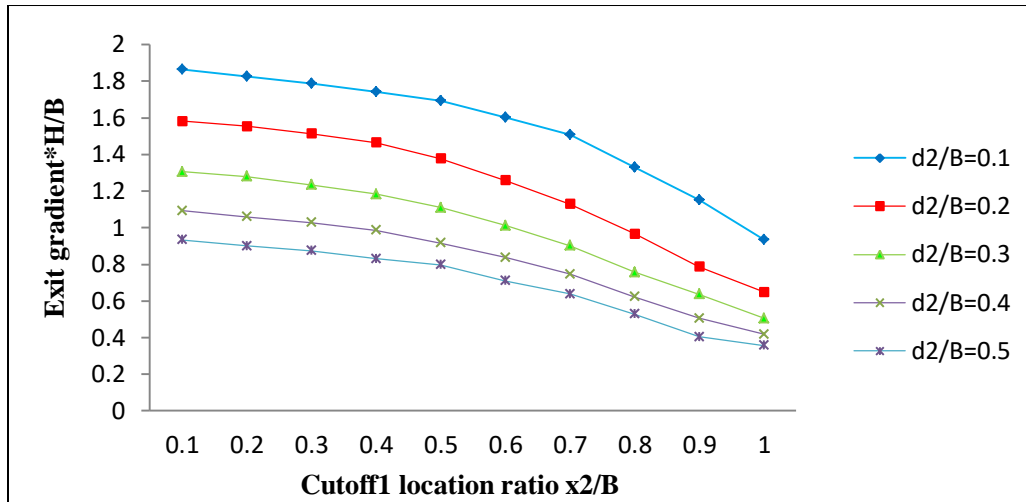


Figure (5-41): Exit gradient versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$.

While the results of the below Figures (5-42) to(5-45) indicate that the minimum uplift force can be noticed when the ratio of the hydraulic conductivity in x direction to the hydraulic conductivity in y direction is equal to one ($K_x/K_y = 1$). This value of the uplift force will increase as the ratio of (K_x/K_y) increased .

Kr=1

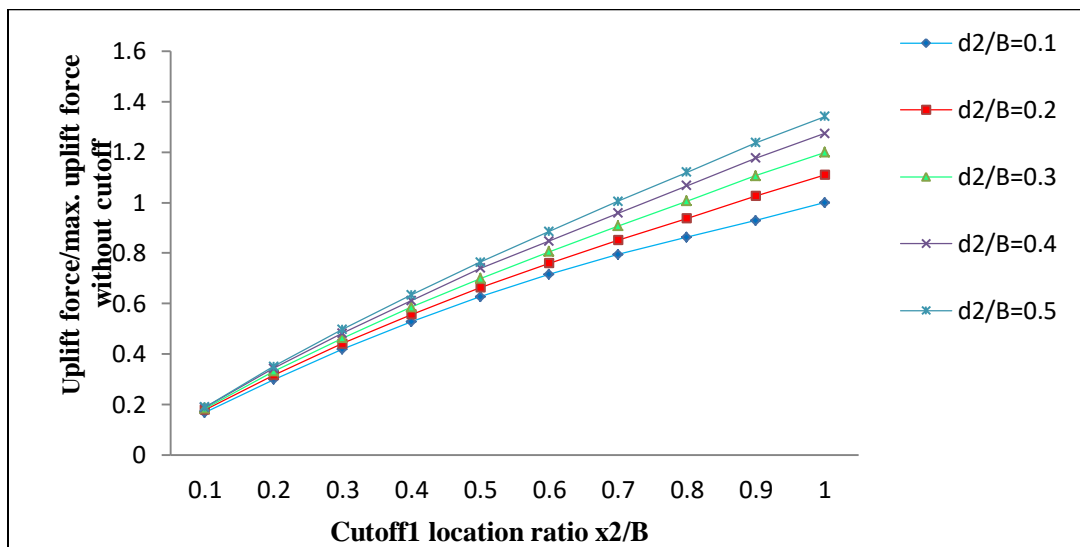


Figure (5-42): uplift force versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$.

Kr=2

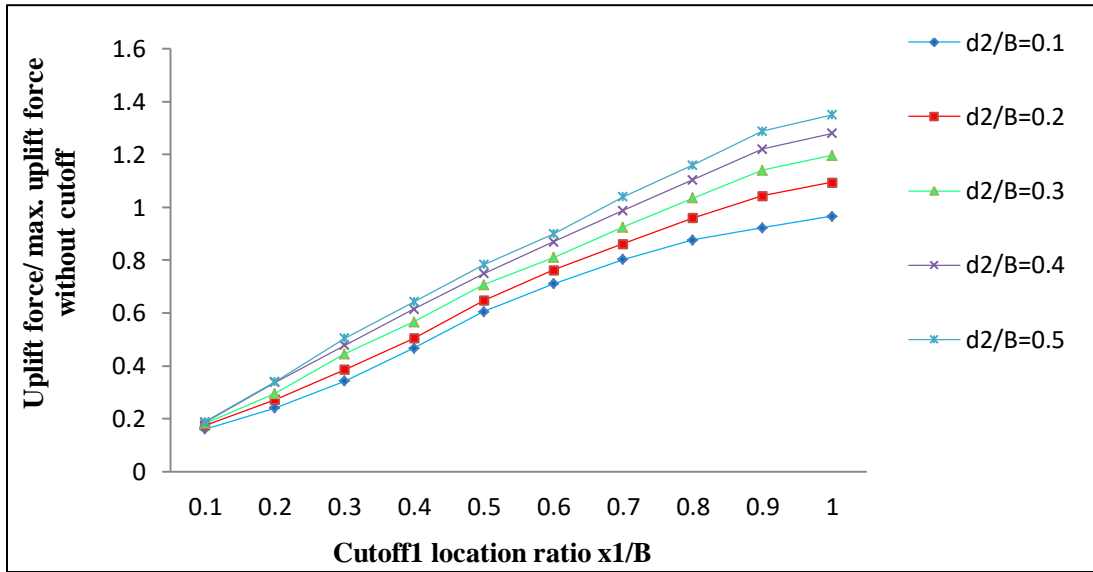


Figure (5-43): uplift force versus cutoff2 location ratio at different values of cutoff depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$.

Kr=4

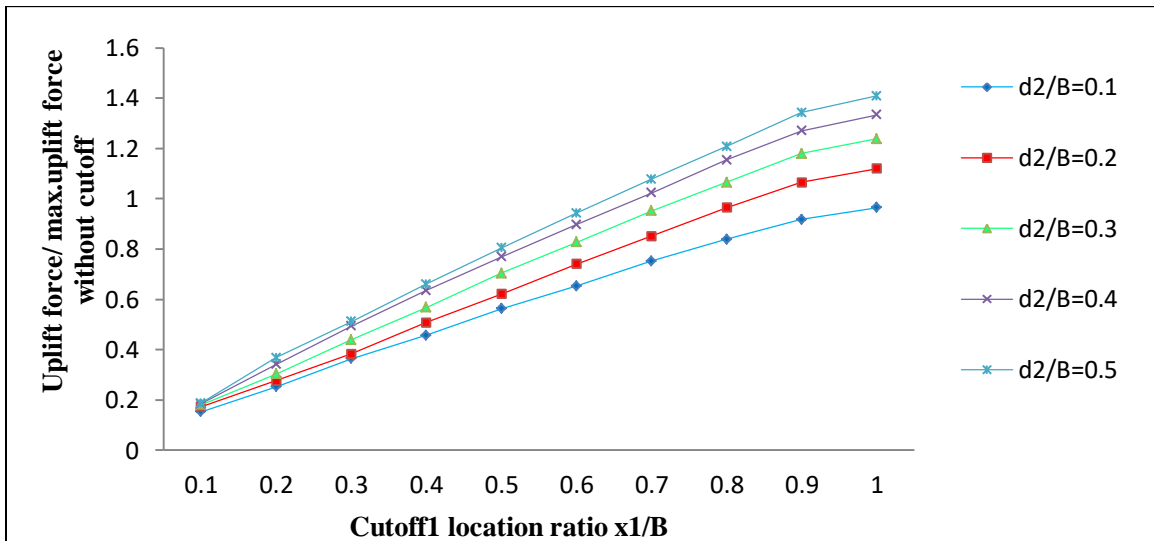


Figure (5-44): uplift force versus cutoff2 location ratio at different values of cutoff depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$.

Kr=8

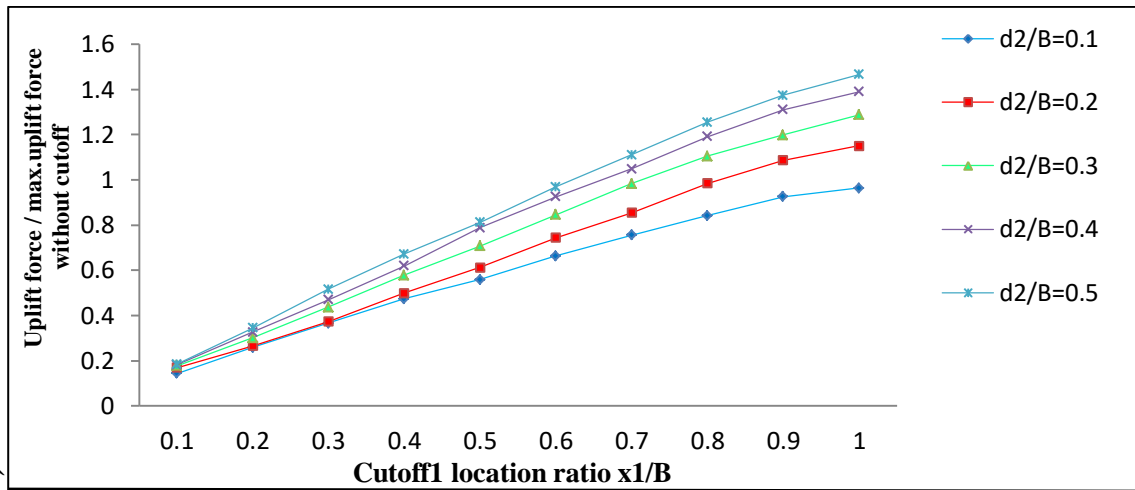


Figure (5-45): uplift force versus cutoff2 location ratio at different values of cut off depths $d_1/B=0.1$, $x_1/B=0$, $H/B=0.75$

5.4 Optimization using GA-FDP model

The GA on MATLAB platform will be used in order to solve the optimization model, which is represented by Figure (4-7). In the optimization formulation model, for a given hydraulic structure scheme, The cost is objective function which computes depend on the constraints (factors of safety against uplift pressure and piping) which is represented by Equations (4.13),(4.14).

c_1 , c_2 are costs coefficients related to construction and excavation of U/S and D/S cutoffs per unit volume (m^3), respectively, which can be expressed by equation as a function of the cutoff depth as shown in Figure(5-46) based on the information of those with experience in this field.

Further, this function was formulated based on the assumption that the cost could not represent a linear relationship with cut off depths, as the requirements, tools and field conditions (Aljuboori and Datta, 2017), to construct cut-offs less than 3m (for example) in depth that are generally different than the depth greater than 10m, etc.

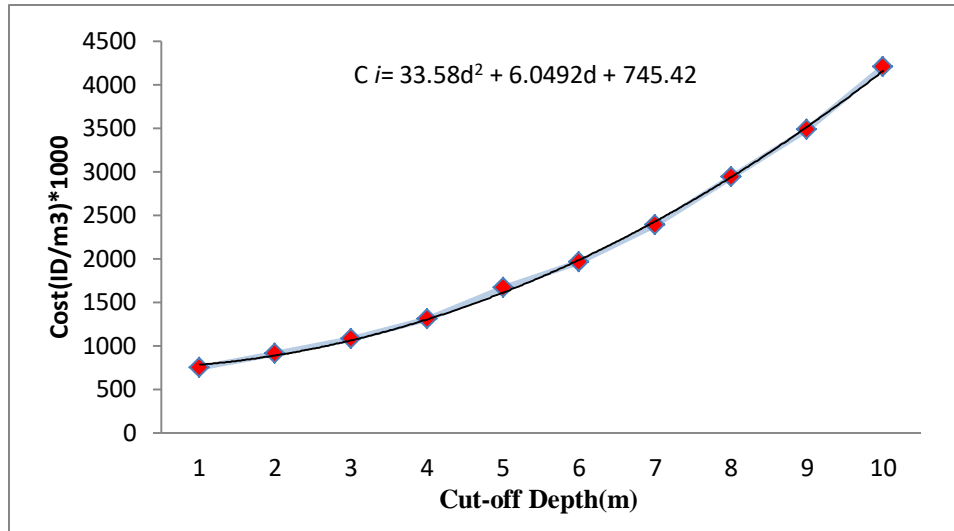
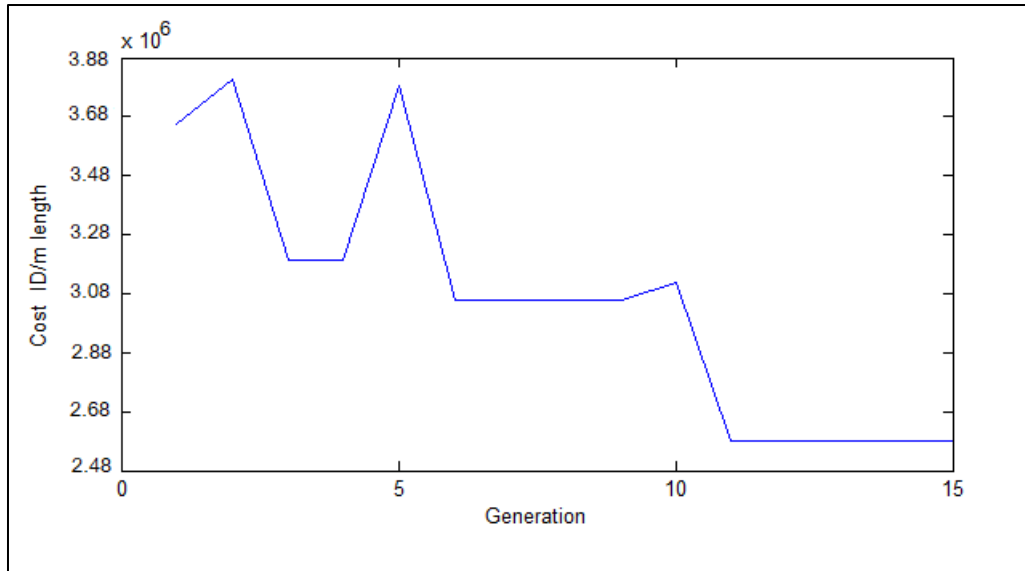


Figure (5-46): The total cost of cutoff per unit volume(m^3) versus cutoff depth (DND-GCKS, 2017).

This model was run for a number of generations and was stopped when there was no development in fitness function value. Figure (5-47) shows the typical convergence curves for the best solution cost, over a number of generations during the evolution process, with $Kr=1$ and $H/B=0.75$ for the seepage problem. This figure shows the total cost for the optimal design obtained after 10 generations.

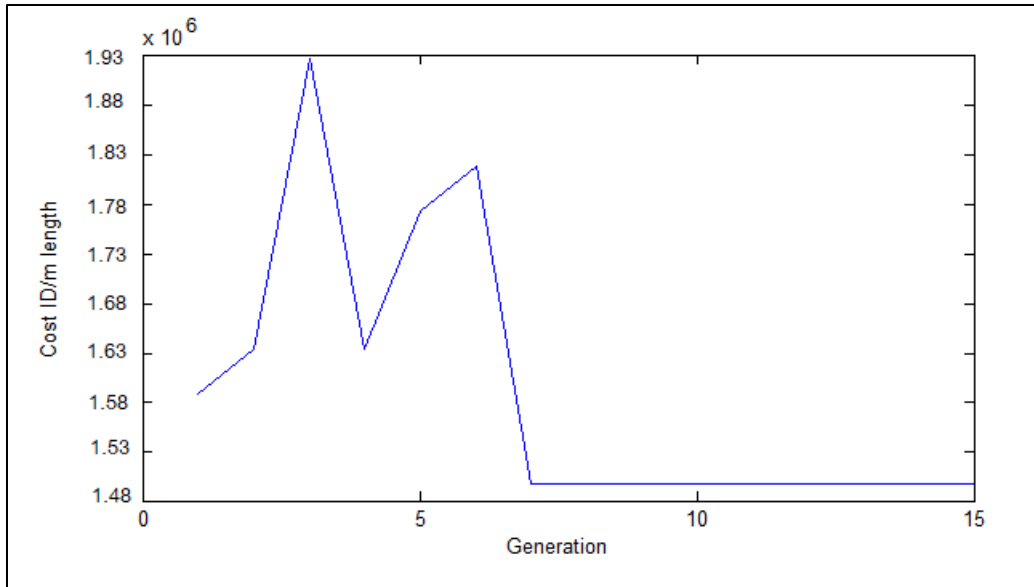
During the operation of optimization, the process of going into a new generation continues until the fitness of the population converges, that is, average fitness of population almost matches the best fitness. This criterion proves the solution to be optimized. The optimized values of relative location of cutoff and its depth are shown in this Table.



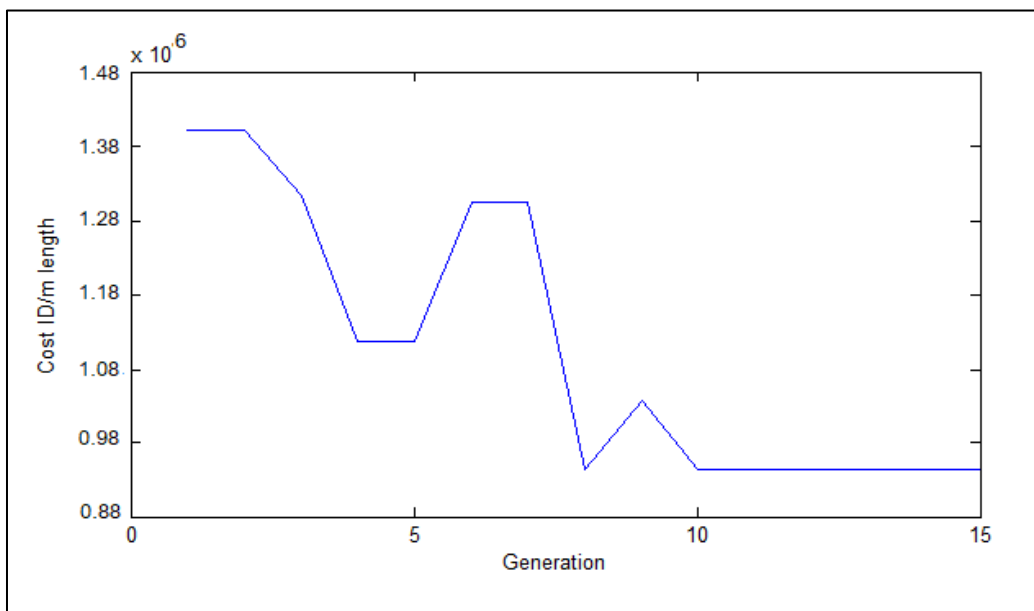
Figure(5-47) :The optimum cost of iterations by the proposed GA-FDP model $K_r=1$ and $H/B=0.75$.

The optimum cutoff location and depth for this solution which achieve the factor of safety against piping and uplift pressure when the cutoff1's location and depth $(0.325, 0.05)B$ respectively, while the cutoff2's location and depth $(0.975, 0.525)B$ respectively on the floor of the structure with min.cost 2.580×10^6 ID/m length.

The Figure (5-48) shows the min. cost for the best solution cost, over a number of generations during the evolution process, with $K_r=1$ and $H/B=0.5$ for the seepage problem. This figure shows the total cost for the optimal design was obtained after 6 generations, while Figure (5-49) shows the min. cost for the best solution cost, over a number of generations during the evolution process, with $K_r=1$ and $H/B=0.25$ for the seepage problem. This figure shows the total cost for the optimal design was obtained after 10 generations.



Figure(5-48) :The optimum cost of iterations by the proposed GA-FDP model
 $K_r=1$ and $H/B=0.5$.



Figure(5-49) :The optimum cost of iterations by proposed GA-FDP model
 $K_r=1$ and $H/B=0.25$.

Table (5-2) includes the results of the optimum cutoff's location and depth for $K_r=1$ which achieve the factor of safety against piping and uplift pressure on the floor of the structure.

Table (5-2): Results of the optimum values for location and depth of cutoffs ratio with cost at various value of head ratio and $K_r=1$

Input		Optimum Output						
Kr	H/B	x1/B	d1/B	x2/B	d2/b	(iexit*H)/B	(F uplift)/F*	cost*10 ⁶ / m length
kr=1	0.25	0.025	0.018	0.925	0.15	0.068925	0.352548828	0.960
	0.4	0.09	0.022	0.925	0.3	0.12428	0.627883112	1.450
	0.5	0.225	0.025	0.945	0.35	0.15885	0.753652735	1.500
	0.6	0.227	0.025	0.95	0.5	0.18732	1.17166157	2.410
	0.75	0.325	0.04	0.975	0.525	0.244875	1.470608223	2.580

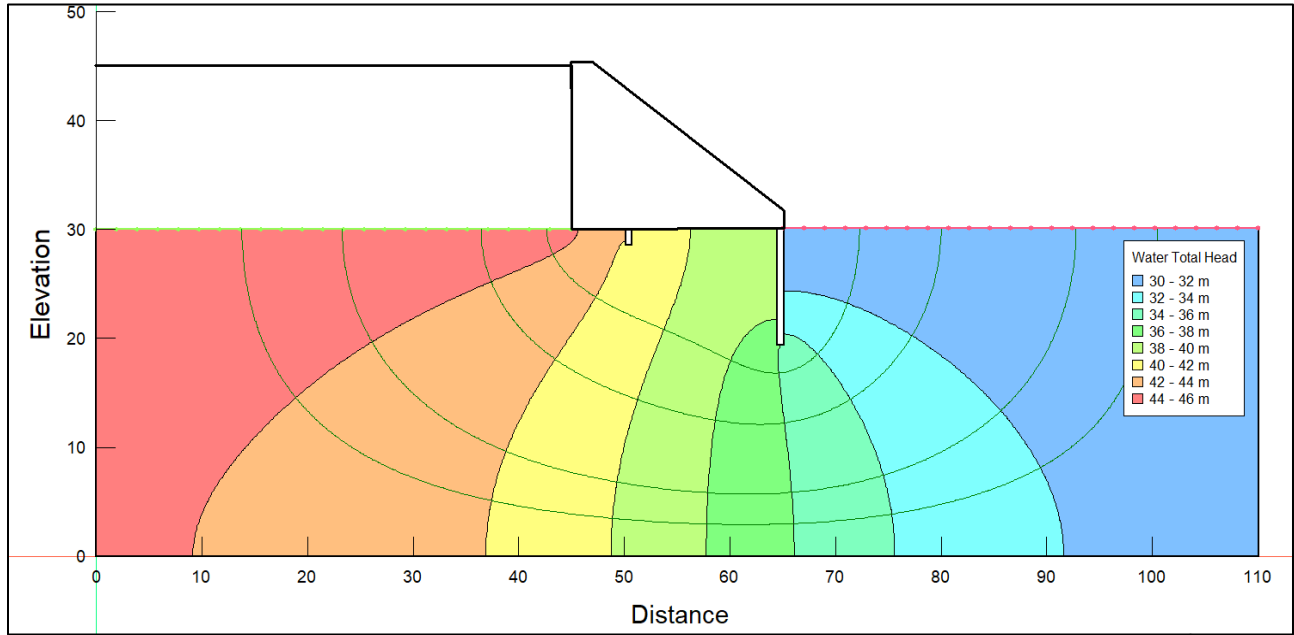
F*: Max. up lift force without cutoff

In order to compare the values of the obtained optimum solution using the GA model with the values obtained using SEEP2D model, the optimum six cases were modeled by SEEP2D. These cases were re-analyzed using the SEEP2D model to find whether the obtained values of specific exit gradient and uplift force under dam by the GA were comparable with those obtained by the SEEP2D solution for the same values of H/B and K_r . The comparison is shown in Table (5-3), which shows good agreement.

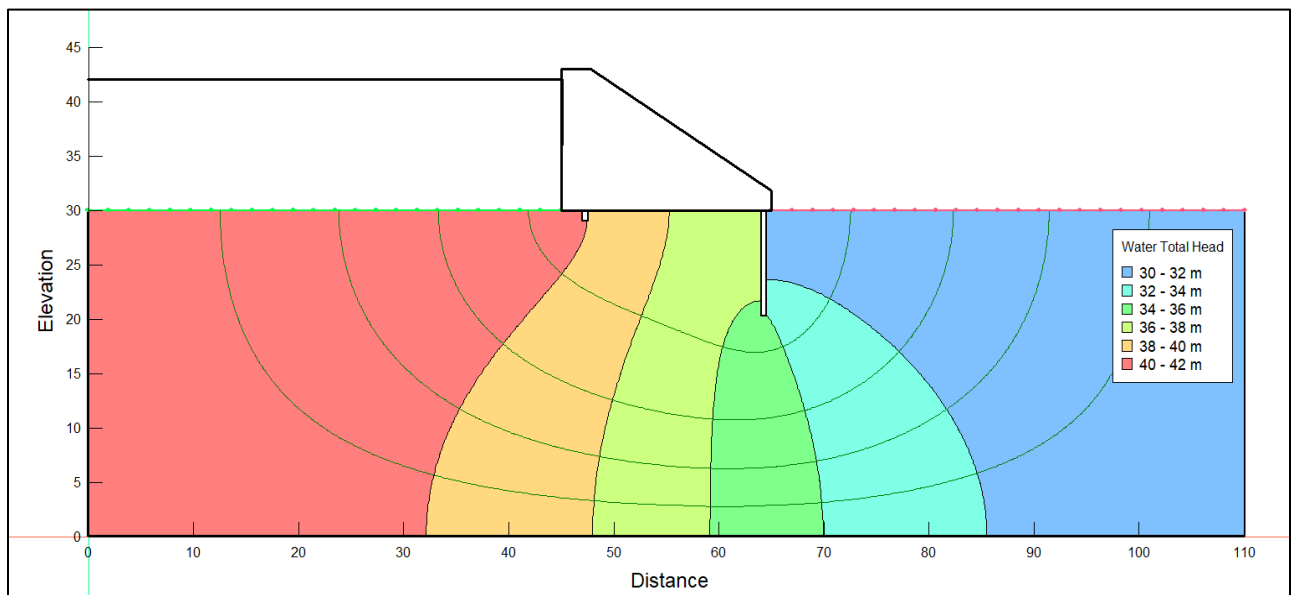
Table (5-3) Comparison of (specific exit gradient and uplift force) values from the optimum solutions for ($K_r=1$)

NO.	H/B	X1/B	d1/B	X2/B	d2/B	GA-FDP Model		SEEP2D Model		% Difference (iexit *H)/B	% Difference (F uplift /max F)
						(iexit *H)/B	(F uplift / F*)	(iexit *H)/B	(Fuplift /max F)		
1	0.25	0.025	0.018	0.925	0.15	0.068925	0.352	0.07005	0.346	-1.63	1.71
2	0.4	0.09	0.022	0.925	0.3	0.12428	0.627	0.12804	0.644	-3	-2.8
3	0.5	0.225	0.025	0.945	0.35	0.15885	0.7536	0.1581	0.797	0.47	-5.7
4	0.6	0.227	0.025	0.95	0.5	0.18732	1.1716	0.18234	1.1422	-0.59	2.5
5	0.75	0.325	0.04	0.975	0.525	0.244875	1.47	0.2475	1.49	-1.07	-1.57

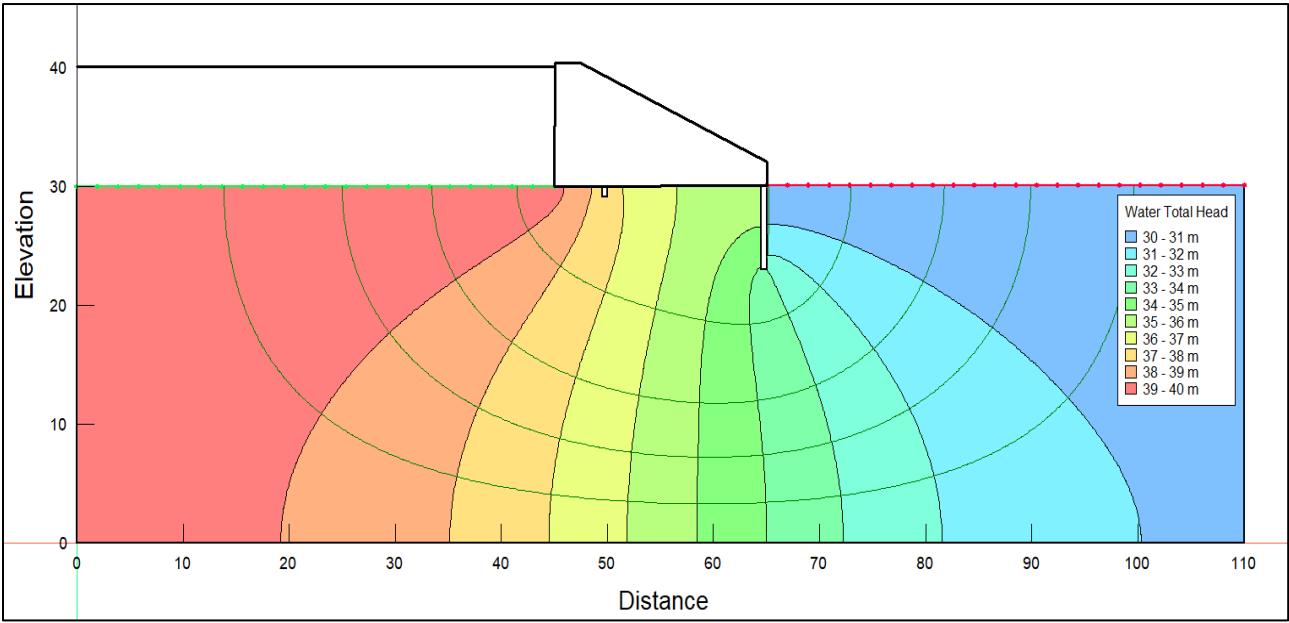
The obtained flow net (flow lines and equipotential lines) for the optimum hydraulic structures model within optimum decision variables (x_1, x_2, d_1 and d_2) of the six cases are shown in Figures (5-50) to(5-54). The values of uplift pressure as well as its distribution under the floor of the structure and the exit gradient value on the downstream side of the structure are calculated.



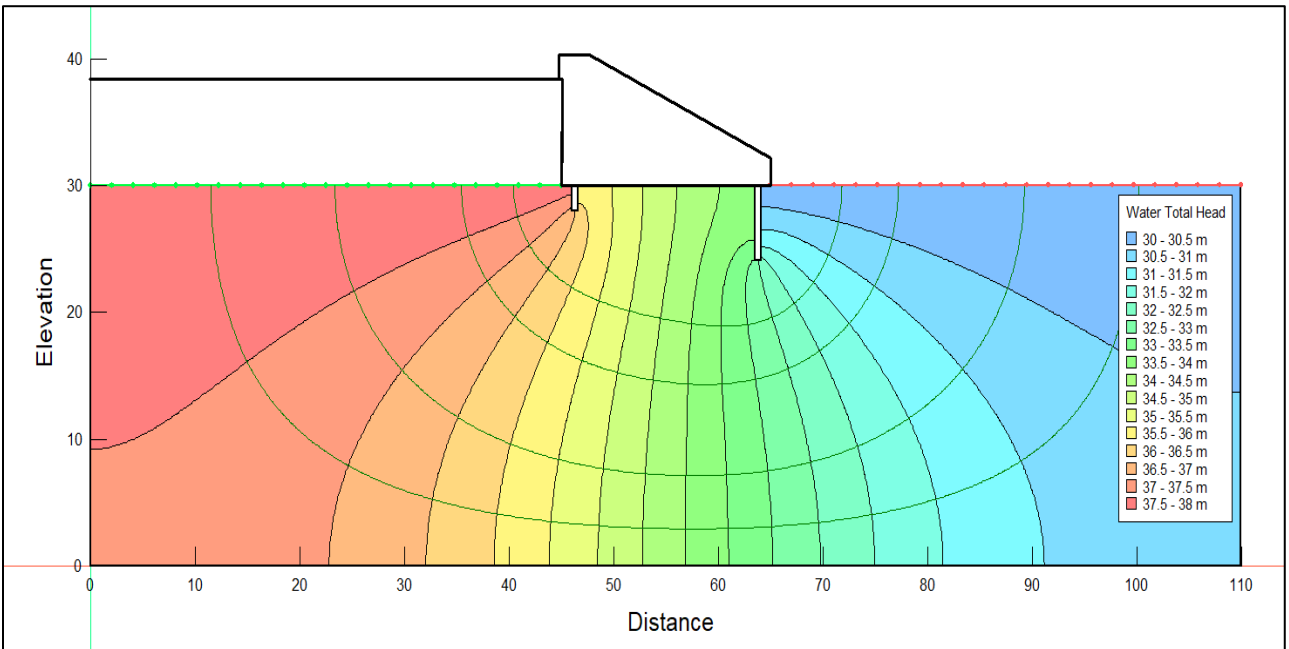
Figure(5-50) The Flow net of the optimum $H/B=0.75$.



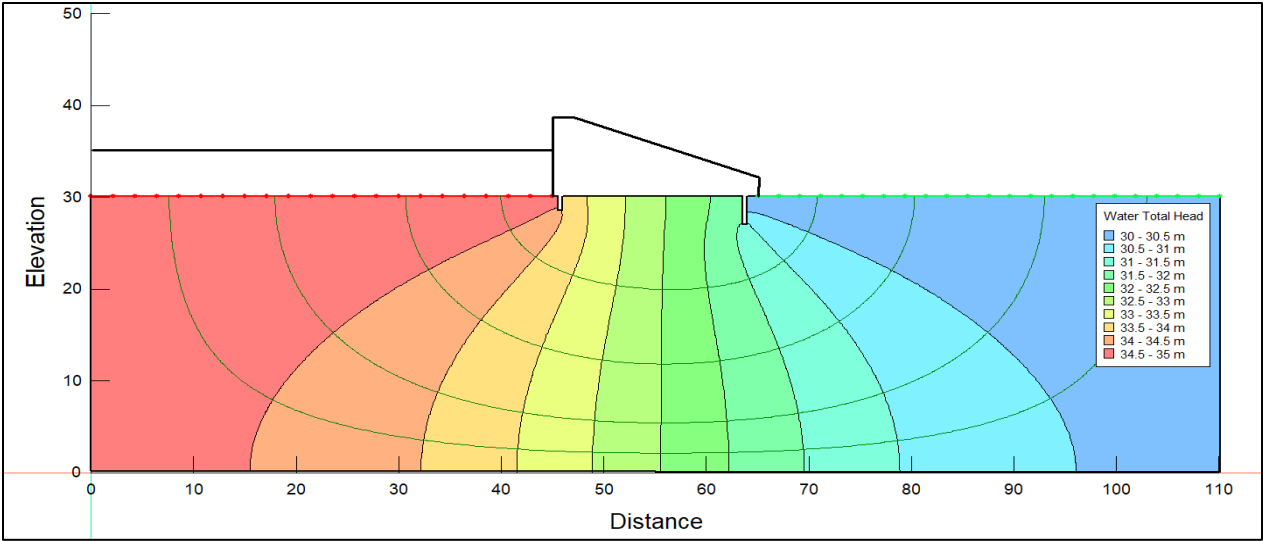
Figure(5-51) The Flow net of the optimum $H/B=0.6$.



Figure(5-52): The Flow net of the optimum $H/B=0.5$.



Figure(5-53): The Flow net of the optimum $H/B=0.4$.



Figure(5-54): The Flow net of the optimum $H/B=0.25$.

For the other ratio of K_r , the min. cost for the best solution and the optimized values of parameters with $K_r=2,4,8$ and different head ratio are shown in Table (5-4).

Table (5-4): Results of the optimum values for location and depth of cutoffs ratio with cost at various value of head and degree of isotropic

Input		Optimum Output						
K_r	H/B	$x1/B$	$d1/B$	$x2/B$	$d2/b$	$(i_{exit} * H)/B$	$(F_{uplift})/F^*$	cost* $10^6/$ m length
kr=2	0.25	0.125	0.18	0.95	0.125	0.06945	0.334352701	1.160
	0.4	0.185	0.235	0.95	0.370	0.12488	0.65993544	1.830
	0.5	0.20	0.25	0.955	0.375	0.15975	0.925874958	1.710
	0.6	0.230	0.285	0.96	0.425	0.18894	0.954376487	2.810
	0.75	0.234	0.3	0.96	0.6	0.245475	1.324226979	3.750
kr=4	0.25	0	0.05	0.95	0.225	0.07055	0.415881753	1.130
	0.4	0.015	0.105	0.95	0.275	0.12644	0.585728848	1.810
	0.5	0.023	0.125	0.965	0.425	0.15995	0.878899083	2.090
	0.6	0.035	0.275	0.975	0.5	0.19008	1.013455657	2.950
	0.75	0.095	0.35	0.975	0.675	0.24575	1.368943255	4.570
kr=8	0.25	0.09	0.05	0.875	0.325	0.07195	0.515141013	1.500
	0.4	0.1	0.055	0.905	0.3	0.12792	0.569819912	2.110
	0.5	0.125	0.15	0.925	0.5	0.1537	1.148691811	3.120
	0.6	0.165	0.3	0.975	0.575	0.1962	1.108392796	3.560
	0.75	0.175	0.575	0.975	0.725	0.246525	1.228610262	6.360

The optimum decision variables ($X1/B, X2/B, d1/B$ and $d2/B$) of the five head ratios for each Kr are shown in Figures (5-55),(5-56),(5-57)and(5-58) respectively.

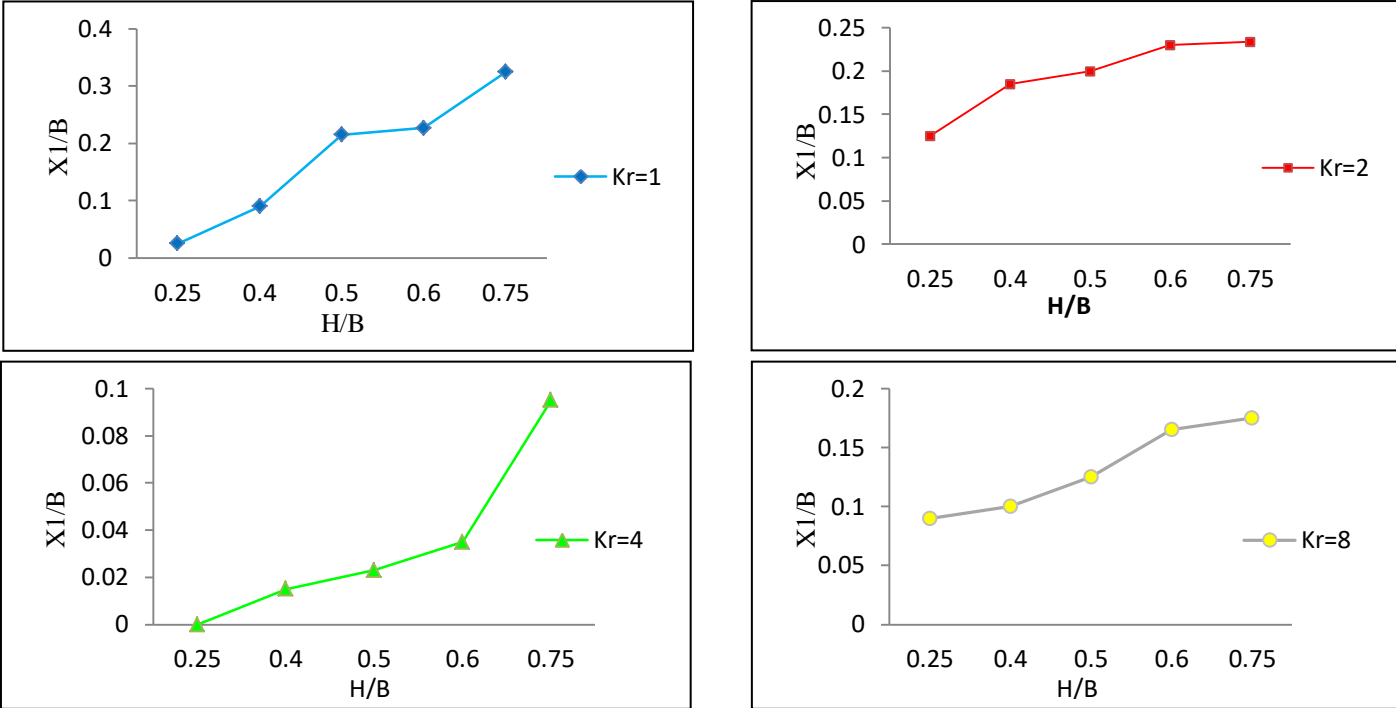
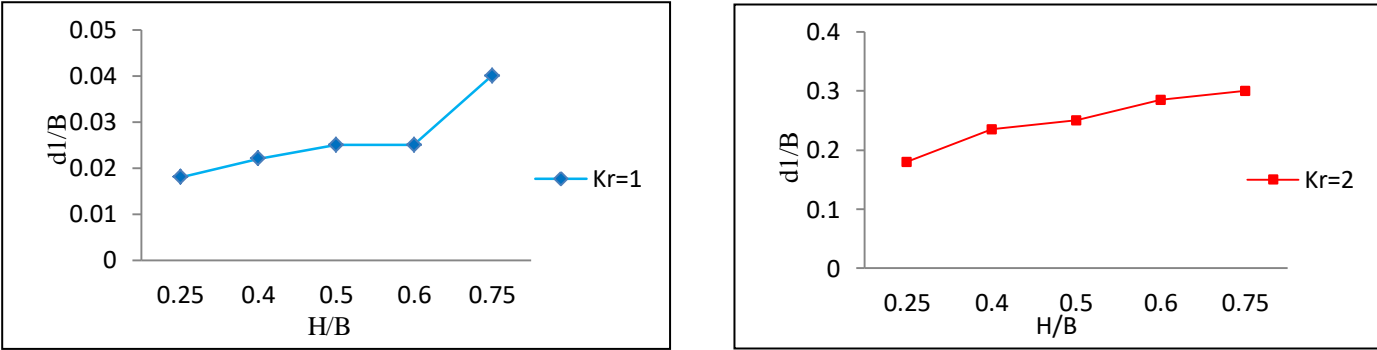


Figure (5-55):The optimum value for location of cutoff 1 ratio with head at different values of Kr



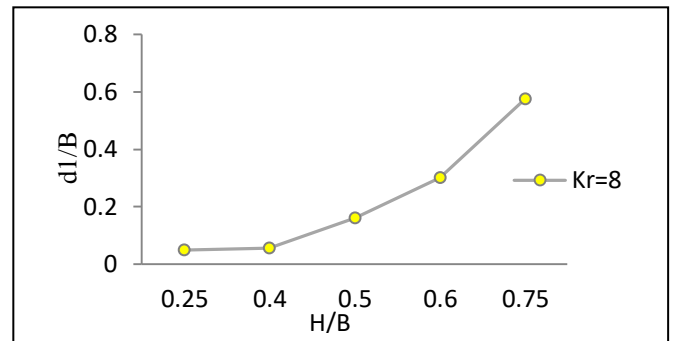
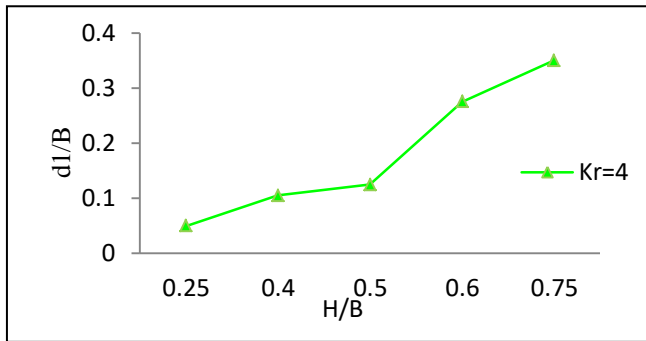


Figure (5-56):The optimum value for depth of cutoff 1 ratio with head at different values of Kr

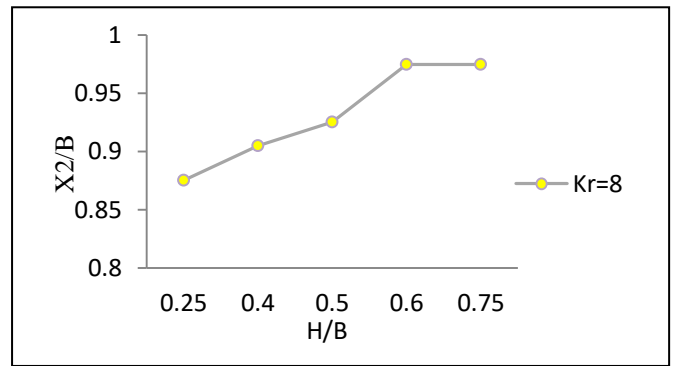
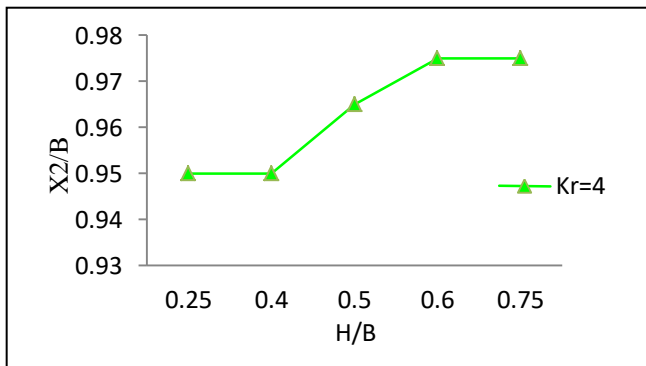
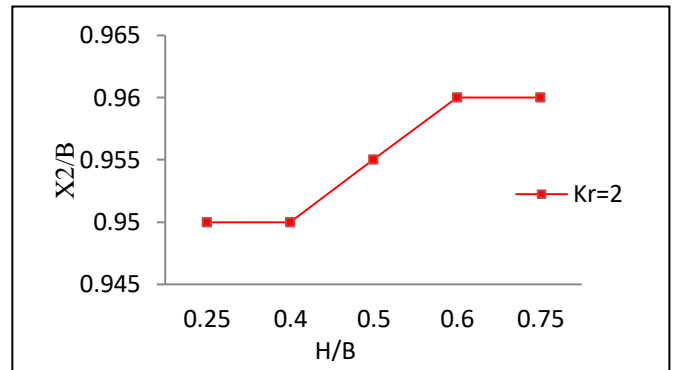
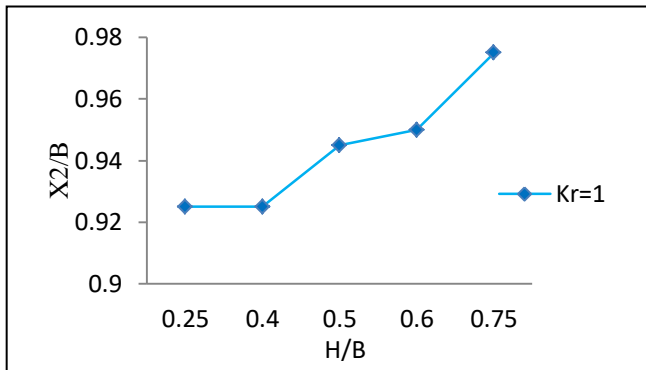
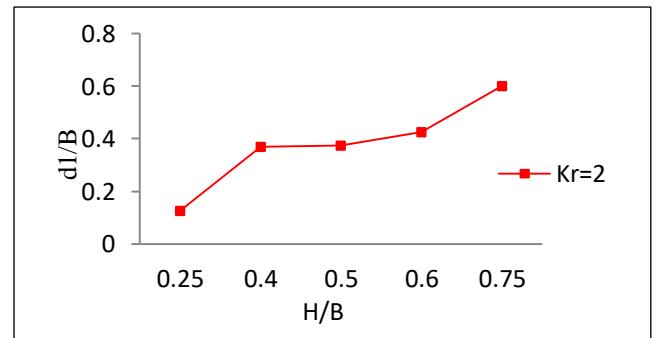
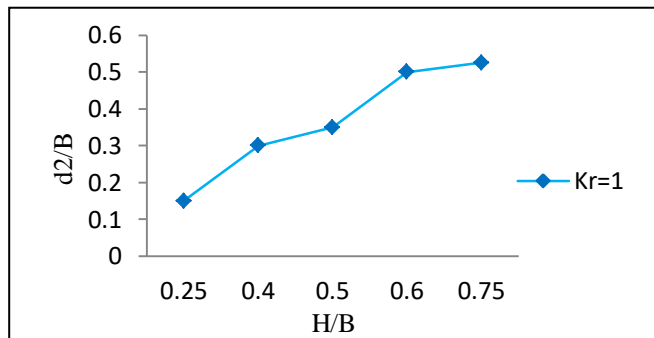


Figure (5-57):The optimum value for location of cutoff 2 ratio with head at different values of Kr



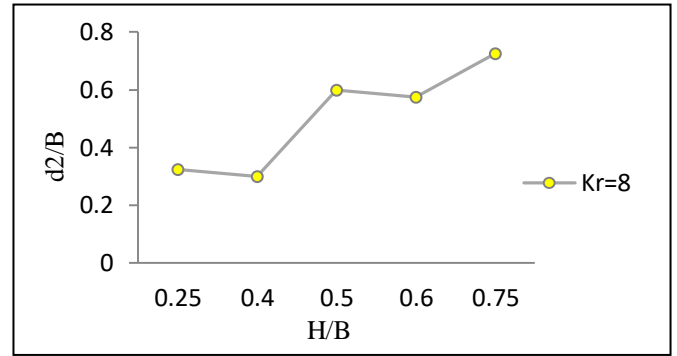
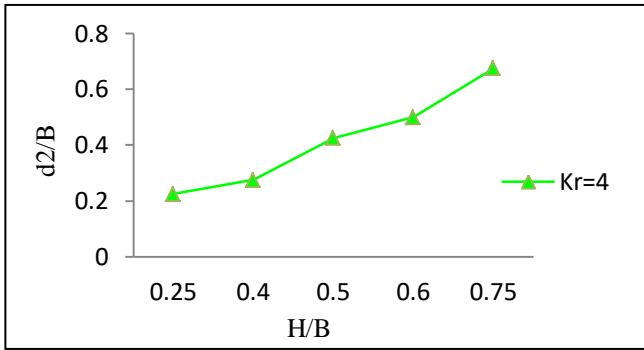


Figure (5-58):The optimum value for depth of cutoff 2 ratio with head at different values of Kr

Figure (5-59) shows the optimum solution obtained by using the GA model for specific exit gradient corresponding to the relative head values. It is clear from the figure that the optimum specific exit gradient increased when increasing the relative head values and the isotropic degree.

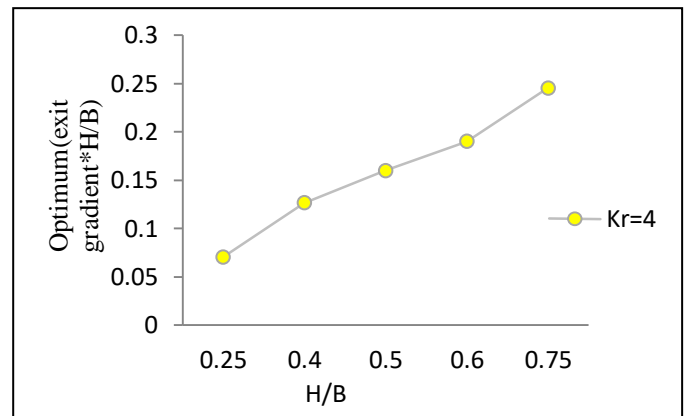
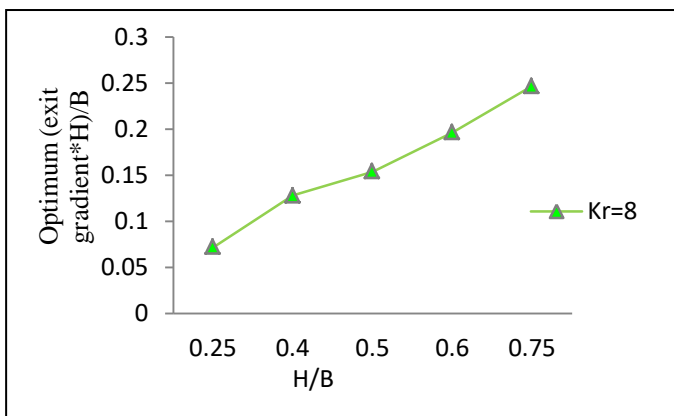
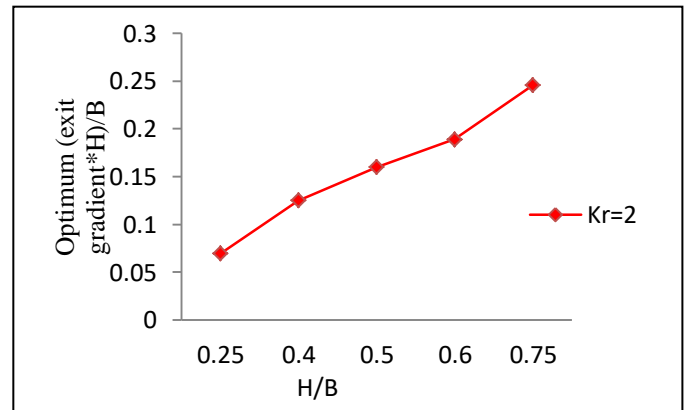
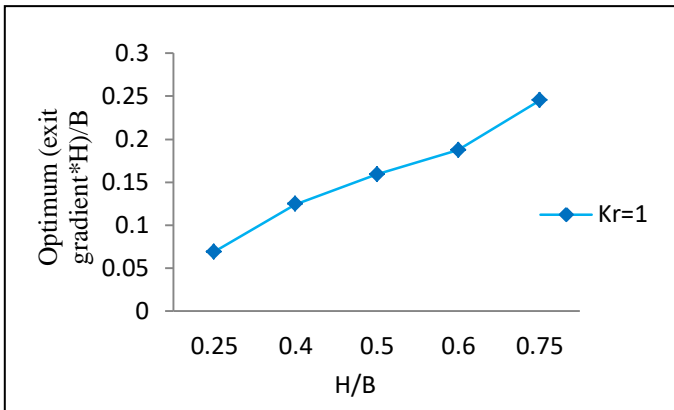


Figure (5-59):The optimum value of exit gradient ratio with head at different values of Kr

While Figure (5-60)) shows the optimum solution for uplift force corresponding to the relative head values. It is clear that the optimum uplift force increased when increasing the relative head values.

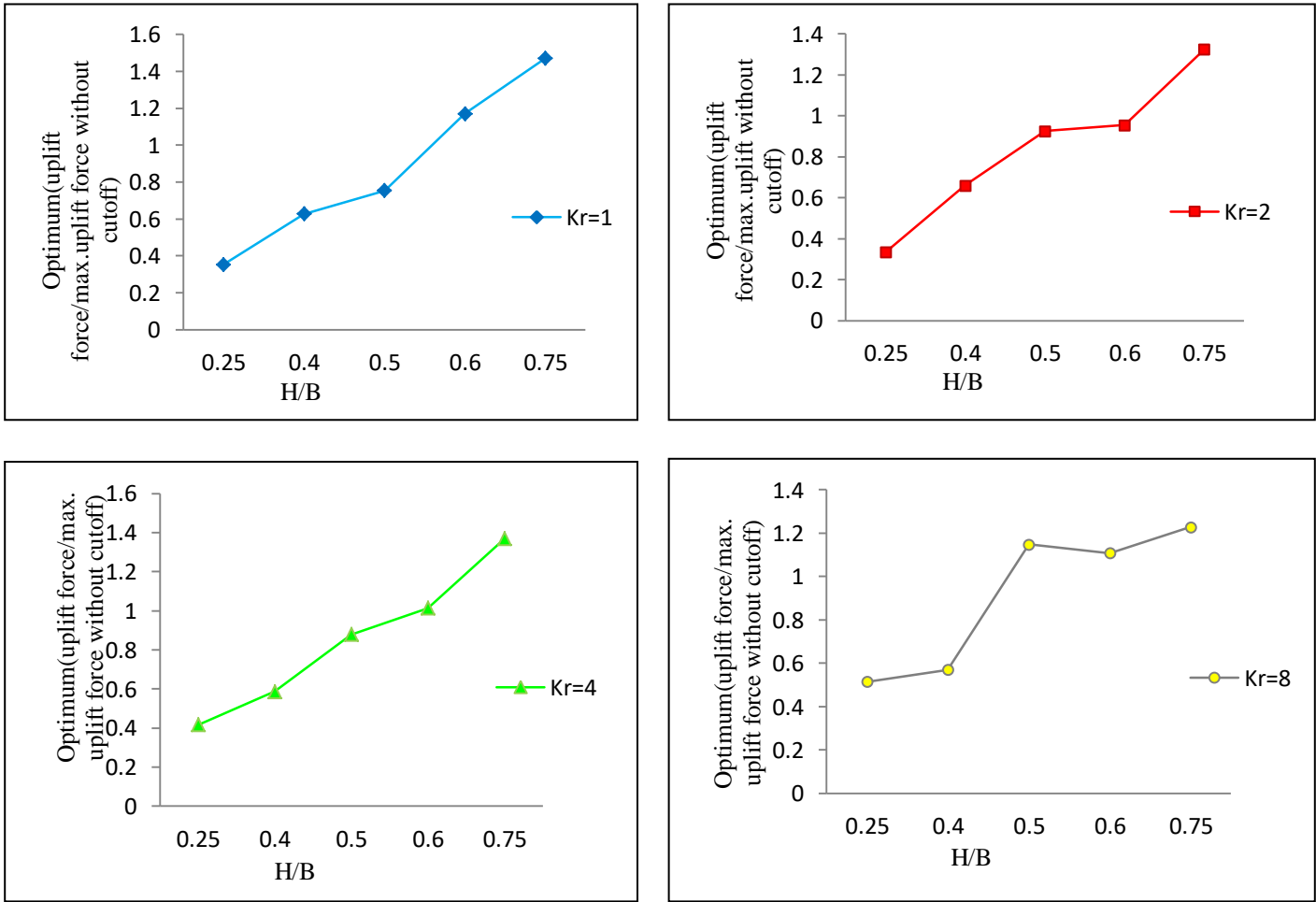


Figure (5-60):The optimum value of uplift force ratio with head at different values of Kr

Figure (5-61) shows the optimum solution obtained by using the GA-FDM model for minimum cost for the structure and cutoff includes the safety for exit gradient and uplift force which corresponding to the relative head values. It is

clear from the figure that the cost increased when increasing the relative head values and the isotropic degree.

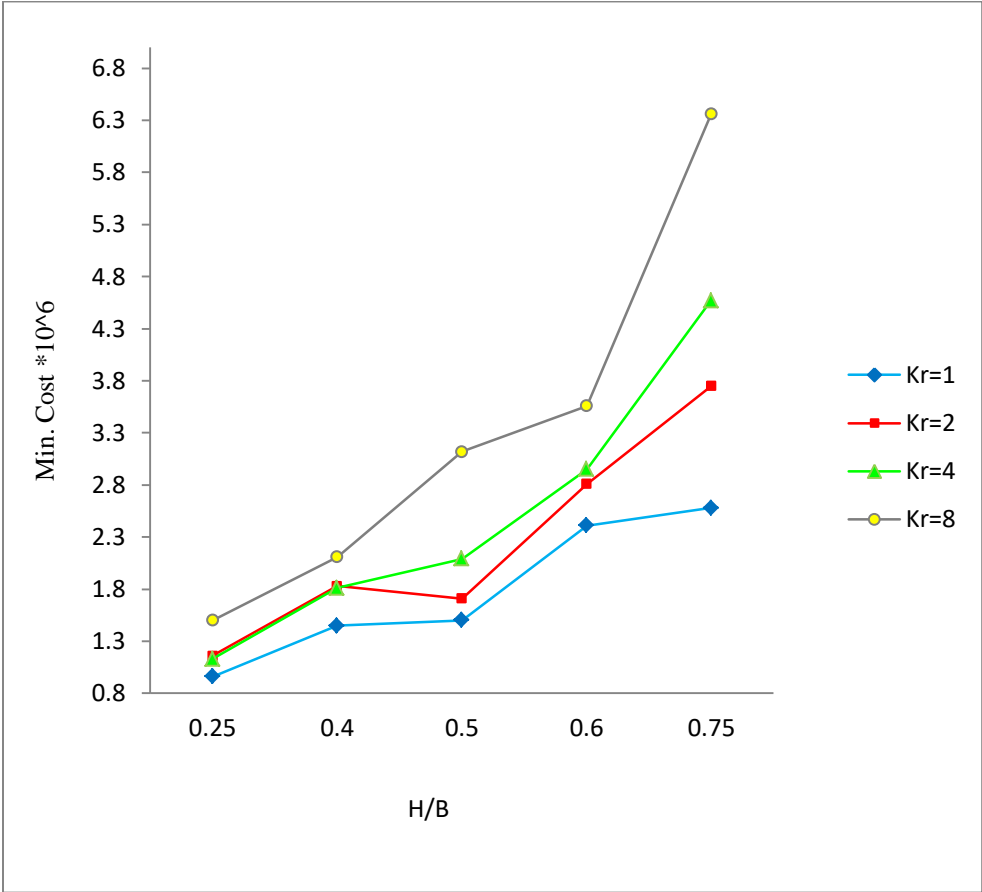


Figure (5-61):The optimum value of minimum cost with head at different values of Kr.

In order to compare the values of the obtained optimum solution using the GA model with the values obtained using finite difference code model for seeping (without cut off, one cutoff at U/S, one cutoff at D/S) these cases are shown in Table(5-5).

Table (5-5): Comparison of (specific exit gradient, uplift force and Min.cost) values between the optimum and with other solutions for relative head =0.75

Kr	case	(d/B)US	(d/B)DS	(iexit*H)/B	(F uplift/F*)	cost*10 ⁶	difference of cost %
Kr= 1	cutoff U/S	1.45	0	0.2298	0.1152	13.670	
	cutoff D/s	0	0.75	0.1837	1.682		not achieve the second constraint
	optimum	0.4	0.45	0.2322	1.0379	2.580	81.12
Kr= 2	cutoff U/S	1.45	0	0.234	0.0937	13.670	
	cutoff D/s	0	0.8	0.2351	1.7159		not achieve the uplift constraint
	optimum	0.3	0.6	0.2455	1.3242	3.750	72.56
Kr=4	cutoff U/S	1.45	0	0.2357	0.0788	13.670	
	cutoff D/s	0	0.825	0.2428	1.7975		not achieve the uplift constraint
	optimum	0.35	0.675	0.2455	1.3689	4.570	66.56
Kr=8	cutoff U/S	1.45	0	0.2432	0.0693	13.670	
	cutoff D/s	0	0.85	0.2485	1.8573		not achieve the uplift constraint
	optimum	0.575	0.725	0.2426	1.2286	6.360	53.47

These cases were modeled to find whether the obtained values of exit gradient and up lift force by the GA _FDP were comparable with factor of safety against piping and floating for the same values of (H/B) and Kr.

This table show three cases for each Kr, The first case that uses cut off at up stream for Kr=1 and continues of increasing the cutoff's depth until achieving the

two constraints with high cost 13.670×10^6 . The second case that uses cutoff at down stream does not achieve the second constraints (FS piping equal to 4 more than 3, FS uplift equal to 1.95 less than 2). The third case is the optimum solution which obtained by GA-FDP model where each of exit gradient and uplift force achieve the factor of safety against piping and uplift with min. cost of 2.580×10^6 .

For $Kr=2$, the first case that uses one cutoff at U/S and continues of increasing the cutoff's depth until achieving the two constraints with high cost of 13.670×10^6 . In the second case that uses cutoff at D/S, continues of increasing the cutoff's depth until the two constraints are not achieved but in this case not achieve the second constraint (FS uplift equal to 1.91 less than 2). The third case is the optimum solution which obtained by GA-FDP model where each of exit gradient and uplift force achieve the factor of safety against piping and uplift with min. cost of 3.750×10^6 .

For $Kr=4$, the second case are the same as in $Kr=2$, also the third case does not achieve the constraint related with uplift force. The optimum solution in fourth case which achieve the two constraints related with exit gradient and uplift force with min. cost of 4.570×10^6 .

For $Kr=8$, the first, and the second cases are the same as in $Kr=4$, The optimum solution in the third case which obtained by GA-FDP model that achieve the two constraints related with exit gradient and uplift force with min. cost of 6.360×10^6 .

It is clear from this table that the results which obtained from GA-FDP model is the optimum solution that achieves the two constraints (safety against exit gradient and uplift force) with minimum cost. The optimum solution which

reduces the cost by 81.12% ,72.56%, 66.56%, 53.47% from the cost of traditional solutions for Kr equal to (1,2,4, and 8), respectively.

Chapter Six

Conclusions and Recommendations

6.1 Conclusions

A Genetic Algorithm with Finite Difference Programming (GA-FDP) model is proposed for the optimal design of safe dam. This proposed model has fulfilled the optimum design task into two stages. Firstly, the Genetic Algorithm (GA) was applied to obtain the location and depth for cutoffs needed for the preliminary design of the Dam. Secondly, Finite difference Programming (FDP) was used to obtain the uplift pressure and exit gradient

A number of conclusions were reached by studying and analyzing the results as follows:

1. The result of verification of the FDP model with SEEP/W of calculating uplift force and exit gradient under the dam indicates a good validation between them, so this model successfully reproduces the outputs.
2. Using the cutoff at the upstream is very effective in reducing the uplift pressure, while the cutoff at the downstream is very effective in reducing the exit gradient. Moreover, the exit gradient decreases with increasing the cutoffs' location ratio and the depth ratio, while the uplift pressure decreases with increasing these ratios until $x=0.5B$, after this the behavior is reverse.
3. The result of finite difference model indicate that the ratio of differential head (H/B) had a significant effect on increasing the exit gradient and uplift force especially when the value of ratio ($H/B = 0.75$).

4. The minimum exit gradient was noticed when the cutoff location ratio at the downstream is of ($x_1/B=1$) with a maximum relative depth of ($d_1/B=0.6$) for $K_r=1$.
5. The minimum uplift pressure was observed when the cutoff location ratio at the upstream is of ($x_1/B=0$) with a minimum relative depth of ($d_1/B=0.1$).
6. The results indicate that the maximum exit gradient is observed when the length of the upstream cutoff's length to the length of downstream cutoff is ($d_1/d_2 = 1$). This maximum value of the exit gradient decreases as the ratio of (d_1/d_2) decreases.
7. Model result revealed that when there is a cut-off at the upstream, the uplift pressure is decreased along the base of floor. Also, the uplift distribution decreases with increasing the depth of cut-off because the cut-off causes an increase in the length of creep, which increases the head loss. Whenever a cut-off is located, a drop in the uplift pressure at that location is observed as expected. Also, the uplift pressure starts decreasing when using different lengths of cut-offs compared with the case of no cut-off; this behavior is reversed beyond the point where $x = 0.5$, this behavior occurs when using U/D cutoffs.
8. The optimum locations of cutoff 1 (X_1/B) that's achieve the safety against exit gradient and uplift force varied from (0 to 0.33) B, (0 to 0.24) B, (0 to 0.1) B and (0 to 0.18) B for K_r equal to (1,2,4, and 8) respectively, while the optimum locations of cutoff2 (X_2/B) varied from (0.875 to 1) B for various values of K_r . This behaviour is due to prevent increasing the uplift pressure and exit gradient of the structure.
9. The optimum depth of cutoff 1 (d_1/B) that's achieve the safety against exit gradient and uplift force varied from (0 to 0.35) B for K_r equal to (1, 2, 4), (0 to 0.6) B for K_r equal to 8 and to the depth of cutoff2 (d_2/B) varied from

(0.15 to 0.5) B, (0.1 to 0.6)B, (0.2 to 0.7)B and (0.3 to 0.7) B for Kr equal to (1,2,4, and 8) respectively .

10. The optimum solution which reduce the cost by 81.12% ,72.56%, 66.56%, 53.47% from the cost of traditional solutions for Kr equal to (1,2,4, and 8) respectively.
11. The results show that the GA-FDP model might be the most efficient for designing dam because its performance proved that it only needs a small number of generation, low effort, and time.

6.2 Recommendations

The following suggestions for future studies are proposed:

1. It is important to work laboratory experiments and check for optimum model.
2. Comparing the results of the study with other tools to control the leakage problem, especially in terms of cost.
3. It is recommended to take the inclination angle for cutoff and study its influence on exit gradient and uplift force.

References

- AL-SUHAILI, R. H. 2009. Analytical Solution for Exit Gradient Variation Downstream of Inclined Sheet Pile. The 6th Engineering Conference of Engineering College, College of Engineering, University of Baghdad, Iraq.
- AL-SUHAILI, R. H. & KARIM, R. A. 2014. Optimal dimensions of small hydraulic structure cutoffs using coupled genetic algorithm and ANN model. *Journal of Engineering*, 20, 1-19.
- ABBAS 1994. Conformal analysis of seepage below a hydraulic structure with an inclined cutoff. *International journal for numerical and analytical methods in geomechanics*, 18, 345-353.
- AHMED, A. A. 2012. Stochastic analysis of seepage under hydraulic structures resting on anisotropic heterogeneous soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 139, 1001-1004.
- AL- SAADI, S. I. K. & AL-FATLAWY, R. A. K. 2007. Effect of Degree of Anisotropy on Exit Gradient under Dams using the Finite Element Method. *M.Sc. Thesis*, Department of Civil Engineering, University of Kufa, Iraq.
- AL-SAAD, S. I. K., AL-DAMARCHI, H. T. N. & AL-ZREJAWI, H. C. 2011. Optimum location and angle of inclination of cut-off to control exit gradient and uplift pressure head under hydraulic structures. *Jordan Journal of Civil Engineering*, 159, 1-12.
- . ALJUBOORI, M. & DATTA, B. 2017. Artificial-Neural-Network-Modeling-and-Genetic-Algorithm-Based-Optimization-of-Hydraulic-Design-Related-to-Seepage-under-Concrete-Gravity-Dams-on-Permeable-Soils, World Academy of Science, Engineering and Technology International Journal of Civil and Environmental Engineering
- AL-JUBOORI, M., DATTA, B. J. J. O. A. W. E. & RESEARCH 2019. Performance evaluation of a genetic algorithm-based linked simulation-optimization model for optimal hydraulic seepage-related design of concrete gravity dams. *Journal of Applied Water Engineering Research*7, 173-197.
- AL-MUSSAWI, W. 2006. Optimum Design of Control Devices for Safe Seepage under Hydraulic Structures. *Journal of Engineering and Development*, 10.
- AL-ROB, M. F. A. A. 2016. Finite Difference and Finite Element Methods for Solving Elliptic Partial Differential Equations.
- ALSENOUSI, K. F. & MOHAMED, H. G. Effects of inclined cutoffs and soil foundation characteristics on seepage beneath hydraulic structures.

Twelfth international water technology conference, IWTC12, Alexandria, Egypt, 2008. 1597-1617.

- AZIZI, S., SALMASI, F., ABBASPOUR, A. & ARVANAGHI, H. 2012. Weep hole and cut-off effect in decreasing of uplift pressure, case study: Yusefkand Mahabad Diversion Dam. *Journal Civil Engineering Urban*, 2, 97-101.
- BEASLEY, D., BULL, D. R. & MARTIN, R. R. 1993. A Sequential Niche Technique for Multimodal Function Optimization. *Evolutionary Computation*, 1, 101-125.
- BENMEBAREK, N., BENMEBAREK, S., KASTNER, R. J. C. & GEOTECHNICS 2005. Numerical studies of seepage failure of sand within a cofferdam. 32, 264-273.
- CASAGRANDE 1940. Seepage through dams, in *Contributions to Soil Mechanics*. Boston Society of Civil Engineers.
- CAUSON, D. & MINGHAM, C. 2010. *Introductory finite difference methods for PDEs*, Bookboon.
- CEDERGREN HARRY, R. J. D., WILEY, F. N. N. J. & SONS, N. Y. 1977. *Seepage*.
- DAS 2008. " *Advanced Soil Mechanics* " Third Edition. Taylor and Francis.
- DND-GCKS 2017. Department of Networks Design in the General Commission of the Karbala Sewers.
- EL-JUMAILY, K. K., AL-BAKRY, H. M. J. J. E. & JOURNAL, T. 2013. Seepage analysis through and under hydraulic structures applying finite volume method. 31, 1719-1731.
- El-Sayed A.F.K and Ali A.R. " Experimental investigation of the effect of multi – Layers of soil under Hydraulic structures on seepage characteristics " bench university 2002
- GEN, M. & CHENG, R. 1999. *Genetic Algorithms and Engineering Optimization*, Wiley.
- GHOBADIAN, R., SHEKARI, H. & KOOCHAK, P. J. W. S. 2019. Model test and numerical investigation of the effect of the impervious layer's slope on seepage characteristics under hydraulic structures. 45, 141-148.
- GOLDBERG, D. E. D., K. 1991. *A Comparative Analysis of Selection Schemes Used in Genetic Algorithms*.
- HANCOCK, P. J. J. B. & ET AL. 2000. A comparison of selection mechanisms. 212-227.
- HARR 1962. *Ground water and seepage*.

- HASSAN, W. H. 2017. Location and Inclination Angle of a Cut-off Wall for Anisotropic foundation under Hydraulic Structures.
- HASSAN, W. H. 2019. Application of a genetic algorithm for the optimization of a location and inclination angle of a cut-off wall for anisotropic foundations under hydraulic structures. *Geotechnical and Geological Engineering* 37, 883-895.
- KARIM, R. A. J. R. O. D. O. I. & DRAINAGE, U. O. B. 1988. Variation of uplift pressure and exit gradient downstream of hydraulic structures. 1-34.
- LAMBE, T. W. & WHITMAN, R. V. 1991. Soil mechanics, John Wiley & Sons.
- MANSURI, B., SALMASI, F. & OGHATI, B. 2014. Effect of location and angle of cutoff wall on uplift pressure in diversion dam. *Geotechnical and Geological Engineering*, 32, 1165-1173.
- MANUAL, G. U. S. 2012. Geo-slope international Ltd. *Calgary, Alberta, Canada T2P 2Y5*.
- MOHARRAMI, A., MORADI, G., BONAB, M. H., KATEBI, J. & MOHARRAMI, G. 2015. Performance of cutoff walls under hydraulic structures against uplift pressure and piping phenomenon. *Geotechnical and Geological Engineering*, 33, 95-103.
- NASI 2005. Eurocode 7: Geotechnical design - Part 1: General rules.
- OBEAD, I. H. J. J. O. B. U. 2009. Design Charts for Sheet Piles-Downstream Blanket Systems. 7.
- POWERS, J. P., CORWIN, A. B., SCHMALL, P. C. & KAECK, W. E. 2007. Construction dewatering and groundwater control: new methods and applications, John Wiley & Sons.
- POWRIE, W. 2018. *Soil mechanics: concepts and applications*, CRC Press.
- RASOOL, M. H. J. K. J. O. E. 2018. Effect of mutual interference piles on seepage properties under hydraulic structures. 9, 273-285.
- SACHPAZIS, C. 2014. Experimental Conceptualisation of the Flow Net System Construction inside the Body of Homogeneous Earth Embankment Dams. *Electronic Journal of Geotechnical Engineering*, 19, 2114-2136.
- SINGH, R. M. Optimal hydraulic structures profiles under uncertain seepage head. World Renewable Energy Congress-Sweden; 8-13 May; 2011; Linköping; Sweden, 2011. Linköping University Electronic Press, 712-718.
- TERZAGHI, K. 1943. Theoretical Soil Mechanics.
- TERZAGHI, K., PECK, R. B. & MESRI, G. 1996. *Soil mechanics in engineering practice*, John Wiley & Sons.

Appendix A- Results of the specific uplift force and exit gradient

A.1- Table of results of ratio permeability in x-direction to permeability of y-direction ($K_x/K_y=1$)

Table (A-1): The Results obtained by using the ratio of ($\frac{K_x}{K_y}=1$) with the ratio of head differential($\frac{H}{B} = 0.25, 0.5, 0.75$)

Run No.	x1/B	d1/B	x2/B	d2/B	H/B	(iexit*H)/B	(f uplift/max f)	case
1	0	0	0	0	0.75	1.27	1.00	without cutoff
2	0	0.1	0	0	0.75	1.17	0.70	one cutoff
3	0.1	0.1	0	0	0.75	1.21	0.77	one cutoff
4	0.2	0.1	0	0	0.75	1.23	0.85	one cutoff
5	0.3	0.1	0	0	0.75	1.23	0.90	one cutoff
6	0.4	0.1	0	0	0.75	1.23	0.95	one cutoff
7	0.5	0.1	0	0	0.75	1.22	1.01	one cutoff
8	0.6	0.1	0	0	0.75	1.21	1.06	one cutoff
9	0.7	0.1	0	0	0.75	1.18	1.11	one cutoff
10	0.8	0.1	0	0	0.75	1.12	1.16	one cutoff
11	0.9	0.1	0	0	0.75	0.92	1.23	one cutoff
12	1	0.1	0	0	0.75	0.65	1.30	one cutoff
13	0	0.2	0	0	0.75	1.10	0.60	one cutoff
14	0.1	0.2	0	0	0.75	1.13	0.71	one cutoff
15	0.2	0.2	0	0	0.75	1.15	0.79	one cutoff
16	0.3	0.2	0	0	0.75	1.15	0.85	one cutoff
17	0.4	0.2	0	0	0.75	1.15	0.94	one cutoff
18	0.5	0.2	0	0	0.75	1.14	1.01	one cutoff
19	0.6	0.2	0	0	0.75	1.11	1.08	one cutoff
20	0.7	0.2	0	0	0.75	1.05	1.15	one cutoff
21	0.8	0.2	0	0	0.75	0.94	1.23	one cutoff
22	0.9	0.2	0	0	0.75	0.70	1.32	one cutoff
23	1	0.2	0	0	0.75	0.46	1.40	one cutoff
24	0	0.3	0	0	0.75	1.02	0.53	one cutoff
25	0.1	0.3	0	0	0.75	1.04	0.64	one cutoff
26	0.2	0.3	0	0	0.75	1.05	0.75	one cutoff
27	0.3	0.3	0	0	0.75	1.07	0.84	one cutoff
28	0.4	0.3	0	0	0.75	1.06	0.94	one cutoff

29	0.5	0.3	0	0	0.75	1.04	1.01	one cutoff
30	0.6	0.3	0	0	0.75	0.97	1.14	one cutoff
31	0.7	0.3	0	0	0.75	0.87	1.19	one cutoff
32	0.8	0.3	0	0	0.75	0.78	1.28	one cutoff
33	0.9	0.3	0	0	0.75	0.56	1.38	one cutoff
34	1	0.3	0	0	0.75	0.36	1.47	one cutoff
35	0	0.4	0	0	0.75	0.95	0.47	one cutoff
36	0.1	0.4	0	0	0.75	0.96	0.58	one cutoff
37	0.2	0.4	0	0	0.75	0.97	0.69	one cutoff
38	0.3	0.4	0	0	0.75	0.98	0.81	one cutoff
39	0.4	0.4	0	0	0.75	0.97	0.93	one cutoff
40	0.5	0.4	0	0	0.75	0.92	1.02	one cutoff
41	0.6	0.4	0	0	0.75	0.88	1.11	one cutoff
42	0.7	0.4	0	0	0.75	0.76	1.28	one cutoff
43	0.8	0.4	0	0	0.75	0.64	1.39	one cutoff
44	0.9	0.4	0	0	0.75	0.47	1.44	one cutoff
45	1	0.4	0	0	0.75	0.30	1.53	one cutoff
46	0	0.5	0	0	0.75	0.88	0.42	one cutoff
47	0.1	0.5	0	0	0.75	0.88	0.57	one cutoff
48	0.2	0.5	0	0	0.75	0.89	0.67	one cutoff
49	0.3	0.5	0	0	0.75	0.89	0.79	one cutoff
50	0.4	0.5	0	0	0.75	0.86	0.89	one cutoff
51	0.5	0.5	0	0	0.75	0.83	1.01	one cutoff
52	0.6	0.5	0	0	0.75	0.78	1.12	one cutoff
53	0.7	0.5	0	0	0.75	0.69	1.27	one cutoff
54	0.8	0.5	0	0	0.75	0.57	1.35	one cutoff
55	0.9	0.5	0	0	0.75	0.40	1.48	one cutoff
56	1	0.5	0	0	0.75	0.26	1.58	one cutoff
57	0	0.6	0	0	0.75	0.81	0.37	one cutoff
58	0.1	0.6	0	0	0.75	0.81	0.50	one cutoff
59	0.2	0.6	0	0	0.75	0.81	0.63	one cutoff
60	0.3	0.6	0	0	0.75	0.81	0.77	one cutoff
61	0.4	0.6	0	0	0.75	0.77	0.85	one cutoff
62	0.5	0.6	0	0	0.75	0.74	0.97	one cutoff
63	0.6	0.6	0	0	0.75	0.69	1.14	one cutoff
64	0.7	0.6	0	0	0.75	0.59	1.31	one cutoff
65	0.8	0.6	0	0	0.75	0.46	1.37	one cutoff
66	0.9	0.6	0	0	0.75	0.35	1.52	one cutoff
67	1	0.6	0	0	0.75	0.27	1.63	one cutoff
68	0	0.1	1	0.1	0.75	0.60	1.00	two cutoff

69	0	0.2	1	0.1	0.75	0.57	0.89	two cutoff
70	0	0.3	1	0.1	0.75	0.53	0.80	two cutoff
71	0	0.4	1	0.1	0.75	0.50	0.73	two cutoff
72	0	0.5	1	0.1	0.75	0.46	0.66	two cutoff
73	0	0.6	1	0.1	0.75	0.43	0.60	two cutoff
74	0	0.1	1	0.2	0.75	0.43	1.11	two cutoff
75	0	0.1	1	0.3	0.75	0.34	1.20	two cutoff
76	0	0.1	1	0.4	0.75	0.29	1.27	two cutoff
77	0	0.1	1	0.5	0.75	0.25	1.34	two cutoff
78	0	0.1	1	0.6	0.75	0.21	1.40	two cutoff
79	0.1	0.1	1	0.1	0.75	0.62	1.07	two cutoff
80	0.2	0.1	1	0.1	0.75	0.63	1.11	two cutoff
81	0.3	0.1	1	0.1	0.75	0.63	1.14	two cutoff
82	0.4	0.1	1	0.1	0.75	0.63	1.16	two cutoff
83	0.5	0.1	1	0.1	0.75	0.63	1.19	two cutoff
84	0.6	0.1	1	0.1	0.75	0.63	1.22	two cutoff
85	0.7	0.1	1	0.1	0.75	0.62	1.25	two cutoff
86	0.8	0.1	1	0.1	0.75	0.62	1.27	two cutoff
87	0.9	0.1	1	0.1	0.75	0.62	1.30	two cutoff
88	0.1	0.2	1	0.1	0.75	0.59	0.98	two cutoff
89	0.2	0.2	1	0.1	0.75	0.60	1.04	two cutoff
90	0.3	0.2	1	0.1	0.75	0.60	1.09	two cutoff
91	0.4	0.2	1	0.1	0.75	0.60	1.14	two cutoff
92	0.5	0.2	1	0.1	0.75	0.60	1.18	two cutoff
93	0.6	0.2	1	0.1	0.75	0.59	1.22	two cutoff
94	0.7	0.2	1	0.1	0.75	0.58	1.26	two cutoff
95	0.8	0.2	1	0.1	0.75	0.56	1.30	two cutoff
96	0.9	0.2	1	0.1	0.75	0.51	1.35	two cutoff
97	0.1	0.3	1	0.1	0.75	0.55	0.90	two cutoff
98	0.2	0.3	1	0.1	0.75	0.56	0.98	two cutoff
99	0.3	0.3	1	0.1	0.75	0.56	1.05	two cutoff
100	0.4	0.3	1	0.1	0.75	0.56	1.11	two cutoff
101	0.5	0.3	1	0.1	0.75	0.56	1.17	two cutoff
102	0.6	0.3	1	0.1	0.75	0.55	1.22	two cutoff
103	0.7	0.3	1	0.1	0.75	0.53	1.28	two cutoff
104	0.8	0.3	1	0.1	0.75	0.49	1.34	two cutoff
105	0.9	0.3	1	0.1	0.75	0.43	1.40	two cutoff
106	0.1	0.4	1	0.1	0.75	0.51	0.83	two cutoff
107	0.2	0.4	1	0.1	0.75	0.52	0.93	two cutoff
108	0.3	0.4	1	0.1	0.75	0.52	1.01	two cutoff

109	0.4	0.4	1	0.1	0.75	0.52	1.08	two cutoff
110	0.5	0.4	1	0.1	0.75	0.51	1.15	two cutoff
111	0.6	0.4	1	0.1	0.75	0.50	1.22	two cutoff
112	0.7	0.4	1	0.1	0.75	0.47	1.29	two cutoff
113	0.8	0.4	1	0.1	0.75	0.43	1.37	two cutoff
114	0.9	0.4	1	0.1	0.75	0.36	1.45	two cutoff
115	0.1	0.5	1	0.1	0.75	0.47	0.77	two cutoff
116	0.2	0.5	1	0.1	0.75	0.48	0.88	two cutoff
117	0.3	0.5	1	0.1	0.75	0.48	0.97	two cutoff
118	0.4	0.5	1	0.1	0.75	0.48	1.06	two cutoff
119	0.5	0.5	1	0.1	0.75	0.46	1.14	two cutoff
120	0.6	0.5	1	0.1	0.75	0.45	1.22	two cutoff
121	0.7	0.5	1	0.1	0.75	0.42	1.31	two cutoff
122	0.8	0.5	1	0.1	0.75	0.37	1.40	two cutoff
123	0.9	0.5	1	0.1	0.75	0.31	1.50	two cutoff
124	0.1	0.6	1	0.1	0.75	0.44	0.72	two cutoff
125	0.2	0.6	1	0.1	0.75	0.44	0.83	two cutoff
126	0.3	0.6	1	0.1	0.75	0.44	0.93	two cutoff
127	0.4	0.6	1	0.1	0.75	0.43	1.03	two cutoff
128	0.5	0.6	1	0.1	0.75	0.42	1.13	two cutoff
129	0.6	0.6	1	0.1	0.75	0.40	1.22	two cutoff
130	0.7	0.6	1	0.1	0.75	0.37	1.32	two cutoff
131	0.8	0.6	1	0.1	0.75	0.33	1.42	two cutoff
132	0.9	0.6	1	0.1	0.75	0.27	1.53	two cutoff
133	0.1	0.1	1	0.2	0.75	0.44	1.18	two cutoff
134	0.2	0.1	1	0.2	0.75	0.45	1.21	two cutoff
135	0.3	0.1	1	0.2	0.75	0.45	1.23	two cutoff
136	0.4	0.1	1	0.2	0.75	0.45	1.25	two cutoff
137	0.5	0.1	1	0.2	0.75	0.45	1.28	two cutoff
138	0.6	0.1	1	0.2	0.75	0.45	1.30	two cutoff
139	0.7	0.1	1	0.2	0.75	0.45	1.32	two cutoff
140	0.8	0.1	1	0.2	0.75	0.45	1.35	two cutoff
141	0.9	0.1	1	0.2	0.75	0.46	1.38	two cutoff
142	0.1	0.1	1	0.3	0.75	0.35	1.29	two cutoff
143	0.2	0.1	1	0.3	0.75	0.35	1.29	two cutoff
144	0.3	0.1	1	0.3	0.75	0.36	1.33	two cutoff
145	0.4	0.1	1	0.3	0.75	0.36	1.33	two cutoff
146	0.5	0.1	1	0.3	0.75	0.36	1.35	two cutoff
147	0.6	0.1	1	0.3	0.75	0.36	1.37	two cutoff
148	0.7	0.1	1	0.3	0.75	0.36	1.39	two cutoff

149	0.8	0.1	1	0.3	0.75	0.36	1.42	two cutoff
150	0.9	0.1	1	0.3	0.75	0.36	1.45	two cutoff
151	0.1	0.1	1	0.4	0.75	0.29	1.33	two cutoff
152	0.2	0.1	1	0.4	0.75	0.30	1.36	two cutoff
153	0.3	0.1	1	0.4	0.75	0.30	1.38	two cutoff
154	0.4	0.1	1	0.4	0.75	0.30	1.39	two cutoff
155	0.5	0.1	1	0.4	0.75	0.30	1.41	two cutoff
156	0.6	0.1	1	0.4	0.75	0.30	1.43	two cutoff
157	0.7	0.1	1	0.4	0.75	0.30	1.46	two cutoff
158	0.8	0.1	1	0.4	0.75	0.30	1.48	two cutoff
159	0.9	0.1	1	0.4	0.75	0.30	1.51	two cutoff
160	0.1	0.1	1	0.5	0.75	0.25	1.39	two cutoff
161	0.2	0.1	1	0.5	0.75	0.25	1.42	two cutoff
162	0.3	0.1	1	0.5	0.75	0.25	1.43	two cutoff
163	0.4	0.1	1	0.5	0.75	0.25	1.45	two cutoff
164	0.5	0.1	1	0.5	0.75	0.25	1.47	two cutoff
165	0.6	0.1	1	0.5	0.75	0.26	1.49	two cutoff
166	0.7	0.1	1	0.5	0.75	0.26	1.51	two cutoff
167	0.8	0.1	1	0.5	0.75	0.26	1.54	two cutoff
168	0.9	0.1	1	0.5	0.75	0.26	1.56	two cutoff
169	0.1	0.1	1	0.6	0.75	0.22	1.45	two cutoff
170	0.2	0.1	1	0.6	0.75	0.22	1.47	two cutoff
171	0.3	0.1	1	0.6	0.75	0.22	1.49	two cutoff
172	0.4	0.1	1	0.6	0.75	0.22	1.50	two cutoff
173	0.5	0.1	1	0.6	0.75	0.22	1.52	two cutoff
174	0.6	0.1	1	0.6	0.75	0.22	1.54	two cutoff
175	0.7	0.1	1	0.6	0.75	0.22	1.56	two cutoff
176	0.8	0.1	1	0.6	0.75	0.22	1.58	two cutoff
177	0.9	0.1	1	0.6	0.75	0.22	1.61	two cutoff
178	0	0.2	1	0.2	0.75	0.41	1.00	two cutoff
179	0.1	0.2	1	0.2	0.75	0.42	1.08	two cutoff
180	0.2	0.2	1	0.2	0.75	0.43	1.14	two cutoff
181	0.3	0.2	1	0.2	0.75	0.43	1.19	two cutoff
182	0.4	0.2	1	0.2	0.75	0.43	1.23	two cutoff
183	0.5	0.2	1	0.2	0.75	0.43	1.26	two cutoff
184	0.6	0.2	1	0.2	0.75	0.43	1.30	two cutoff
185	0.7	0.2	1	0.2	0.75	0.43	1.33	two cutoff
186	0.8	0.2	1	0.2	0.75	0.43	1.36	two cutoff
187	0.9	0.2	1	0.2	0.75	0.44	1.39	two cutoff
188	0	0.3	1	0.3	0.75	0.31	1.00	two cutoff

189	0.1	0.3	1	0.3	0.75	0.32	1.09	two cutoff
190	0.2	0.3	1	0.3	0.75	0.33	1.16	two cutoff
191	0.3	0.3	1	0.3	0.75	0.33	1.22	two cutoff
192	0.4	0.3	1	0.3	0.75	0.33	1.27	two cutoff
193	0.5	0.3	1	0.3	0.75	0.33	1.31	two cutoff
194	0.6	0.3	1	0.3	0.75	0.34	1.35	two cutoff
195	0.7	0.3	1	0.3	0.75	0.34	1.39	two cutoff
196	0.8	0.3	1	0.3	0.75	0.34	1.43	two cutoff
197	0.9	0.3	1	0.3	0.75	0.35	1.46	two cutoff
198	0	0.4	1	0.4	0.75	0.25	1.00	two cutoff
199	0.1	0.4	1	0.4	0.75	0.26	1.09	two cutoff
200	0.2	0.4	1	0.4	0.75	0.26	1.17	two cutoff
201	0.3	0.4	1	0.4	0.75	0.27	1.24	two cutoff
202	0.4	0.4	1	0.4	0.75	0.27	1.29	two cutoff
203	0.5	0.4	1	0.4	0.75	0.27	1.34	two cutoff
204	0.6	0.4	1	0.4	0.75	0.27	1.39	two cutoff
205	0.7	0.4	1	0.4	0.75	0.28	1.44	two cutoff
206	0.8	0.4	1	0.4	0.75	0.28	1.48	two cutoff
207	0.9	0.4	1	0.4	0.75	0.29	1.51	two cutoff
208	0	0.5	1	0.5	0.75	0.21	1.00	two cutoff
209	0.1	0.5	1	0.5	0.75	0.21	1.09	two cutoff
210	0.2	0.5	1	0.5	0.75	0.22	1.18	two cutoff
211	0.3	0.5	1	0.5	0.75	0.22	1.25	two cutoff
212	0.4	0.5	1	0.5	0.75	0.23	1.31	two cutoff
213	0.5	0.5	1	0.5	0.75	0.23	1.37	two cutoff
214	0.6	0.5	1	0.5	0.75	0.23	1.42	two cutoff
215	0.7	0.5	1	0.5	0.75	0.25	1.56	two cutoff
216	0.8	0.5	1	0.5	0.75	0.24	1.52	two cutoff
217	0.9	0.5	1	0.5	0.75	0.25	1.56	two cutoff
218	0	0.1	0.1	0.1	0.75	1.16	0.17	two cutoff
219	0	0.1	0.2	0.1	0.75	1.15	0.30	two cutoff
220	0	0.1	0.3	0.1	0.75	1.15	0.42	two cutoff
221	0	0.1	0.4	0.1	0.75	1.14	0.53	two cutoff
222	0	0.1	0.5	0.1	0.75	1.14	0.63	two cutoff
223	0	0.1	0.6	0.1	0.75	1.12	0.72	two cutoff
224	0	0.1	0.7	0.1	0.75	1.10	0.79	two cutoff
225	0	0.1	0.8	0.1	0.75	1.04	0.86	two cutoff
226	0	0.1	0.9	0.1	0.75	0.86	0.93	two cutoff
227	0	0.1	1	0.1	0.75	0.60	1.00	two cutoff
228	0	0.2	0.1	0.1	0.75	1.09	0.14	two cutoff

229	0	0.2	0.2	0.1	0.75	1.08	0.26	two cutoff
230	0	0.2	0.3	0.1	0.75	1.08	0.37	two cutoff
231	0	0.2	0.4	0.1	0.75	1.07	0.47	two cutoff
232	0	0.2	0.5	0.1	0.75	1.07	0.56	two cutoff
233	0	0.2	0.6	0.1	0.75	1.05	0.63	two cutoff
234	0	0.2	0.7	0.1	0.75	1.03	0.70	two cutoff
235	0	0.2	0.8	0.1	0.75	0.97	0.76	two cutoff
236	0	0.2	0.9	0.1	0.75	0.81	0.82	two cutoff
237	0	0.2	1	0.1	0.75	0.57	0.89	two cutoff
238	0	0.3	0.1	0.1	0.75	1.02	0.13	two cutoff
239	0	0.3	0.2	0.1	0.75	1.01	0.23	two cutoff
240	0	0.3	0.3	0.1	0.75	1.01	0.33	two cutoff
241	0	0.3	0.4	0.1	0.75	1.00	0.42	two cutoff
242	0	0.3	0.5	0.1	0.75	1.00	0.50	two cutoff
243	0	0.3	0.6	0.1	0.75	0.98	0.57	two cutoff
244	0	0.3	0.7	0.1	0.75	0.96	0.63	two cutoff
245	0	0.3	0.8	0.1	0.75	0.91	0.68	two cutoff
246	0	0.3	0.9	0.1	0.75	0.75	0.74	two cutoff
247	0	0.3	1	0.1	0.75	0.53	0.80	two cutoff
248	0	0.4	0.1	0.1	0.75	0.94	0.11	two cutoff
249	0	0.4	0.2	0.1	0.75	0.94	0.20	two cutoff
250	0	0.4	0.3	0.1	0.75	0.94	0.29	two cutoff
251	0	0.4	0.4	0.1	0.75	0.93	0.37	two cutoff
252	0	0.4	0.5	0.1	0.75	0.93	0.44	two cutoff
253	0	0.4	0.6	0.1	0.75	0.92	0.51	two cutoff
254	0	0.4	0.7	0.1	0.75	0.90	0.57	two cutoff
255	0	0.4	0.8	0.1	0.75	0.85	0.62	two cutoff
256	0	0.4	0.9	0.1	0.75	0.70	0.67	two cutoff
257	0	0.4	1	0.1	0.75	0.95	0.73	two cutoff
258	0	0.5	0.1	0.1	0.75	0.87	0.10	two cutoff
259	0	0.5	0.2	0.1	0.75	0.87	0.18	two cutoff
260	0	0.5	0.3	0.1	0.75	0.87	0.26	two cutoff
261	0	0.5	0.4	0.1	0.75	0.86	0.33	two cutoff
262	0	0.5	0.5	0.1	0.75	0.86	0.40	two cutoff
263	0	0.5	0.6	0.1	0.75	0.85	0.46	two cutoff
264	0	0.5	0.7	0.1	0.75	0.83	0.51	two cutoff
265	0	0.5	0.8	0.1	0.75	0.79	0.56	two cutoff
266	0	0.5	0.9	0.1	0.75	0.65	0.60	two cutoff
267	0	0.5	1	0.1	0.75	0.46	0.66	two cutoff
268	0	0.1	0.1	0.2	0.75	1.11	0.18	two cutoff

269	0	0.1	0.2	0.2	0.75	1.11	0.32	two cutoff
270	0	0.1	0.3	0.2	0.75	1.10	0.44	two cutoff
271	0	0.1	0.4	0.2	0.75	1.09	0.56	two cutoff
272	0	0.1	0.5	0.2	0.75	1.07	0.66	two cutoff
273	0	0.1	0.6	0.2	0.75	1.04	0.76	two cutoff
274	0	0.1	0.7	0.2	0.75	0.98	0.85	two cutoff
275	0	0.1	0.8	0.2	0.75	0.88	0.94	two cutoff
276	0	0.1	0.9	0.2	0.75	0.65	1.03	two cutoff
277	0	0.1	1	0.2	0.75	0.43	1.11	two cutoff
278	0	0.1	0.1	0.3	0.75	1.04	0.18	two cutoff
279	0	0.1	0.2	0.3	0.75	1.04	0.33	two cutoff
280	0	0.1	0.3	0.3	0.75	1.03	0.46	two cutoff
281	0	0.1	0.4	0.3	0.75	1.01	0.59	two cutoff
282	0	0.1	0.5	0.3	0.75	0.98	0.70	two cutoff
283	0	0.1	0.6	0.3	0.75	0.94	0.81	two cutoff
284	0	0.1	0.7	0.3	0.75	0.86	0.91	two cutoff
285	0	0.1	0.8	0.3	0.75	0.74	1.01	two cutoff
286	0	0.1	0.9	0.3	0.75	0.53	1.11	two cutoff
287	0	0.1	1	0.3	0.75	0.34	1.20	two cutoff
288	0	0.1	0.1	0.4	0.75	0.96	0.19	two cutoff
289	0	0.1	0.2	0.4	0.75	0.96	0.34	two cutoff
290	0	0.1	0.3	0.4	0.75	0.95	0.48	two cutoff
291	0	0.1	0.4	0.4	0.75	0.93	0.61	two cutoff
292	0	0.1	0.5	0.4	0.75	0.89	0.74	two cutoff
293	0	0.1	0.6	0.4	0.75	0.84	0.85	two cutoff
294	0	0.1	0.7	0.4	0.75	0.76	0.96	two cutoff
295	0	0.1	0.8	0.4	0.75	0.63	1.07	two cutoff
296	0	0.1	0.9	0.4	0.75	0.44	1.18	two cutoff
297	0	0.1	1	0.4	0.75	0.29	1.27	two cutoff
298	0	0.1	0.1	0.5	0.75	0.89	0.19	two cutoff
299	0	0.1	0.2	0.5	0.75	0.88	0.35	two cutoff
300	0	0.1	0.3	0.5	0.75	0.87	0.50	two cutoff
301	0	0.1	0.4	0.5	0.75	0.84	0.63	two cutoff
302	0	0.1	0.5	0.5	0.75	0.81	0.76	two cutoff
303	0	0.1	0.6	0.5	0.75	0.75	0.89	two cutoff
304	0	0.1	0.7	0.5	0.75	0.67	1.00	two cutoff
305	0	0.1	0.8	0.5	0.75	0.55	1.12	two cutoff
306	0	0.1	0.9	0.5	0.75	0.38	1.24	two cutoff
307	0	0.1	1	0.5	0.75	0.25	1.34	two cutoff
308	0	0.1	0.1	0.6	0.75	0.81	0.19	two cutoff

309	0	0.1	0.2	0.6	0.75	0.80	0.31	two cutoff
310	0	0.1	0.3	0.6	0.75	0.79	0.51	two cutoff
311	0	0.1	0.4	0.6	0.75	0.76	0.60	two cutoff
312	0	0.1	0.5	0.6	0.75	0.72	0.79	two cutoff
313	0	0.1	0.6	0.6	0.75	0.64	0.90	two cutoff
314	0	0.1	0.7	0.6	0.75	0.59	1.04	two cutoff
315	0	0.1	0.8	0.6	0.75	0.46	1.23	two cutoff
316	0	0.1	0.9	0.6	0.75	0.33	1.29	two cutoff
317	0	0.1	1	0.6	0.75	0.21	1.40	two cutoff
318	0	0.2	0.1	0.2	0.75	1.07	0.16	two cutoff
319	0	0.2	0.2	0.2	0.75	1.06	0.28	two cutoff
320	0	0.2	0.3	0.2	0.75	1.04	0.39	two cutoff
321	0	0.2	0.4	0.2	0.75	1.03	0.50	two cutoff
322	0	0.2	0.5	0.2	0.75	1.01	0.59	two cutoff
323	0	0.2	0.6	0.2	0.75	0.98	0.68	two cutoff
324	0	0.2	0.7	0.2	0.75	0.93	0.76	two cutoff
325	0	0.2	0.8	0.2	0.75	0.83	0.84	two cutoff
326	0	0.2	0.9	0.2	0.75	0.62	0.92	two cutoff
327	0	0.2	1	0.2	0.75	0.41	1.00	two cutoff
328	0	0.3	0.1	0.3	0.75	0.99	0.15	two cutoff
329	0	0.3	0.2	0.3	0.75	0.97	0.27	two cutoff
330	0	0.3	0.3	0.3	0.75	0.95	0.37	two cutoff
331	0	0.3	0.4	0.3	0.75	0.92	0.48	two cutoff
332	0	0.3	0.5	0.3	0.75	0.89	0.57	two cutoff
333	0	0.3	0.6	0.3	0.75	0.85	0.66	two cutoff
334	0	0.3	0.7	0.3	0.75	0.78	0.74	two cutoff
335	0	0.3	0.8	0.3	0.75	0.67	0.83	two cutoff
336	0	0.3	0.9	0.3	0.75	0.48	0.92	two cutoff
337	0	0.3	1	0.3	0.75	0.31	1.00	two cutoff
338	0	0.4	0.1	0.4	0.75	0.91	0.14	two cutoff
339	0	0.4	0.2	0.4	0.75	0.88	0.26	two cutoff
340	0	0.4	0.3	0.4	0.75	0.85	0.36	two cutoff
341	0	0.4	0.4	0.4	0.75	0.82	0.46	two cutoff
342	0	0.4	0.5	0.4	0.75	0.78	0.56	two cutoff
343	0	0.4	0.6	0.4	0.75	0.73	0.65	two cutoff
344	0	0.4	0.7	0.4	0.75	0.66	0.73	two cutoff
345	0	0.4	0.8	0.4	0.75	0.55	0.82	two cutoff
346	0	0.4	0.9	0.4	0.75	0.39	0.92	two cutoff
347	0	0.4	1	0.4	0.75	0.25	1.00	two cutoff
348	0	0.5	0.1	0.5	0.75	0.83	0.14	two cutoff

349	0	0.5	0.2	0.5	0.75	0.80	0.25	two cutoff
350	0	0.5	0.3	0.5	0.75	0.77	0.35	two cutoff
351	0	0.5	0.4	0.5	0.75	0.73	0.45	two cutoff
352	0	0.5	0.5	0.5	0.75	0.69	0.55	two cutoff
353	0	0.5	0.6	0.5	0.75	0.64	0.64	two cutoff
354	0	0.5	0.7	0.5	0.75	0.56	0.73	two cutoff
355	0	0.5	0.8	0.5	0.75	0.46	0.82	two cutoff
356	0	0.5	0.9	0.5	0.75	0.32	0.91	two cutoff
357	0	0.5	1	0.5	0.75	0.21	1.00	two cutoff
358	0	0.6	0.1	0.6	0.75	0.76	0.13	two cutoff
359	0	0.6	0.2	0.6	0.75	0.73	0.24	two cutoff
360	0	0.6	0.3	0.6	0.75	0.69	0.34	two cutoff
361	0	0.6	0.4	0.6	0.75	0.65	0.44	two cutoff
362	0	0.6	0.5	0.6	0.75	0.61	0.54	two cutoff
363	0	0.6	0.6	0.6	0.75	0.55	0.63	two cutoff
364	0	0.6	0.7	0.6	0.75	0.49	0.72	two cutoff
365	0	0.6	0.8	0.6	0.75	0.40	0.82	two cutoff
366	0	0.6	0.9	0.6	0.75	0.40	0.82	two cutoff
367	0	0.6	1	0.6	0.75	0.18	1.00	two cutoff
369	0	0.1	0	0	0.5	0.52	0.47	one cutoff
370	0.1	0.1	0	0	0.5	0.54	0.53	one cutoff
371	0.2	0.1	0	0	0.5	0.54	0.57	one cutoff
372	0.3	0.1	0	0	0.5	0.55	0.60	one cutoff
373	0.4	0.1	0	0	0.5	0.55	0.64	one cutoff
374	0.5	0.1	0	0	0.5	0.54	0.67	one cutoff
375	0.6	0.1	0	0	0.5	0.54	0.71	one cutoff
376	0.7	0.1	0	0	0.5	0.52	0.74	one cutoff
377	0.8	0.1	0	0	0.5	0.50	0.78	one cutoff
378	0.9	0.1	0	0	0.5	0.41	0.82	one cutoff
379	1	0.1	0	0	0.5	0.29	0.87	one cutoff
380	0	0.2	0	0	0.5	0.49	0.40	one cutoff
381	0.1	0.2	0	0	0.5	0.50	0.47	one cutoff
382	0.2	0.2	0	0	0.5	0.51	0.53	one cutoff
383	0.3	0.2	0	0	0.5	0.52	0.58	one cutoff
384	0.4	0.2	0	0	0.5	0.51	0.63	one cutoff
385	0.5	0.2	0	0	0.5	0.51	0.67	one cutoff
386	0.6	0.2	0	0	0.5	0.49	0.72	one cutoff
387	0.7	0.2	0	0	0.5	0.47	0.77	one cutoff
388	0.8	0.2	0	0	0.5	0.42	0.82	one cutoff
389	0.9	0.2	0	0	0.5	0.31	0.88	one cutoff

390	1	0.2	0	0	0.5	0.20	0.94	one cutoff
391	0	0.3	0	0	0.5	0.45	0.35	one cutoff
392	0.1	0.3	0	0	0.5	0.47	0.43	one cutoff
393	0.2	0.3	0	0	0.5	0.48	0.50	one cutoff
394	0.3	0.3	0	0	0.5	0.48	0.56	one cutoff
395	0.4	0.3	0	0	0.5	0.47	0.62	one cutoff
396	0.5	0.3	0	0	0.5	0.46	0.67	one cutoff
397	0.6	0.3	0	0	0.5	0.44	0.73	one cutoff
398	0.7	0.3	0	0	0.5	0.41	0.79	one cutoff
399	0.8	0.3	0	0	0.5	0.35	0.85	one cutoff
400	0.9	0.3	0	0	0.5	0.25	0.92	one cutoff
401	1	0.3	0	0	0.5	0.17	0.98	one cutoff
402	0	0.4	0	0	0.5	0.42	0.31	one cutoff
403	0.1	0.4	0	0	0.5	0.43	0.39	one cutoff
404	0.2	0.4	0	0	0.5	0.44	0.47	one cutoff
405	0.3	0.4	0	0	0.5	0.44	0.54	one cutoff
406	0.4	0.4	0	0	0.5	0.43	0.61	one cutoff
407	0.5	0.4	0	0	0.5	0.41	0.67	one cutoff
408	0.6	0.4	0	0	0.5	0.39	0.74	one cutoff
409	0.7	0.4	0	0	0.5	0.35	0.81	one cutoff
410	0.8	0.4	0	0	0.5	0.30	0.88	one cutoff
411	0.9	0.4	0	0	0.5	0.21	0.96	one cutoff
412	1	0.4	0	0	0.5	0.13	1.02	one cutoff
413	0	0.5	0	0	0.5	0.39	0.28	one cutoff
414	0.1	0.5	0	0	0.5	0.40	0.37	one cutoff
415	0.2	0.5	0	0	0.5	0.40	0.45	one cutoff
416	0.3	0.5	0	0	0.5	0.40	0.53	one cutoff
417	0.4	0.5	0	0	0.5	0.39	0.60	one cutoff
418	0.5	0.5	0	0	0.5	0.37	0.68	one cutoff
419	0.6	0.5	0	0	0.5	0.35	0.75	one cutoff
420	0.7	0.5	0	0	0.5	0.31	0.83	one cutoff
421	0.8	0.5	0	0	0.5	0.26	0.90	one cutoff
422	0.9	0.5	0	0	0.5	0.18	0.99	one cutoff
423	1	0.5	0	0	0.5	0.11	1.06	one cutoff
424	0	0.6	0	0	0.5	0.36	0.25	one cutoff
425	0.1	0.6	0	0	0.5	0.36	0.34	one cutoff
426	0.2	0.6	0	0	0.5	0.36	0.43	one cutoff
427	0.3	0.6	0	0	0.5	0.36	0.51	one cutoff
428	0.4	0.6	0	0	0.5	0.35	0.60	one cutoff
429	0.5	0.6	0	0	0.5	0.35	0.68	one cutoff

430	0.6	0.6	0	0	0.5	0.31	0.76	one cutoff
431	0.7	0.6	0	0	0.5	0.27	0.84	one cutoff
432	0.8	0.6	0	0	0.5	0.22	0.93	one cutoff
433	0.9	0.6	0	0	0.5	0.15	1.01	one cutoff
434	1	0.6	0	0	0.5	0.10	1.08	one cutoff
435	0	0.1	1	0.1	0.5	0.27	0.67	two cutoff
436	0	0.2	1	0.1	0.5	0.25	0.59	two cutoff
437	0	0.3	1	0.1	0.5	0.24	0.53	two cutoff
438	0	0.4	1	0.1	0.5	0.22	0.48	two cutoff
439	0	0.5	1	0.1	0.5	0.21	0.44	two cutoff
440	0	0.1	1	0.2	0.5	0.19	0.74	two cutoff
441	0	0.1	1	0.3	0.5	0.15	0.80	two cutoff
442	0	0.1	1	0.4	0.5	0.13	0.85	two cutoff
443	0	0.1	1	0.5	0.5	0.11	0.89	two cutoff
444	0.1	0.1	1	0.1	0.5	0.28	0.71	two cutoff
445	0.2	0.1	1	0.1	0.5	0.28	0.74	two cutoff
446	0.3	0.1	1	0.1	0.5	0.28	0.76	two cutoff
447	0.4	0.1	1	0.1	0.5	0.28	0.78	two cutoff
448	0.5	0.1	1	0.1	0.5	0.28	0.79	two cutoff
449	0.6	0.1	1	0.1	0.5	0.28	0.81	two cutoff
450	0.7	0.1	1	0.1	0.5	0.28	0.83	two cutoff
451	0.8	0.1	1	0.1	0.5	0.28	0.85	two cutoff
452	0.9	0.1	1	0.1	0.5	0.28	0.86	two cutoff
453	0.1	0.2	1	0.1	0.5	0.26	0.65	two cutoff
454	0.2	0.2	1	0.1	0.5	0.27	0.70	two cutoff
455	0.3	0.2	1	0.1	0.5	0.27	0.73	two cutoff
456	0.4	0.2	1	0.1	0.5	0.27	0.76	two cutoff
457	0.5	0.2	1	0.1	0.5	0.27	0.79	two cutoff
458	0.6	0.2	1	0.1	0.5	0.26	0.81	two cutoff
459	0.7	0.2	1	0.1	0.5	0.26	0.84	two cutoff
460	0.8	0.2	1	0.1	0.5	0.25	0.87	two cutoff
461	0.9	0.2	1	0.1	0.5	0.23	0.90	two cutoff
462	0.1	0.3	1	0.1	0.5	0.24	0.60	two cutoff
463	0.2	0.3	1	0.1	0.5	0.25	0.65	two cutoff
464	0.3	0.3	1	0.1	0.5	0.25	0.70	two cutoff
465	0.4	0.3	1	0.1	0.5	0.25	0.74	two cutoff
466	0.5	0.3	1	0.1	0.5	0.25	0.78	two cutoff
467	0.6	0.3	1	0.1	0.5	0.24	0.81	two cutoff
468	0.7	0.3	1	0.1	0.5	0.23	0.85	two cutoff
469	0.8	0.3	1	0.1	0.5	0.22	0.89	two cutoff

470	0.9	0.3	1	0.1	0.5	0.19	0.94	two cutoff
471	0.1	0.4	1	0.1	0.5	0.23	0.56	two cutoff
472	0.2	0.4	1	0.1	0.5	0.23	0.62	two cutoff
473	0.3	0.4	1	0.1	0.5	0.23	0.67	two cutoff
474	0.4	0.4	1	0.1	0.5	0.23	0.72	two cutoff
475	0.5	0.4	1	0.1	0.5	0.23	0.77	two cutoff
476	0.6	0.4	1	0.1	0.5	0.22	0.81	two cutoff
477	0.7	0.4	1	0.1	0.5	0.21	0.86	two cutoff
478	0.8	0.4	1	0.1	0.5	0.19	0.91	two cutoff
479	0.9	0.4	1	0.1	0.5	0.16	0.97	two cutoff
480	0.1	0.5	1	0.1	0.5	0.21	0.52	two cutoff
481	0.2	0.5	1	0.1	0.5	0.21	0.58	two cutoff
482	0.3	0.5	1	0.1	0.5	0.21	0.65	two cutoff
483	0.4	0.5	1	0.1	0.5	0.21	0.70	two cutoff
484	0.5	0.5	1	0.1	0.5	0.21	0.76	two cutoff
485	0.6	0.5	1	0.1	0.5	0.20	0.81	two cutoff
486	0.7	0.5	1	0.1	0.5	0.19	0.87	two cutoff
487	0.8	0.5	1	0.1	0.5	0.17	0.93	two cutoff
488	0.9	0.5	1	0.1	0.5	0.14	1.00	two cutoff
489	0.1	0.1	1	0.2	0.5	0.20	0.78	two cutoff
490	0.2	0.1	1	0.2	0.5	0.20	0.80	two cutoff
491	0.3	0.1	1	0.2	0.5	0.20	0.82	two cutoff
492	0.4	0.1	1	0.2	0.5	0.20	0.83	two cutoff
493	0.5	0.1	1	0.2	0.5	0.20	0.85	two cutoff
494	0.6	0.1	1	0.2	0.5	0.20	0.87	two cutoff
495	0.7	0.1	1	0.2	0.5	0.20	0.88	two cutoff
496	0.8	0.1	1	0.2	0.5	0.20	0.90	two cutoff
497	0.9	0.1	1	0.2	0.5	0.20	0.92	two cutoff
498	0.1	0.1	1	0.3	0.5	0.16	0.84	two cutoff
499	0.2	0.1	1	0.3	0.5	0.16	0.86	two cutoff
500	0.3	0.1	1	0.3	0.5	0.16	0.87	two cutoff
501	0.4	0.1	1	0.3	0.5	0.16	0.89	two cutoff
502	0.5	0.1	1	0.3	0.5	0.16	0.90	two cutoff
503	0.6	0.1	1	0.3	0.5	0.16	0.91	two cutoff
504	0.7	0.1	1	0.3	0.5	0.16	0.93	two cutoff
505	0.8	0.1	1	0.3	0.5	0.16	0.95	two cutoff
506	0.9	0.1	1	0.3	0.5	0.16	0.97	two cutoff
507	0.1	0.1	1	0.4	0.5	0.13	0.89	two cutoff
508	0.2	0.1	1	0.4	0.5	0.13	0.90	two cutoff
509	0.3	0.1	1	0.4	0.5	0.13	0.92	two cutoff

510	0.4	0.1	1	0.4	0.5	0.13	0.93	two cutoff
511	0.5	0.1	1	0.4	0.5	0.13	0.94	two cutoff
512	0.6	0.1	1	0.4	0.5	0.13	0.96	two cutoff
513	0.7	0.1	1	0.4	0.5	0.13	0.97	two cutoff
514	0.8	0.1	1	0.4	0.5	0.13	0.99	two cutoff
515	0.9	0.1	1	0.4	0.5	0.13	1.01	two cutoff
516	0.1	0.1	1	0.5	0.5	0.11	0.93	two cutoff
517	0.2	0.1	1	0.5	0.5	0.11	0.95	two cutoff
518	0.3	0.1	1	0.5	0.5	0.11	0.96	two cutoff
519	0.4	0.1	1	0.5	0.5	0.11	0.97	two cutoff
520	0.5	0.1	1	0.5	0.5	0.11	0.98	two cutoff
521	0.6	0.1	1	0.5	0.5	0.11	0.99	two cutoff
522	0.7	0.1	1	0.5	0.5	0.11	1.01	two cutoff
523	0.8	0.1	1	0.5	0.5	0.11	1.02	two cutoff
524	0.9	0.1	1	0.5	0.5	0.11	1.04	two cutoff
525	0	0.2	1	0.2	0.5	0.18	0.67	two cutoff
526	0.1	0.2	1	0.2	0.5	0.19	0.72	two cutoff
527	0.2	0.2	1	0.2	0.5	0.19	0.76	two cutoff
528	0.3	0.2	1	0.2	0.5	0.19	0.79	two cutoff
529	0.4	0.2	1	0.2	0.5	0.19	0.82	two cutoff
530	0.5	0.2	1	0.2	0.5	0.19	0.84	two cutoff
531	0.6	0.2	1	0.2	0.5	0.19	0.86	two cutoff
532	0.7	0.2	1	0.2	0.5	0.19	0.89	two cutoff
533	0.8	0.2	1	0.2	0.5	0.19	0.91	two cutoff
534	0.9	0.2	1	0.2	0.5	0.19	0.93	two cutoff
535	0	0.3	1	0.3	0.5	0.14	0.67	two cutoff
536	0.1	0.3	1	0.3	0.5	0.14	0.73	two cutoff
537	0.2	0.3	1	0.3	0.5	0.14	0.77	two cutoff
538	0.3	0.3	1	0.3	0.5	0.15	0.81	two cutoff
539	0.4	0.3	1	0.3	0.5	0.15	0.84	two cutoff
540	0.5	0.3	1	0.3	0.5	0.15	0.87	two cutoff
541	0.6	0.3	1	0.3	0.5	0.15	0.90	two cutoff
542	0.7	0.3	1	0.3	0.5	0.16	0.92	two cutoff
543	0.8	0.3	1	0.3	0.5	0.15	0.95	two cutoff
544	0.9	0.3	1	0.3	0.5	0.15	0.97	two cutoff
545	0	0.4	1	0.4	0.5	0.11	0.67	two cutoff
546	0.1	0.4	1	0.4	0.5	0.12	0.73	two cutoff
547	0.2	0.4	1	0.4	0.5	0.12	0.78	two cutoff
548	0.3	0.4	1	0.4	0.5	0.12	0.80	two cutoff
549	0.4	0.4	1	0.4	0.5	0.12	0.86	two cutoff

550	0.5	0.4	1	0.4	0.5	0.12	0.90	two cutoff
551	0.6	0.4	1	0.4	0.5	0.12	0.92	two cutoff
552	0.7	0.4	1	0.4	0.5	0.12	0.95	two cutoff
553	0.8	0.4	1	0.4	0.5	0.12	0.97	two cutoff
554	0.9	0.4	1	0.4	0.5	0.13	1.01	two cutoff
555	0	0.5	1	0.5	0.5	0.09	0.67	two cutoff
556	0.1	0.5	1	0.5	0.5	0.09	0.75	two cutoff
557	0.2	0.5	1	0.5	0.5	0.10	0.82	two cutoff
558	0.3	0.5	1	0.5	0.5	0.10	0.83	two cutoff
559	0.4	0.5	1	0.5	0.5	0.10	0.88	two cutoff
560	0.5	0.5	1	0.5	0.5	0.10	0.90	two cutoff
561	0.6	0.5	1	0.5	0.5	0.10	0.95	two cutoff
562	0.7	0.5	1	0.5	0.5	0.10	0.99	two cutoff
563	0.8	0.5	1	0.5	0.5	0.11	1.01	two cutoff
564	0.9	0.5	1	0.5	0.5	0.11	1.04	two cutoff
565	0	0.1	0.1	0.1	0.5	0.98	0.21	two cutoff
566	0	0.1	0.2	0.1	0.5	0.82	0.28	two cutoff
567	0	0.1	0.3	0.1	0.5	0.66	0.30	two cutoff
568	0	0.1	0.4	0.1	0.5	0.51	0.35	two cutoff
569	0	0.1	0.5	0.1	0.5	0.50	0.42	two cutoff
570	0	0.1	0.6	0.1	0.5	0.50	0.48	two cutoff
571	0	0.1	0.7	0.1	0.5	0.48	0.54	two cutoff
572	0	0.1	0.8	0.1	0.5	0.46	0.57	two cutoff
573	0	0.1	0.9	0.1	0.5	0.38	0.63	two cutoff
574	0	0.1	1	0.1	0.5	0.27	0.67	two cutoff
575	0	0.2	0.1	0.1	0.5	0.48	0.10	two cutoff
576	0	0.2	0.2	0.1	0.5	0.48	0.10	two cutoff
577	0	0.2	0.3	0.1	0.5	0.48	0.25	two cutoff
578	0	0.2	0.4	0.1	0.5	0.48	0.29	two cutoff
579	0	0.2	0.5	0.1	0.5	0.47	0.37	two cutoff
580	0	0.2	0.6	0.1	0.5	0.47	0.42	two cutoff
581	0	0.2	0.7	0.1	0.5	0.47	0.47	two cutoff
582	0	0.2	0.8	0.1	0.5	0.43	0.51	two cutoff
583	0	0.2	0.9	0.1	0.5	0.36	0.55	two cutoff
584	0	0.2	1	0.1	0.5	0.25	0.59	two cutoff
585	0	0.3	0.1	0.1	0.5	0.45	0.08	two cutoff
586	0	0.3	0.2	0.1	0.5	0.45	0.15	two cutoff
587	0	0.3	0.3	0.1	0.5	0.45	0.22	two cutoff
588	0	0.3	0.4	0.1	0.5	0.45	0.28	two cutoff
589	0	0.3	0.5	0.1	0.5	0.44	0.33	two cutoff

590	0	0.3	0.6	0.1	0.5	0.44	0.38	two cutoff
591	0	0.3	0.7	0.1	0.5	0.43	0.41	two cutoff
592	0	0.3	0.8	0.1	0.5	0.40	0.46	two cutoff
593	0	0.3	0.9	0.1	0.5	0.30	0.49	two cutoff
594	0	0.3	1	0.1	0.5	0.24	0.53	two cutoff
595	0	0.4	0.1	0.1	0.5	0.42	0.08	two cutoff
596	0	0.4	0.2	0.1	0.5	0.42	0.13	two cutoff
597	0	0.4	0.3	0.1	0.5	0.42	0.19	two cutoff
598	0	0.4	0.4	0.1	0.5	0.41	0.25	two cutoff
599	0	0.4	0.5	0.1	0.5	0.41	0.32	two cutoff
600	0	0.4	0.6	0.1	0.5	0.41	0.34	two cutoff
601	0	0.4	0.7	0.1	0.5	0.40	0.38	two cutoff
602	0	0.4	0.8	0.1	0.5	0.36	0.42	two cutoff
603	0	0.4	0.9	0.1	0.5	0.31	0.44	two cutoff
604	0	0.4	1	0.1	0.5	0.22	0.48	two cutoff
605	0	0.5	0.1	0.1	0.5	0.39	0.07	two cutoff
606	0	0.5	0.2	0.1	0.5	0.39	0.13	two cutoff
607	0	0.5	0.3	0.1	0.5	0.39	0.17	two cutoff
608	0	0.5	0.4	0.1	0.5	0.39	0.21	two cutoff
609	0	0.5	0.5	0.1	0.5	0.38	0.27	two cutoff
610	0	0.5	0.6	0.1	0.5	0.38	0.31	two cutoff
611	0	0.5	0.7	0.1	0.5	0.36	0.34	two cutoff
612	0	0.5	0.8	0.1	0.5	0.35	0.37	two cutoff
613	0	0.5	0.9	0.1	0.5	0.27	0.41	two cutoff
614	0	0.5	1	0.1	0.5	0.21	0.44	two cutoff
615	0	0.1	0.1	0.2	0.5	0.49	0.12	two cutoff
616	0	0.1	0.2	0.2	0.5	0.49	0.22	two cutoff
617	0	0.1	0.3	0.2	0.5	0.49	0.29	two cutoff
618	0	0.1	0.4	0.2	0.5	0.48	0.37	two cutoff
619	0	0.1	0.5	0.2	0.5	0.48	0.44	two cutoff
620	0	0.1	0.6	0.2	0.5	0.46	0.51	two cutoff
621	0	0.1	0.7	0.2	0.5	0.44	0.60	two cutoff
622	0	0.1	0.8	0.2	0.5	0.39	0.62	two cutoff
623	0	0.1	0.9	0.2	0.5	0.27	0.67	two cutoff
624	0	0.1	1	0.2	0.5	0.15	0.80	two cutoff
625	0	0.1	0.1	0.3	0.5	0.46	0.12	two cutoff
626	0	0.1	0.2	0.3	0.5	0.46	0.21	two cutoff
627	0	0.1	0.3	0.3	0.5	0.46	0.31	two cutoff
628	0	0.1	0.4	0.3	0.5	0.45	0.36	two cutoff
629	0	0.1	0.5	0.3	0.5	0.44	0.47	two cutoff

630	0	0.1	0.6	0.3	0.5	0.42	0.54	two cutoff
631	0	0.1	0.7	0.3	0.5	0.39	0.58	two cutoff
632	0	0.1	0.8	0.3	0.5	0.33	0.67	two cutoff
633	0	0.1	0.9	0.3	0.5	0.23	0.68	two cutoff
634	0	0.1	1	0.3	0.5	0.15	0.80	two cutoff
635	0	0.1	0.1	0.4	0.5	0.43	0.23	two cutoff
636	0	0.1	0.2	0.4	0.5	0.43	0.26	two cutoff
637	0	0.1	0.3	0.4	0.5	0.42	0.32	two cutoff
638	0	0.1	0.4	0.4	0.5	0.41	0.47	two cutoff
639	0	0.1	0.5	0.4	0.5	0.40	0.49	two cutoff
640	0	0.1	0.6	0.4	0.5	0.37	0.57	two cutoff
641	0	0.1	0.7	0.4	0.5	0.34	0.63	two cutoff
642	0	0.1	0.8	0.4	0.5	0.28	0.71	two cutoff
643	0	0.1	0.9	0.4	0.5	0.19	0.75	two cutoff
644	0	0.1	1	0.4	0.5	0.13	0.85	two cutoff
645	0	0.1	0.1	0.5	0.5	0.39	0.13	two cutoff
646	0	0.1	0.2	0.5	0.5	0.39	0.18	two cutoff
647	0	0.1	0.3	0.5	0.5	0.39	0.33	two cutoff
648	0	0.1	0.4	0.5	0.5	0.37	0.40	two cutoff
649	0	0.1	0.5	0.5	0.5	0.36	0.51	two cutoff
650	0	0.1	0.6	0.5	0.5	0.33	0.59	two cutoff
651	0	0.1	0.7	0.5	0.5	0.29	0.65	two cutoff
652	0	0.1	0.8	0.5	0.5	0.24	0.75	two cutoff
653	0	0.1	0.9	0.5	0.5	0.18	0.83	two cutoff
654	0	0.1	1	0.5	0.5	0.11	0.89	two cutoff
655	0	0.1	0.1	0.6	0.5	0.36	0.13	two cutoff
656	0	0.1	0.2	0.6	0.5	0.36	0.22	two cutoff
657	0	0.1	0.3	0.6	0.5	0.35	0.34	two cutoff
658	0	0.1	0.4	0.6	0.5	0.33	0.44	two cutoff
659	0	0.1	0.5	0.6	0.5	0.32	0.53	two cutoff
660	0	0.1	0.6	0.6	0.5	0.29	0.63	two cutoff
661	0	0.1	0.7	0.6	0.5	0.26	0.70	two cutoff
662	0	0.1	0.8	0.6	0.5	0.21	0.78	two cutoff
663	0	0.1	0.9	0.6	0.5	0.15	0.86	two cutoff
664	0	0.1	1	0.6	0.5	0.10	0.93	two cutoff
665	0	0.2	0.1	0.2	0.5	0.48	0.10	two cutoff
666	0	0.2	0.2	0.2	0.5	0.47	0.18	two cutoff
667	0	0.2	0.3	0.2	0.5	0.46	0.26	two cutoff
668	0	0.2	0.4	0.2	0.5	0.45	0.30	two cutoff
669	0	0.2	0.5	0.2	0.5	0.45	0.39	two cutoff

670	0	0.2	0.6	0.2	0.5	0.44	0.45	two cutoff
671	0	0.2	0.7	0.2	0.5	0.40	0.51	two cutoff
672	0	0.2	0.8	0.2	0.5	0.37	0.56	two cutoff
673	0	0.2	0.9	0.2	0.5	0.26	0.61	two cutoff
674	0	0.2	1	0.2	0.5	0.18	0.67	two cutoff
675	0	0.3	0.1	0.3	0.5	0.44	0.10	two cutoff
676	0	0.3	0.2	0.3	0.5	0.43	0.20	two cutoff
677	0	0.3	0.3	0.3	0.5	0.42	0.25	two cutoff
678	0	0.3	0.4	0.3	0.5	0.41	0.29	two cutoff
679	0	0.3	0.5	0.3	0.5	0.40	0.38	two cutoff
680	0	0.3	0.6	0.3	0.5	0.38	0.44	two cutoff
681	0	0.3	0.7	0.3	0.5	0.35	0.50	two cutoff
682	0	0.3	0.8	0.3	0.5	0.30	0.55	two cutoff
683	0	0.3	0.9	0.3	0.5	0.21	0.61	two cutoff
684	0	0.3	1	0.3	0.5	0.14	0.67	two cutoff
685	0	0.4	0.1	0.4	0.5	0.40	0.09	two cutoff
686	0	0.4	0.2	0.4	0.5	0.39	0.16	two cutoff
687	0	0.4	0.3	0.4	0.5	0.38	0.24	two cutoff
688	0	0.4	0.4	0.4	0.5	0.37	0.31	two cutoff
689	0	0.4	0.5	0.4	0.5	0.34	0.40	two cutoff
690	0	0.4	0.6	0.4	0.5	0.33	0.43	two cutoff
691	0	0.4	0.7	0.4	0.5	0.27	0.51	two cutoff
692	0	0.4	0.8	0.4	0.5	0.24	0.55	two cutoff
693	0	0.4	0.9	0.4	0.5	0.18	0.59	two cutoff
694	0	0.4	1	0.4	0.5	0.11	0.67	two cutoff
695	0	0.5	0.1	0.5	0.5	0.37	0.09	two cutoff
696	0	0.5	0.2	0.5	0.5	0.36	0.14	two cutoff
697	0	0.5	0.3	0.5	0.5	0.34	0.23	two cutoff
698	0	0.5	0.4	0.5	0.5	0.33	0.30	two cutoff
699	0	0.5	0.5	0.5	0.5	0.31	0.36	two cutoff
700	0	0.5	0.6	0.5	0.5	0.28	0.43	two cutoff
701	0	0.5	0.7	0.5	0.5	0.25	0.50	two cutoff
702	0	0.5	0.8	0.5	0.5	0.21	0.55	two cutoff
703	0	0.5	0.9	0.5	0.5	0.15	0.61	two cutoff
704	0	0.5	1	0.5	0.5	0.09	0.67	two cutoff
706	0	0.1	0	0	0.25	0.13	0.23	one cutoff
707	0.1	0.1	0	0	0.25	0.13	0.27	one cutoff
708	0.2	0.1	0	0	0.25	0.14	0.28	one cutoff
709	0.3	0.1	0	0	0.25	0.14	0.30	one cutoff
710	0.4	0.1	0	0	0.25	0.14	0.32	one cutoff

711	0.5	0.1	0	0	0.25	0.14	0.34	one cutoff
712	0.6	0.1	0	0	0.25	0.13	0.35	one cutoff
713	0.7	0.1	0	0	0.25	0.13	0.37	one cutoff
714	0.8	0.1	0	0	0.25	0.12	0.40	one cutoff
715	0.9	0.1	0	0	0.25	0.10	0.41	one cutoff
716	1	0.1	0	0	0.25	0.07	0.43	one cutoff
717	0	0.2	0	0	0.25	0.12	0.20	one cutoff
718	0.1	0.2	0	0	0.25	0.12	0.23	one cutoff
719	0.2	0.2	0	0	0.25	0.13	0.26	one cutoff
720	0.3	0.2	0	0	0.25	0.13	0.28	one cutoff
721	0.4	0.2	0	0	0.25	0.13	0.31	one cutoff
722	0.5	0.2	0	0	0.25	0.13	0.34	one cutoff
723	0.6	0.2	0	0	0.25	0.12	0.36	one cutoff
724	0.7	0.2	0	0	0.25	0.12	0.38	one cutoff
725	0.8	0.2	0	0	0.25	0.10	0.41	one cutoff
726	0.9	0.2	0	0	0.25	0.08	0.44	one cutoff
727	1	0.2	0	0	0.25	0.05	0.47	one cutoff
728	0	0.3	0	0	0.25	0.11	0.18	one cutoff
729	0.1	0.3	0	0	0.25	0.12	0.21	one cutoff
730	0.2	0.3	0	0	0.25	0.12	0.25	one cutoff
731	0.3	0.3	0	0	0.25	0.12	0.28	one cutoff
732	0.4	0.3	0	0	0.25	0.12	0.31	one cutoff
733	0.5	0.3	0	0	0.25	0.11	0.33	one cutoff
734	0.6	0.3	0	0	0.25	0.11	0.37	one cutoff
735	0.7	0.3	0	0	0.25	0.10	0.40	one cutoff
736	0.8	0.3	0	0	0.25	0.09	0.43	one cutoff
737	0.9	0.3	0	0	0.25	0.06	0.46	one cutoff
738	1	0.3	0	0	0.25	0.04	0.49	one cutoff
739	0	0.4	0	0	0.25	0.11	0.16	one cutoff
740	0.1	0.4	0	0	0.25	0.11	0.20	one cutoff
741	0.2	0.4	0	0	0.25	0.11	0.24	one cutoff
742	0.3	0.4	0	0	0.25	0.11	0.27	one cutoff
743	0.4	0.4	0	0	0.25	0.11	0.30	one cutoff
744	0.5	0.4	0	0	0.25	0.10	0.34	one cutoff
745	0.6	0.4	0	0	0.25	0.10	0.37	one cutoff
746	0.7	0.4	0	0	0.25	0.09	0.41	one cutoff
747	0.8	0.4	0	0	0.25	0.07	0.44	one cutoff
748	0.9	0.4	0	0	0.25	0.05	0.48	one cutoff
749	1	0.4	0	0	0.25	0.03	0.51	one cutoff
750	0	0.5	0	0	0.25	0.10	0.14	one cutoff

751	0.1	0.5	0	0	0.25	0.10	0.19	one cutoff
752	0.2	0.5	0	0	0.25	0.10	0.22	one cutoff
753	0.3	0.5	0	0	0.25	0.10	0.26	one cutoff
754	0.4	0.5	0	0	0.25	0.10	0.30	one cutoff
755	0.5	0.5	0	0	0.25	0.09	0.34	one cutoff
756	0.6	0.5	0	0	0.25	0.09	0.37	one cutoff
757	0.7	0.5	0	0	0.25	0.08	0.42	one cutoff
758	0.8	0.5	0	0	0.25	0.06	0.45	one cutoff
759	0.9	0.5	0	0	0.25	0.04	0.49	one cutoff
760	1	0.5	0	0	0.25	0.03	0.53	one cutoff
761	0	0.6	0	0	0.25	0.09	0.12	one cutoff
762	0.1	0.6	0	0	0.25	0.09	0.17	one cutoff
763	0.2	0.6	0	0	0.25	0.09	0.21	one cutoff
764	0.3	0.6	0	0	0.25	0.09	0.26	one cutoff
765	0.4	0.6	0	0	0.25	0.09	0.30	one cutoff
766	0.5	0.6	0	0	0.25	0.08	0.33	one cutoff
767	0.6	0.6	0	0	0.25	0.08	0.38	one cutoff
768	0.7	0.6	0	0	0.25	0.07	0.42	one cutoff
769	0.8	0.6	0	0	0.25	0.06	0.46	one cutoff
770	0.9	0.6	0	0	0.25	0.04	0.51	one cutoff
771	1	0.6	0	0	0.25	0.02	0.54	one cutoff
772	0	0.1	1	0.1	0.25	0.07	0.33	two cutoff
773	0	0.2	1	0.1	0.25	0.06	0.30	two cutoff
774	0	0.3	1	0.1	0.25	0.06	0.27	two cutoff
775	0	0.4	1	0.1	0.25	0.05	0.25	two cutoff
776	0	0.5	1	0.1	0.25	0.05	0.22	two cutoff
777	0	0.1	1	0.2	0.25	0.05	0.37	two cutoff
778	0	0.1	1	0.3	0.25	0.04	0.45	two cutoff
779	0	0.1	1	0.4	0.25	0.03	0.42	two cutoff
780	0	0.1	1	0.5	0.25	0.03	0.45	two cutoff
781	0.1	0.1	1	0.1	0.25	0.07	0.36	two cutoff
782	0.2	0.1	1	0.1	0.25	0.07	0.37	two cutoff
783	0.3	0.1	1	0.1	0.25	0.07	0.38	two cutoff
784	0.4	0.1	1	0.1	0.25	0.07	0.39	two cutoff
785	0.5	0.1	1	0.1	0.25	0.07	0.40	two cutoff
786	0.6	0.1	1	0.1	0.25	0.07	0.41	two cutoff
787	0.7	0.1	1	0.1	0.25	0.07	0.42	two cutoff
788	0.8	0.1	1	0.1	0.25	0.07	0.42	two cutoff
789	0.9	0.1	1	0.1	0.25	0.07	0.43	two cutoff
790	0.1	0.2	1	0.1	0.25	0.07	0.33	two cutoff

791	0.2	0.2	1	0.1	0.25	0.07	0.34	two cutoff
792	0.3	0.2	1	0.1	0.25	0.07	0.36	two cutoff
793	0.4	0.2	1	0.1	0.25	0.07	0.37	two cutoff
794	0.5	0.2	1	0.1	0.25	0.07	0.39	two cutoff
795	0.6	0.2	1	0.1	0.25	0.07	0.40	two cutoff
796	0.7	0.2	1	0.1	0.25	0.06	0.42	two cutoff
797	0.8	0.2	1	0.1	0.25	0.06	0.43	two cutoff
798	0.9	0.2	1	0.1	0.25	0.06	0.45	two cutoff
799	0.1	0.3	1	0.1	0.25	0.06	0.30	two cutoff
800	0.2	0.3	1	0.1	0.25	0.06	0.33	two cutoff
801	0.3	0.3	1	0.1	0.25	0.06	0.35	two cutoff
802	0.4	0.3	1	0.1	0.25	0.06	0.37	two cutoff
803	0.5	0.3	1	0.1	0.25	0.06	0.39	two cutoff
804	0.6	0.3	1	0.1	0.25	0.06	0.41	two cutoff
805	0.7	0.3	1	0.1	0.25	0.06	0.43	two cutoff
806	0.8	0.3	1	0.1	0.25	0.05	0.46	two cutoff
807	0.9	0.3	1	0.1	0.25	0.05	0.47	two cutoff
808	0.1	0.4	1	0.1	0.25	0.06	0.28	two cutoff
809	0.2	0.4	1	0.1	0.25	0.06	0.32	two cutoff
810	0.3	0.4	1	0.1	0.25	0.06	0.34	two cutoff
811	0.4	0.4	1	0.1	0.25	0.06	0.36	two cutoff
812	0.5	0.4	1	0.1	0.25	0.06	0.38	two cutoff
813	0.6	0.4	1	0.1	0.25	0.06	0.41	two cutoff
814	0.7	0.4	1	0.1	0.25	0.05	0.43	two cutoff
815	0.8	0.4	1	0.1	0.25	0.05	0.46	two cutoff
816	0.9	0.4	1	0.1	0.25	0.04	0.48	two cutoff
817	0.1	0.5	1	0.1	0.25	0.05	0.26	two cutoff
818	0.2	0.5	1	0.1	0.25	0.05	0.28	two cutoff
819	0.3	0.5	1	0.1	0.25	0.05	0.32	two cutoff
820	0.4	0.5	1	0.1	0.25	0.05	0.34	two cutoff
821	0.5	0.5	1	0.1	0.25	0.05	0.38	two cutoff
822	0.6	0.5	1	0.1	0.25	0.05	0.34	two cutoff
823	0.7	0.5	1	0.1	0.25	0.05	0.44	two cutoff
824	0.8	0.5	1	0.1	0.25	0.04	0.47	two cutoff
825	0.9	0.5	1	0.1	0.25	0.03	0.50	two cutoff
826	0.1	0.1	1	0.2	0.25	0.05	0.39	two cutoff
827	0.2	0.1	1	0.2	0.25	0.05	0.41	two cutoff
828	0.3	0.1	1	0.2	0.25	0.05	0.41	two cutoff
829	0.4	0.1	1	0.2	0.25	0.05	0.42	two cutoff
830	0.5	0.1	1	0.2	0.25	0.05	0.43	two cutoff

831	0.6	0.1	1	0.2	0.25	0.05	0.43	two cutoff
832	0.7	0.1	1	0.2	0.25	0.05	0.44	two cutoff
833	0.8	0.1	1	0.2	0.25	0.05	0.45	two cutoff
834	0.9	0.1	1	0.2	0.25	0.05	0.46	two cutoff
835	0.1	0.1	1	0.3	0.25	0.04	0.42	two cutoff
836	0.2	0.1	1	0.3	0.25	0.04	0.43	two cutoff
837	0.3	0.1	1	0.3	0.25	0.04	0.44	two cutoff
838	0.4	0.1	1	0.3	0.25	0.04	0.44	two cutoff
839	0.5	0.1	1	0.3	0.25	0.04	0.45	two cutoff
840	0.6	0.1	1	0.3	0.25	0.04	0.46	two cutoff
841	0.7	0.1	1	0.3	0.25	0.04	0.47	two cutoff
842	0.8	0.1	1	0.3	0.25	0.04	0.47	two cutoff
843	0.9	0.1	1	0.3	0.25	0.04	0.48	two cutoff
844	0.1	0.1	1	0.4	0.25	0.03	0.44	two cutoff
845	0.2	0.1	1	0.4	0.25	0.03	0.45	two cutoff
846	0.3	0.1	1	0.4	0.25	0.03	0.46	two cutoff
847	0.4	0.1	1	0.4	0.25	0.03	0.46	two cutoff
848	0.5	0.1	1	0.4	0.25	0.03	0.47	two cutoff
849	0.6	0.1	1	0.4	0.25	0.03	0.48	two cutoff
850	0.7	0.1	1	0.4	0.25	0.03	0.49	two cutoff
851	0.8	0.1	1	0.4	0.25	0.03	0.50	two cutoff
852	0.9	0.1	1	0.4	0.25	0.03	0.50	two cutoff
853	0.1	0.1	1	0.5	0.25	0.03	0.46	two cutoff
854	0.2	0.1	1	0.5	0.25	0.03	0.47	two cutoff
855	0.3	0.1	1	0.5	0.25	0.03	0.48	two cutoff
856	0.4	0.1	1	0.5	0.25	0.03	0.49	two cutoff
857	0.5	0.1	1	0.5	0.25	0.03	0.49	two cutoff
858	0.6	0.1	1	0.5	0.25	0.03	0.50	two cutoff
859	0.7	0.1	1	0.5	0.25	0.03	0.50	two cutoff
860	0.8	0.1	1	0.5	0.25	0.03	0.51	two cutoff
861	0.9	0.1	1	0.5	0.25	0.03	0.52	two cutoff
862	0	0.2	1	0.2	0.25	0.05	0.33	two cutoff
863	0.1	0.2	1	0.2	0.25	0.05	0.36	two cutoff
864	0.2	0.2	1	0.2	0.25	0.05	0.38	two cutoff
865	0.3	0.2	1	0.2	0.25	0.05	0.40	two cutoff
866	0.4	0.2	1	0.2	0.25	0.05	0.41	two cutoff
867	0.5	0.2	1	0.2	0.25	0.05	0.42	two cutoff
868	0.6	0.2	1	0.2	0.25	0.05	0.43	two cutoff
869	0.7	0.2	1	0.2	0.25	0.05	0.44	two cutoff
870	0.8	0.2	1	0.2	0.25	0.05	0.45	two cutoff

871	0.9	0.2	1	0.2	0.25	0.05	0.46	two cutoff
872	0	0.3	1	0.3	0.25	0.03	0.33	two cutoff
873	0.1	0.3	1	0.3	0.25	0.04	0.36	two cutoff
874	0.2	0.3	1	0.3	0.25	0.04	0.39	two cutoff
875	0.3	0.3	1	0.3	0.25	0.04	0.41	two cutoff
876	0.4	0.3	1	0.3	0.25	0.04	0.42	two cutoff
877	0.5	0.3	1	0.3	0.25	0.04	0.37	two cutoff
878	0.6	0.3	1	0.3	0.25	0.04	0.45	two cutoff
879	0.7	0.3	1	0.3	0.25	0.04	0.46	two cutoff
880	0.8	0.3	1	0.3	0.25	0.04	0.48	two cutoff
881	0.9	0.3	1	0.3	0.25	0.04	0.49	two cutoff
882	0	0.4	1	0.4	0.25	0.03	0.33	two cutoff
883	0.1	0.4	1	0.4	0.25	0.03	0.36	two cutoff
884	0.2	0.4	1	0.4	0.25	0.03	0.40	two cutoff
885	0.3	0.4	1	0.4	0.25	0.03	0.41	two cutoff
886	0.4	0.4	1	0.4	0.25	0.03	0.43	two cutoff
887	0.5	0.4	1	0.4	0.25	0.03	0.45	two cutoff
888	0.6	0.4	1	0.4	0.25	0.03	0.46	two cutoff
889	0.7	0.4	1	0.4	0.25	0.03	0.48	two cutoff
890	0.8	0.4	1	0.4	0.25	0.03	0.49	two cutoff
891	0.9	0.4	1	0.4	0.25	0.04	0.31	two cutoff
892	0	0.5	1	0.5	0.25	0.03	0.33	two cutoff
893	0.1	0.5	1	0.5	0.25	0.02	0.35	two cutoff
894	0.2	0.5	1	0.5	0.25	0.02	0.40	two cutoff
895	0.3	0.5	1	0.5	0.25	0.02	0.42	two cutoff
896	0.4	0.5	1	0.5	0.25	0.03	0.44	two cutoff
897	0.5	0.5	1	0.5	0.25	0.03	0.46	two cutoff
898	0.6	0.5	1	0.5	0.25	0.03	0.47	two cutoff
899	0.7	0.5	1	0.5	0.25	0.03	0.50	two cutoff
900	0.8	0.5	1	0.5	0.25	0.03	0.51	two cutoff
901	0.9	0.5	1	0.5	0.25	0.03	0.52	two cutoff
902	0	0.1	0.1	0.1	0.25	0.13	0.06	two cutoff
903	0	0.1	0.2	0.1	0.25	0.13	0.10	two cutoff
904	0	0.1	0.3	0.1	0.25	0.13	0.14	two cutoff
905	0	0.1	0.4	0.1	0.25	0.13	0.20	two cutoff
906	0	0.1	0.5	0.1	0.25	0.13	0.21	two cutoff
907	0	0.1	0.6	0.1	0.25	0.12	0.23	two cutoff
908	0	0.1	0.7	0.1	0.25	0.12	0.26	two cutoff
909	0	0.1	0.8	0.1	0.25	0.11	0.28	two cutoff
910	0	0.1	0.9	0.1	0.25	0.10	0.31	two cutoff

911	0	0.1	1	0.1	0.25	0.07	0.33	two cutoff
912	0	0.2	0.1	0.1	0.25	0.12	0.05	two cutoff
913	0	0.2	0.2	0.1	0.25	0.12	0.08	two cutoff
914	0	0.2	0.3	0.1	0.25	0.12	0.12	two cutoff
915	0	0.2	0.4	0.1	0.25	0.12	0.15	two cutoff
916	0	0.2	0.5	0.1	0.25	0.12	0.19	two cutoff
917	0	0.2	0.6	0.1	0.25	0.12	0.20	two cutoff
918	0	0.2	0.7	0.1	0.25	0.11	0.23	two cutoff
919	0	0.2	0.8	0.1	0.25	0.10	0.26	two cutoff
920	0	0.2	0.9	0.1	0.25	0.09	0.27	two cutoff
921	0	0.2	1	0.1	0.25	0.06	0.30	two cutoff
922	0	0.3	0.1	0.1	0.25	0.11	0.04	two cutoff
923	0	0.3	0.2	0.1	0.25	0.11	0.07	two cutoff
924	0	0.3	0.3	0.1	0.25	0.11	0.11	two cutoff
925	0	0.3	0.4	0.1	0.25	0.11	0.13	two cutoff
926	0	0.3	0.5	0.1	0.25	0.11	0.17	two cutoff
927	0	0.3	0.6	0.1	0.25	0.11	0.20	two cutoff
928	0	0.3	0.7	0.1	0.25	0.11	0.21	two cutoff
929	0	0.3	0.8	0.1	0.25	0.10	0.24	two cutoff
930	0	0.3	0.9	0.1	0.25	0.08	0.25	two cutoff
931	0	0.3	1	0.1	0.25	0.06	0.27	two cutoff
932	0	0.4	0.1	0.1	0.25	0.10	0.04	two cutoff
933	0	0.4	0.2	0.1	0.25	0.10	0.07	two cutoff
934	0	0.4	0.3	0.1	0.25	0.10	0.10	two cutoff
935	0	0.4	0.4	0.1	0.25	0.10	0.13	two cutoff
936	0	0.4	0.5	0.1	0.25	0.10	0.15	two cutoff
937	0	0.4	0.6	0.1	0.25	0.10	0.17	two cutoff
938	0	0.4	0.7	0.1	0.25	0.10	0.19	two cutoff
939	0	0.4	0.8	0.1	0.25	0.09	0.21	two cutoff
940	0	0.4	0.9	0.1	0.25	0.08	0.22	two cutoff
941	0	0.4	1	0.1	0.25	0.06	0.24	two cutoff
942	0	0.5	0.1	0.1	0.25	0.10	0.03	two cutoff
943	0	0.5	0.2	0.1	0.25	0.10	0.07	two cutoff
944	0	0.5	0.3	0.1	0.25	0.10	0.09	two cutoff
945	0	0.5	0.4	0.1	0.25	0.10	0.11	two cutoff
946	0	0.5	0.5	0.1	0.25	0.10	0.13	two cutoff
947	0	0.5	0.6	0.1	0.25	0.09	0.15	two cutoff
948	0	0.5	0.7	0.1	0.25	0.09	0.17	two cutoff
949	0	0.5	0.8	0.1	0.25	0.08	0.19	two cutoff
950	0	0.5	0.9	0.1	0.25	0.07	0.20	two cutoff

951	0	0.5	1	0.1	0.25	0.05	0.22	two cutoff
952	0	0.1	0.1	0.2	0.25	0.12	0.06	two cutoff
953	0	0.1	0.2	0.2	0.25	0.12	0.10	two cutoff
954	0	0.1	0.3	0.2	0.25	0.12	0.15	two cutoff
955	0	0.1	0.4	0.2	0.25	0.12	0.20	two cutoff
956	0	0.1	0.5	0.2	0.25	0.12	0.22	two cutoff
957	0	0.1	0.6	0.2	0.25	0.11	0.26	two cutoff
958	0	0.1	0.7	0.2	0.25	0.11	0.28	two cutoff
959	0	0.1	0.8	0.2	0.25	0.09	0.33	two cutoff
960	0	0.1	0.9	0.2	0.25	0.07	0.34	two cutoff
961	0	0.1	1	0.2	0.25	0.05	0.37	two cutoff
962	0	0.1	0.1	0.3	0.25	0.12	0.06	two cutoff
963	0	0.1	0.2	0.3	0.25	0.11	0.12	two cutoff
964	0	0.1	0.3	0.3	0.25	0.11	0.15	two cutoff
965	0	0.1	0.4	0.3	0.25	0.11	0.20	two cutoff
966	0	0.1	0.5	0.3	0.25	0.11	0.23	two cutoff
967	0	0.1	0.6	0.3	0.25	0.10	0.23	two cutoff
968	0	0.1	0.7	0.3	0.25	0.10	0.30	two cutoff
969	0	0.1	0.8	0.3	0.25	0.08	0.33	two cutoff
970	0	0.1	0.9	0.3	0.25	0.06	0.37	two cutoff
971	0	0.1	1	0.3	0.25	0.04	0.40	two cutoff
972	0	0.1	0.1	0.4	0.25	0.11	0.06	two cutoff
973	0	0.1	0.2	0.4	0.25	0.11	0.13	two cutoff
974	0	0.1	0.3	0.4	0.25	0.11	0.16	two cutoff
975	0	0.1	0.4	0.4	0.25	0.10	0.20	two cutoff
976	0	0.1	0.5	0.4	0.25	0.10	0.24	two cutoff
977	0	0.1	0.6	0.4	0.25	0.09	0.28	two cutoff
978	0	0.1	0.7	0.4	0.25	0.08	0.32	two cutoff
979	0	0.1	0.8	0.4	0.25	0.07	0.35	two cutoff
980	0	0.1	0.9	0.4	0.25	0.05	0.39	two cutoff
981	0	0.1	1	0.4	0.25	0.03	0.42	two cutoff
982	0	0.1	0.1	0.5	0.25	0.10	0.06	two cutoff
983	0	0.1	0.2	0.5	0.25	0.10	0.11	two cutoff
984	0	0.1	0.3	0.5	0.25	0.10	0.17	two cutoff
985	0	0.1	0.4	0.5	0.25	0.09	0.21	two cutoff
986	0	0.1	0.5	0.5	0.25	0.09	0.25	two cutoff
987	0	0.1	0.6	0.5	0.25	0.08	0.29	two cutoff
988	0	0.1	0.7	0.5	0.25	0.07	0.33	two cutoff
989	0	0.1	0.8	0.5	0.25	0.05	0.37	two cutoff
990	0	0.1	0.9	0.5	0.25	0.04	0.41	two cutoff

991	0	0.1	1	0.5	0.25	0.03	0.45	two cutoff
992	0	0.1	0.1	0.6	0.25	0.09	0.06	two cutoff
993	0	0.1	0.2	0.6	0.25	0.09	0.11	two cutoff
994	0	0.1	0.3	0.6	0.25	0.09	0.17	two cutoff
995	0	0.1	0.4	0.6	0.25	0.09	0.22	two cutoff
996	0	0.1	0.5	0.6	0.25	0.08	0.26	two cutoff
997	0	0.1	0.6	0.6	0.25	0.08	0.30	two cutoff
998	0	0.1	0.7	0.6	0.25	0.07	0.35	two cutoff
999	0	0.1	0.8	0.6	0.25	0.05	0.40	two cutoff
1000	0	0.1	0.9	0.6	0.25	0.04	0.43	two cutoff
1001	0	0.1	1	0.6	0.25	0.02	0.47	two cutoff
1002	0	0.2	0.1	0.2	0.25	0.12	0.05	two cutoff
1003	0	0.2	0.2	0.2	0.25	0.12	0.10	two cutoff
1004	0	0.2	0.3	0.2	0.25	0.12	0.13	two cutoff
1005	0	0.2	0.4	0.2	0.25	0.11	0.17	two cutoff
1006	0	0.2	0.5	0.2	0.25	0.11	0.20	two cutoff
1007	0	0.2	0.6	0.2	0.25	0.11	0.22	two cutoff
1008	0	0.2	0.7	0.2	0.25	0.10	0.25	two cutoff
1009	0	0.2	0.8	0.2	0.25	0.09	0.28	two cutoff
1010	0	0.2	0.9	0.2	0.25	0.07	0.31	two cutoff
1011	0	0.2	1	0.2	0.25	0.05	0.33	two cutoff
1012	0	0.3	0.1	0.3	0.25	0.11	0.05	two cutoff
1013	0	0.3	0.2	0.3	0.25	0.11	0.08	two cutoff
1014	0	0.3	0.3	0.3	0.25	0.11	0.12	two cutoff
1015	0	0.3	0.4	0.3	0.25	0.10	0.15	two cutoff
1016	0	0.3	0.5	0.3	0.25	0.10	0.19	two cutoff
1017	0	0.3	0.6	0.3	0.25	0.09	0.21	two cutoff
1018	0	0.3	0.7	0.3	0.25	0.09	0.25	two cutoff
1019	0	0.3	0.8	0.3	0.25	0.07	0.28	two cutoff
1020	0	0.3	0.9	0.3	0.25	0.05	0.31	two cutoff
1021	0	0.3	1	0.3	0.25	0.03	0.02	two cutoff
1022	0	0.4	0.1	0.4	0.25	0.10	0.05	two cutoff
1023	0	0.4	0.2	0.4	0.25	0.10	0.07	two cutoff
1024	0	0.4	0.3	0.4	0.25	0.09	0.12	two cutoff
1025	0	0.4	0.4	0.4	0.25	0.09	0.15	two cutoff
1026	0	0.4	0.5	0.4	0.25	0.09	0.19	two cutoff
1027	0	0.4	0.6	0.4	0.25	0.08	0.21	two cutoff
1028	0	0.4	0.7	0.4	0.25	0.07	0.24	two cutoff
1029	0	0.4	0.8	0.4	0.25	0.05	0.29	two cutoff
1030	0	0.4	0.9	0.4	0.25	0.04	0.31	two cutoff

1031	0	0.4	1	0.4	0.25	0.03	0.33	two cutoff
1032	0	0.5	0.1	0.5	0.25	0.09	0.05	two cutoff
1033	0	0.5	0.2	0.5	0.25	0.09	0.08	two cutoff
1034	0	0.5	0.3	0.5	0.25	0.09	0.12	two cutoff
1035	0	0.5	0.4	0.5	0.25	0.08	0.15	two cutoff
1036	0	0.5	0.5	0.5	0.25	0.08	0.18	two cutoff
1037	0	0.5	0.6	0.5	0.25	0.07	0.21	two cutoff
1038	0	0.5	0.7	0.5	0.25	0.06	0.24	two cutoff
1039	0	0.5	0.8	0.5	0.25	0.07	0.31	two cutoff
1040	0	0.5	0.9	0.5	0.25	0.04	0.30	two cutoff
1041	0	0.5	1	0.5	0.25	0.02	0.33	two cutoff

A.2- Table of results the specific uplift force and exit gradient ratio of permeability in x-direction to permeability of y-direction ($K_x/K_y=2$).

Table (A-2): The Results obtained by using the ratio of ($\frac{K_x}{K_y}=2$) with the ratio of head differential ($\frac{H}{B}=0.75$)

Run No.	x1/B	d1/B	x2/B	d2/B	H/B	(iexit*H)/B	(f uplift/max f)	case
1	0	0.1	1	0.1	0.75	0.74	0.97	two cutoff
2	0	0.2	1	0.1	0.75	0.69	0.84	two cutoff
3	0	0.3	1	0.1	0.75	0.63	0.73	two cutoff
4	0	0.4	1	0.1	0.75	0.59	0.63	two cutoff
5	0	0.5	1	0.1	0.75	0.53	0.58	two cutoff
6	0	0.6	1	0.1	0.75	0.48	0.52	two cutoff
7	0	0.1	1	0.1	0.75	1.46	0.77	two cutoff
8	0	0.1	1	0.2	0.75	0.53	1.09	two cutoff
9	0	0.1	1	0.3	0.75	0.49	1.17	two cutoff
10	0	0.1	1	0.4	0.75	0.34	1.28	two cutoff
11	0	0.1	1	0.5	0.75	0.30	1.31	two cutoff
12	0	0.1	1	0.6	0.75	0.25	1.41	two cutoff
13	0.1	0.1	1	0.1	0.75	0.73	1.08	two cutoff
14	0.2	0.1	1	0.1	0.75	0.74	1.11	two cutoff
15	0.3	0.1	1	0.1	0.75	0.75	1.17	two cutoff

16	0.4	0.1	1	0.1	0.75	0.75	1.20	two cutoff
17	0.5	0.1	1	0.1	0.75	0.75	1.24	two cutoff
18	0.6	0.1	1	0.1	0.75	0.74	1.26	two cutoff
19	0.7	0.1	1	0.1	0.75	0.67	1.30	two cutoff
20	0.8	0.1	1	0.1	0.75	0.74	1.32	two cutoff
21	0.9	0.1	1	0.1	0.75	0.74	1.35	two cutoff
22	0.1	0.2	1	0.1	0.75	0.71	0.94	two cutoff
23	0.2	0.2	1	0.1	0.75	0.74	1.03	two cutoff
24	0.3	0.2	1	0.1	0.75	0.76	1.16	two cutoff
25	0.4	0.2	1	0.1	0.75	0.78	1.31	two cutoff
26	0.5	0.2	1	0.1	0.75	0.69	1.21	two cutoff
27	0.6	0.2	1	0.1	0.75	0.68	1.25	two cutoff
28	0.7	0.2	1	0.1	0.75	0.67	1.30	two cutoff
29	0.8	0.2	1	0.1	0.75	0.65	1.36	two cutoff
30	0.9	0.2	1	0.1	0.75	0.59	1.41	two cutoff
31	0.1	0.3	1	0.1	0.75	0.62	0.87	two cutoff
32	0.2	0.3	1	0.1	0.75	0.62	0.95	two cutoff
33	0.3	0.3	1	0.1	0.75	0.63	1.04	two cutoff
34	0.4	0.3	1	0.1	0.75	0.63	1.12	two cutoff
35	0.5	0.3	1	0.1	0.75	0.62	1.18	two cutoff
36	0.6	0.3	1	0.1	0.75	0.61	1.25	two cutoff
37	0.7	0.3	1	0.1	0.75	0.59	1.35	two cutoff
38	0.8	0.3	1	0.1	0.75	0.54	1.39	two cutoff
39	0.9	0.3	1	0.1	0.75	0.48	1.47	two cutoff
40	0.1	0.4	1	0.1	0.75	0.56	0.79	two cutoff
41	0.2	0.4	1	0.1	0.75	0.56	0.90	two cutoff
42	0.3	0.4	1	0.1	0.75	0.57	0.99	two cutoff
43	0.4	0.4	1	0.1	0.75	0.56	1.08	two cutoff
44	0.5	0.4	1	0.1	0.75	0.55	1.26	two cutoff
45	0.6	0.4	1	0.1	0.75	0.53	1.25	two cutoff
46	0.7	0.4	1	0.1	0.75	0.50	1.33	two cutoff
47	0.8	0.4	1	0.1	0.75	0.47	1.48	two cutoff
48	0.9	0.4	1	0.1	0.75	0.40	1.52	two cutoff
49	0.1	0.5	1	0.1	0.75	0.53	0.70	two cutoff
50	0.2	0.5	1	0.1	0.75	0.52	0.82	two cutoff
51	0.3	0.5	1	0.1	0.75	0.51	0.94	two cutoff
52	0.4	0.5	1	0.1	0.75	0.48	0.99	two cutoff
53	0.5	0.5	1	0.1	0.75	0.46	1.14	two cutoff
54	0.6	0.5	1	0.1	0.75	0.45	1.24	two cutoff
55	0.7	0.5	1	0.1	0.75	0.44	1.34	two cutoff

56	0.8	0.5	1	0.1	0.75	0.38	1.45	two cutoff
57	0.9	0.5	1	0.1	0.75	0.34	1.56	two cutoff
58	0.1	0.6	1	0.1	0.75	0.46	0.67	two cutoff
59	0.2	0.6	1	0.1	0.75	0.46	0.77	two cutoff
60	0.3	0.6	1	0.1	0.75	0.46	0.90	two cutoff
61	0.4	0.6	1	0.1	0.75	0.44	1.00	two cutoff
62	0.5	0.6	1	0.1	0.75	0.43	1.13	two cutoff
63	0.6	0.6	1	0.1	0.75	0.39	1.17	two cutoff
64	0.7	0.6	1	0.1	0.75	0.38	1.35	two cutoff
65	0.8	0.6	1	0.1	0.75	0.34	1.46	two cutoff
66	0.9	0.6	1	0.1	0.75	0.29	1.60	two cutoff
67	0.1	0.1	1	0.2	0.75	0.51	1.21	two cutoff
68	0.2	0.1	1	0.2	0.75	0.52	1.24	two cutoff
69	0.3	0.1	1	0.2	0.75	0.52	1.28	two cutoff
70	0.4	0.1	1	0.2	0.75	0.53	1.31	two cutoff
71	0.5	0.1	1	0.2	0.75	0.53	1.34	two cutoff
72	0.6	0.1	1	0.2	0.75	0.53	1.35	two cutoff
73	0.7	0.1	1	0.2	0.75	0.53	1.39	two cutoff
74	0.8	0.1	1	0.2	0.75	0.53	0.00	two cutoff
75	0.9	0.1	1	0.2	0.75	0.54	1.45	two cutoff
76	0.1	0.1	1	0.3	0.75	0.40	1.31	two cutoff
77	0.2	0.1	1	0.3	0.75	0.41	1.35	two cutoff
78	0.3	0.1	1	0.3	0.75	0.41	1.37	two cutoff
79	0.4	0.1	1	0.3	0.75	0.41	1.40	two cutoff
80	0.5	0.1	1	0.3	0.75	0.41	1.42	two cutoff
81	0.6	0.1	1	0.3	0.75	0.42	1.45	two cutoff
82	0.7	0.1	1	0.3	0.75	0.42	1.47	two cutoff
83	0.8	0.1	1	0.3	0.75	0.42	1.50	two cutoff
84	0.9	0.1	1	0.3	0.75	0.42	1.53	two cutoff
85	0.1	0.1	1	0.4	0.75	0.33	1.39	two cutoff
86	0.2	0.1	1	0.4	0.75	0.34	1.42	two cutoff
87	0.3	0.1	1	0.4	0.75	0.34	1.45	two cutoff
88	0.4	0.1	1	0.4	0.75	0.34	1.48	two cutoff
89	0.5	0.1	1	0.4	0.75	0.34	1.49	two cutoff
90	0.6	0.1	1	0.4	0.75	0.34	1.52	two cutoff
91	0.7	0.1	1	0.4	0.75	0.34	1.54	two cutoff
92	0.8	0.1	1	0.4	0.75	0.34	1.57	two cutoff
93	0.9	0.1	1	0.4	0.75	0.34	1.59	two cutoff
94	0.1	0.1	1	0.5	0.75	0.28	1.46	two cutoff
95	0.2	0.1	1	0.5	0.75	0.29	1.49	two cutoff

96	0.3	0.1	1	0.5	0.75	0.29	1.50	two cutoff
97	0.4	0.1	1	0.5	0.75	0.29	1.53	two cutoff
98	0.5	0.1	1	0.5	0.75	0.29	15.64	two cutoff
99	0.6	0.1	1	0.5	0.75	0.29	1.58	two cutoff
100	0.7	0.1	1	0.5	0.75	0.29	1.59	two cutoff
101	0.8	0.1	1	0.5	0.75	0.29	1.62	two cutoff
102	0.9	0.1	1	0.5	0.75	0.29	1.64	two cutoff
103	0.1	0.1	1	0.6	0.75	0.25	1.52	two cutoff
104	0.2	0.1	1	0.6	0.75	0.25	1.55	two cutoff
105	0.3	0.1	1	0.6	0.75	0.25	1.57	two cutoff
106	0.4	0.1	1	0.6	0.75	0.25	1.59	two cutoff
107	0.5	0.1	1	0.6	0.75	0.25	1.61	two cutoff
108	0.6	0.1	1	0.6	0.75	0.25	1.63	two cutoff
109	0.7	0.1	1	0.6	0.75	0.25	1.64	two cutoff
110	0.8	0.1	1	0.6	0.75	0.25	1.67	two cutoff
111	0.9	0.1	1	0.6	0.75	0.25	1.68	two cutoff
112	0	0.2	1	0.2	0.75	0.49	0.96	two cutoff
113	0.1	0.2	1	0.2	0.75	0.49	1.04	two cutoff
114	0.2	0.2	1	0.2	0.75	0.49	1.16	two cutoff
115	0.3	0.2	1	0.2	0.75	0.50	1.22	two cutoff
116	0.4	0.2	1	0.2	0.75	0.50	1.26	two cutoff
117	0.5	0.2	1	0.2	0.75	0.50	1.32	two cutoff
118	0.6	0.2	1	0.2	0.75	0.50	1.35	two cutoff
119	0.7	0.2	1	0.2	0.75	0.51	2.04	two cutoff
120	0.8	0.2	1	0.2	0.75	0.51	1.42	two cutoff
121	0.9	0.2	1	0.2	0.75	0.52	1.45	two cutoff
122	0	0.3	1	0.3	0.75	0.37	0.96	two cutoff
123	0.1	0.3	1	0.3	0.75	0.37	1.08	two cutoff
124	0.2	0.3	1	0.3	0.75	0.37	1.17	two cutoff
125	0.3	0.3	1	0.3	0.75	0.38	1.24	two cutoff
126	0.4	0.3	1	0.3	0.75	0.38	1.30	two cutoff
127	0.5	0.3	1	0.3	0.75	0.38	1.36	two cutoff
128	0.6	0.3	1	0.3	0.75	0.39	1.40	two cutoff
129	0.7	0.3	1	0.3	0.75	0.39	1.43	two cutoff
130	0.8	0.3	1	0.3	0.75	0.40	1.49	two cutoff
131	0.9	0.3	1	0.3	0.75	0.40	1.53	two cutoff
132	0	0.4	1	0.4	0.75	0.30	0.96	two cutoff
133	0.1	0.4	1	0.4	0.75	0.29	1.06	two cutoff
134	0.2	0.4	1	0.4	0.75	0.29	1.18	two cutoff
135	0.3	0.4	1	0.4	0.75	0.30	1.26	two cutoff

136	0.4	0.4	1	0.4	0.75	0.30	1.32	two cutoff
137	0.5	0.4	1	0.4	0.75	0.31	1.38	two cutoff
138	0.6	0.4	1	0.4	0.75	0.31	1.44	two cutoff
139	0.7	0.4	1	0.4	0.75	0.32	1.49	two cutoff
140	0.8	0.4	1	0.4	0.75	0.32	1.55	two cutoff
141	0.9	0.4	1	0.4	0.75	0.33	1.59	two cutoff
142	0	0.5	1	0.5	0.75	0.25	0.96	two cutoff
143	0.1	0.5	1	0.5	0.75	0.25	1.11	two cutoff
144	0.2	0.5	1	0.5	0.75	0.24	1.19	two cutoff
145	0.3	0.5	1	0.5	0.75	0.25	1.25	two cutoff
146	0.4	0.5	1	0.5	0.75	0.25	1.34	two cutoff
147	0.5	0.5	1	0.5	0.75	0.26	1.40	two cutoff
148	0.6	0.5	1	0.5	0.75	0.26	1.47	two cutoff
149	0.7	0.5	1	0.5	0.75	0.27	1.52	two cutoff
150	0.8	0.5	1	0.5	0.75	0.27	1.58	two cutoff
151	0.9	0.5	1	0.5	0.75	0.28	1.63	two cutoff
152	0	0.1	0.1	0.1	0.75	1.41	0.16	two cutoff
153	0	0.1	0.2	0.1	0.75	1.40	0.27	two cutoff
154	0	0.1	0.3	0.1	0.75	1.39	0.40	two cutoff
155	0	0.1	0.4	0.1	0.75	1.37	0.47	two cutoff
156	0	0.1	0.5	0.1	0.75	1.36	0.61	two cutoff
157	0	0.1	0.6	0.1	0.75	1.34	0.71	two cutoff
158	0	0.1	0.7	0.1	0.75	1.29	0.77	two cutoff
159	0	0.1	0.8	0.1	0.75	1.16	0.88	two cutoff
160	0	0.1	0.9	0.1	0.75	0.95	0.92	two cutoff
161	0	0.1	1	0.1	0.75	0.74	0.97	two cutoff
162	0	0.2	0.1	0.1	0.75	1.27	0.15	two cutoff
163	0	0.2	0.2	0.1	0.75	1.27	0.22	two cutoff
164	0	0.2	0.3	0.1	0.75	1.28	0.34	two cutoff
165	0	0.2	0.4	0.1	0.75	1.26	0.42	two cutoff
166	0	0.2	0.5	0.1	0.75	1.25	0.52	two cutoff
167	0	0.2	0.6	0.1	0.75	1.22	0.57	two cutoff
168	0	0.2	0.7	0.1	0.75	1.18	0.66	two cutoff
169	0	0.2	0.8	0.1	0.75	0.99	0.70	two cutoff
170	0	0.2	0.9	0.1	0.75	0.87	0.80	two cutoff
171	0	0.2	1	0.1	0.75	0.69	0.84	two cutoff
172	0	0.3	0.1	0.1	0.75	1.17	0.11	two cutoff
173	0	0.3	0.2	0.1	0.75	1.16	0.20	two cutoff
174	0	0.3	0.3	0.1	0.75	1.16	0.29	two cutoff
175	0	0.3	0.4	0.1	0.75	1.15	0.37	two cutoff

176	0	0.3	0.5	0.1	0.75	1.13	0.43	two cutoff
177	0	0.3	0.6	0.1	0.75	1.12	0.51	two cutoff
178	0	0.3	0.7	0.1	0.75	1.07	0.56	two cutoff
179	0	0.3	0.8	0.1	0.75	1.00	0.63	two cutoff
180	0	0.3	0.9	0.1	0.75	0.80	0.70	two cutoff
181	0	0.3	1	0.1	0.75	0.63	0.73	two cutoff
182	0	0.4	0.1	0.1	0.75	1.06	0.10	two cutoff
183	0	0.4	0.2	0.1	0.75	1.06	0.17	two cutoff
184	0	0.4	0.3	0.1	0.75	1.05	0.25	two cutoff
185	0	0.4	0.4	0.1	0.75	1.04	0.30	two cutoff
186	0	0.4	0.5	0.1	0.75	1.03	0.39	two cutoff
187	0	0.4	0.6	0.1	0.75	1.01	0.43	two cutoff
188	0	0.4	0.7	0.1	0.75	0.98	0.50	two cutoff
189	0	0.4	0.8	0.1	0.75	0.84	0.56	two cutoff
190	0	0.4	0.9	0.1	0.75	0.73	0.61	two cutoff
191	0	0.4	1	0.1	0.75	0.58	0.65	two cutoff
192	0	0.5	0.1	0.1	0.75	0.96	0.09	two cutoff
193	0	0.5	0.2	0.1	0.75	0.95	0.16	two cutoff
194	0	0.5	0.3	0.1	0.75	0.95	0.22	two cutoff
195	0	0.5	0.4	0.1	0.75	0.94	0.29	two cutoff
196	0	0.5	0.5	0.1	0.75	0.93	0.34	two cutoff
197	0	0.5	0.6	0.1	0.75	0.91	0.36	two cutoff
198	0	0.5	0.7	0.1	0.75	0.89	0.44	two cutoff
199	0	0.5	0.8	0.1	0.75	0.71	0.50	two cutoff
200	0	0.5	0.9	0.1	0.75	0.53	0.58	two cutoff
201	0	0.5	1	0.1	0.75	0.53	0.58	two cutoff
202	0	0.1	0.1	0.2	0.75	1.32	0.18	two cutoff
203	0	0.1	0.2	0.2	0.75	1.30	0.24	two cutoff
204	0	0.1	0.3	0.2	0.75	1.29	0.44	two cutoff
205	0	0.1	0.4	0.2	0.75	1.27	0.54	two cutoff
206	0	0.1	0.5	0.2	0.75	1.23	0.66	two cutoff
207	0	0.1	0.6	0.2	0.75	1.15	0.76	two cutoff
208	0	0.1	0.7	0.2	0.75	1.09	0.85	two cutoff
209	0	0.1	0.8	0.2	0.75	0.84	0.95	two cutoff
210	0	0.1	0.9	0.2	0.75	0.69	1.04	two cutoff
211	0	0.1	1	0.2	0.75	0.53	1.09	two cutoff
212	0	0.1	0.1	0.3	0.75	1.19	0.18	two cutoff
213	0	0.1	0.2	0.3	0.75	1.18	0.30	two cutoff
214	0	0.1	0.3	0.3	0.75	1.17	0.47	two cutoff
215	0	0.1	0.4	0.3	0.75	1.13	0.57	two cutoff

216	0	0.1	0.5	0.3	0.75	1.09	0.71	two cutoff
217	0	0.1	0.6	0.3	0.75	1.04	0.78	two cutoff
218	0	0.1	0.7	0.3	0.75	0.92	0.93	two cutoff
219	0	0.1	0.8	0.3	0.75	0.73	1.04	two cutoff
220	0	0.1	0.9	0.3	0.75	0.55	1.14	two cutoff
221	0	0.1	1	0.3	0.75	0.41	1.20	two cutoff
222	0	0.1	0.1	0.4	0.75	1.07	0.19	two cutoff
223	0	0.1	0.2	0.4	0.75	1.05	0.29	two cutoff
224	0	0.1	0.3	0.4	0.75	1.04	0.49	two cutoff
225	0	0.1	0.4	0.4	0.75	0.99	0.64	two cutoff
226	0	0.1	0.5	0.4	0.75	0.95	0.75	two cutoff
227	0	0.1	0.6	0.4	0.75	0.84	0.90	two cutoff
228	0	0.1	0.7	0.4	0.75	0.78	0.99	two cutoff
229	0	0.1	0.8	0.4	0.75	0.65	1.10	two cutoff
230	0	0.1	0.9	0.4	0.75	0.46	1.22	two cutoff
231	0	0.1	1	0.4	0.75	0.34	1.28	two cutoff
232	0	0.1	0.1	0.5	0.75	0.96	0.19	two cutoff
233	0	0.1	0.2	0.5	0.75	0.94	0.28	two cutoff
234	0	0.1	0.3	0.5	0.75	0.92	0.51	two cutoff
235	0	0.1	0.4	0.5	0.75	0.90	0.64	two cutoff
236	0	0.1	0.5	0.5	0.75	0.83	0.78	two cutoff
237	0	0.1	0.6	0.5	0.75	0.72	0.90	two cutoff
238	0	0.1	0.7	0.5	0.75	0.67	1.04	two cutoff
239	0	0.1	0.8	0.5	0.75	0.49	1.16	two cutoff
240	0	0.1	0.9	0.5	0.75	0.39	1.29	two cutoff
241	0	0.1	1	0.5	0.75	0.29	1.35	two cutoff
242	0	0.2	0.1	0.2	0.75	1.27	0.15	two cutoff
243	0	0.2	0.2	0.2	0.75	1.24	0.26	two cutoff
244	0	0.2	0.3	0.2	0.75	1.22	0.35	two cutoff
245	0	0.2	0.4	0.2	0.75	1.19	0.47	two cutoff
246	0	0.2	0.5	0.2	0.75	1.14	0.56	two cutoff
247	0	0.2	0.6	0.2	0.75	1.10	0.66	two cutoff
248	0	0.2	0.7	0.2	0.75	1.01	0.74	two cutoff
249	0	0.2	0.8	0.2	0.75	0.88	0.83	two cutoff
250	0	0.2	0.9	0.2	0.75	0.65	0.92	two cutoff
251	0	0.2	1	0.2	0.75	0.49	0.96	two cutoff
252	0	0.3	0.1	0.3	0.75	1.14	0.14	two cutoff
253	0	0.3	0.2	0.3	0.75	1.12	0.23	two cutoff
254	0	0.3	0.3	0.3	0.75	1.07	0.36	two cutoff
255	0	0.3	0.4	0.3	0.75	1.03	0.51	two cutoff

256	0	0.3	0.5	0.3	0.75	0.97	0.55	two cutoff
257	0	0.3	0.6	0.3	0.75	0.89	0.66	two cutoff
258	0	0.3	0.7	0.3	0.75	0.82	0.73	two cutoff
259	0	0.3	0.8	0.3	0.75	0.69	0.82	two cutoff
260	0	0.3	0.9	0.3	0.75	0.49	0.91	two cutoff
261	0	0.3	1	0.3	0.75	0.37	0.96	two cutoff
262	0	0.4	0.1	0.4	0.75	1.02	0.13	two cutoff
263	0	0.4	0.2	0.4	0.75	0.98	0.24	two cutoff
264	0	0.4	0.3	0.4	0.75	0.93	0.34	two cutoff
265	0	0.4	0.4	0.4	0.75	0.87	0.44	two cutoff
266	0	0.4	0.5	0.4	0.75	0.83	0.54	two cutoff
267	0	0.4	0.6	0.4	0.75	0.73	0.63	two cutoff
268	0	0.4	0.7	0.4	0.75	0.67	0.72	two cutoff
269	0	0.4	0.8	0.4	0.75	0.46	0.79	two cutoff
270	0	0.4	0.9	0.4	0.75	0.39	0.91	two cutoff
271	0	0.4	1	0.4	0.75	0.30	0.96	two cutoff
272	0	0.5	0.1	0.5	0.75	0.91	0.13	two cutoff
273	0	0.5	0.2	0.5	0.75	0.85	0.21	two cutoff
274	0	0.5	0.3	0.5	0.75	0.82	0.34	two cutoff
275	0	0.5	0.4	0.5	0.75	0.76	0.44	two cutoff
276	0	0.5	0.5	0.5	0.75	0.72	0.53	two cutoff
277	0	0.5	0.6	0.5	0.75	0.62	0.59	two cutoff
278	0	0.5	0.7	0.5	0.75	0.57	0.72	two cutoff
279	0	0.5	0.8	0.5	0.75	0.48	0.85	two cutoff
280	0	0.5	0.9	0.5	0.75	0.33	0.91	two cutoff
281	0	0.5	1	0.5	0.75	0.25	0.96	two cutoff
282	0	0.6	0.1	0.6	0.75	0.81	0.13	two cutoff
283	0	0.6	0.2	0.6	0.75	0.74	0.22	two cutoff
284	0	0.6	0.3	0.6	0.75	0.73	0.33	two cutoff
285	0	0.6	0.4	0.6	0.75	0.64	0.44	two cutoff
286	0	0.6	0.5	0.6	0.75	0.62	0.52	two cutoff
287	0	0.6	0.6	0.6	0.75	0.56	0.62	two cutoff
288	0	0.6	0.7	0.6	0.75	0.48	0.74	two cutoff
289	0	0.6	0.8	0.6	0.75	0.40	0.81	two cutoff
290	0	0.6	0.9	0.6	0.75	0.28	0.91	two cutoff
291	0	0.6	1	0.6	0.75	0.21	0.96	two cutoff

A.3- Table of results the specific uplift force and exit gradient ratio of permeability in x-direction to permeability of y-direction ($K_x/K_y=4$)

Table (A-3): The Results obtained by using the ratio of ($\frac{K_x}{K_y}=4$) with ratio of head differential ($\frac{H}{B} = 0.75$)

Run No.	x1/B	d1/B	x2/B	d2/B	H/B	(iexit*H)/B	(f uplift/max f)	case
1	0	0.1	1	0.1	0.75	0.84	0.12	two cutoff
2	0	0.2	1	0.1	0.75	0.75	0.26	two cutoff
3	0	0.3	1	0.1	0.75	0.68	0.44	two cutoff
4	0	0.4	1	0.1	0.75	0.60	0.66	two cutoff
5	0	0.5	1	0.1	0.75	0.54	0.93	two cutoff
6	0	0.6	1	0.1	0.75	0.48	1.25	two cutoff
7	0	0.1	1	0.1	0.75	1.82	0.05	two cutoff
8	0	0.1	1	0.2	0.75	0.59	0.17	two cutoff
9	0	0.1	1	0.3	0.75	0.46	0.22	two cutoff
10	0	0.1	1	0.4	0.75	0.38	0.26	two cutoff
11	0	0.1	1	0.5	0.75	0.32	0.31	two cutoff
12	0	0.1	1	0.6	0.75	0.28	0.36	two cutoff
13	0.1	0.1	1	0.1	0.75	0.84	0.12	two cutoff
14	0.2	0.1	1	0.1	0.75	0.85	0.12	two cutoff
15	0.3	0.1	1	0.1	0.75	0.86	0.12	two cutoff
16	0.4	0.1	1	0.1	0.75	0.87	0.12	two cutoff
17	0.5	0.1	1	0.1	0.75	0.87	0.12	two cutoff
18	0.6	0.1	1	0.1	0.75	0.87	0.12	two cutoff
19	0.7	0.1	1	0.1	0.75	0.87	0.11	two cutoff
20	0.8	0.1	1	0.1	0.75	0.87	0.11	two cutoff
21	0.9	0.1	1	0.1	0.75	0.88	0.11	two cutoff
22	0.1	0.2	1	0.1	0.75	0.76	0.26	two cutoff
23	0.2	0.2	1	0.1	0.75	0.77	0.26	two cutoff
24	0.3	0.2	1	0.1	0.75	0.78	0.26	two cutoff
25	0.4	0.2	1	0.1	0.75	0.78	0.26	two cutoff
26	0.5	0.2	1	0.1	0.75	0.78	0.26	two cutoff
27	0.6	0.2	1	0.1	0.75	0.77	0.26	two cutoff
28	0.7	0.2	1	0.1	0.75	0.75	0.27	two cutoff
29	0.8	0.2	1	0.1	0.75	0.73	0.27	two cutoff
30	0.9	0.2	1	0.1	0.75	0.68	0.30	two cutoff

31	0.1	0.3	1	0.1	0.75	0.67	0.45	two cutoff
32	0.2	0.3	1	0.1	0.75	0.68	0.44	two cutoff
33	0.3	0.3	1	0.1	0.75	0.68	0.44	two cutoff
34	0.4	0.3	1	0.1	0.75	0.68	0.44	two cutoff
35	0.5	0.3	1	0.1	0.75	0.67	0.45	two cutoff
36	0.6	0.3	1	0.1	0.75	0.66	0.46	two cutoff
37	0.7	0.3	1	0.1	0.75	0.62	0.48	two cutoff
38	0.8	0.3	1	0.1	0.75	0.59	0.51	two cutoff
39	0.9	0.3	1	0.1	0.75	0.53	0.57	two cutoff
40	0.1	0.4	1	0.1	0.75	0.60	0.67	two cutoff
41	0.2	0.4	1	0.1	0.75	0.60	0.67	two cutoff
42	0.3	0.4	1	0.1	0.75	0.60	0.67	two cutoff
43	0.4	0.4	1	0.1	0.75	0.59	0.68	two cutoff
44	0.5	0.4	1	0.1	0.75	0.57	0.70	two cutoff
45	0.6	0.4	1	0.1	0.75	0.56	0.72	two cutoff
46	0.7	0.4	1	0.1	0.75	0.53	0.76	two cutoff
47	0.8	0.4	1	0.1	0.75	0.49	0.82	two cutoff
48	0.9	0.4	1	0.1	0.75	0.43	0.92	two cutoff
49	0.1	0.5	1	0.1	0.75	0.53	0.94	two cutoff
50	0.2	0.5	1	0.1	0.75	0.53	0.95	two cutoff
51	0.3	0.5	1	0.1	0.75	0.53	0.95	two cutoff
52	0.4	0.5	1	0.1	0.75	0.52	0.97	two cutoff
53	0.5	0.5	1	0.1	0.75	0.49	1.02	two cutoff
54	0.6	0.5	1	0.1	0.75	0.48	1.04	two cutoff
55	0.7	0.5	1	0.1	0.75	0.46	1.09	two cutoff
56	0.8	0.5	1	0.1	0.75	0.42	1.20	two cutoff
57	0.9	0.5	1	0.1	0.75	0.37	1.36	two cutoff
58	0.1	0.6	1	0.1	0.75	0.47	1.27	two cutoff
59	0.2	0.6	1	0.1	0.75	0.47	1.28	two cutoff
60	0.3	0.6	1	0.1	0.75	0.46	1.29	two cutoff
61	0.4	0.6	1	0.1	0.75	0.45	1.32	two cutoff
62	0.5	0.6	1	0.1	0.75	0.43	1.40	two cutoff
63	0.6	0.6	1	0.1	0.75	0.42	1.44	two cutoff
64	0.7	0.6	1	0.1	0.75	0.38	1.56	two cutoff
65	0.8	0.6	1	0.1	0.75	0.36	1.67	two cutoff
66	0.9	0.6	1	0.1	0.75	0.32	1.89	two cutoff
67	0.1	0.1	1	0.2	0.75	0.59	0.17	two cutoff
68	0.2	0.1	1	0.2	0.75	0.59	0.17	two cutoff
69	0.3	0.1	1	0.2	0.75	0.60	0.17	two cutoff
70	0.4	0.1	1	0.2	0.75	0.60	0.17	two cutoff
71	0.5	0.1	1	0.2	0.75	0.61	0.16	two cutoff
72	0.6	0.1	1	0.2	0.75	0.61	0.16	two cutoff
73	0.7	0.1	1	0.2	0.75	0.62	0.16	two cutoff

74	0.8	0.1	1	0.2	0.75	0.62	0.16	two cutoff
75	0.9	0.1	1	0.2	0.75	0.62	0.16	two cutoff
76	0.1	0.1	1	0.3	0.75	0.46	0.22	two cutoff
77	0.2	0.1	1	0.3	0.75	0.46	0.22	two cutoff
78	0.3	0.1	1	0.3	0.75	0.47	0.21	two cutoff
79	0.4	0.1	1	0.3	0.75	0.47	0.21	two cutoff
80	0.5	0.1	1	0.3	0.75	0.47	0.21	two cutoff
81	0.6	0.1	1	0.3	0.75	0.48	0.21	two cutoff
82	0.7	0.1	1	0.3	0.75	0.48	0.21	two cutoff
83	0.8	0.1	1	0.3	0.75	0.48	0.21	two cutoff
84	0.9	0.1	1	0.3	0.75	0.48	0.21	two cutoff
85	0.1	0.1	1	0.4	0.75	0.38	0.27	two cutoff
86	0.2	0.1	1	0.4	0.75	0.38	0.26	two cutoff
87	0.3	0.1	1	0.4	0.75	0.38	0.26	two cutoff
88	0.4	0.1	1	0.4	0.75	0.39	0.26	two cutoff
89	0.5	0.1	1	0.4	0.75	0.39	0.26	two cutoff
90	0.6	0.1	1	0.4	0.75	0.39	0.26	two cutoff
91	0.7	0.1	1	0.4	0.75	0.39	0.26	two cutoff
92	0.8	0.1	1	0.4	0.75	0.39	0.26	two cutoff
93	0.9	0.1	1	0.4	0.75	0.39	0.26	two cutoff
94	0.1	0.1	1	0.5	0.75	0.32	0.31	two cutoff
95	0.2	0.1	1	0.5	0.75	0.32	0.31	two cutoff
96	0.3	0.1	1	0.5	0.75	0.33	0.31	two cutoff
97	0.4	0.1	1	0.5	0.75	0.33	0.31	two cutoff
98	0.5	0.1	1	0.5	0.75	0.33	0.31	two cutoff
99	0.6	0.1	1	0.5	0.75	0.33	0.30	two cutoff
100	0.7	0.1	1	0.5	0.75	31.82	0.00	two cutoff
101	0.8	0.1	1	0.5	0.75	32.15	0.00	two cutoff
102	0.9	0.1	1	0.5	0.75	0.33	0.30	two cutoff
103	0.1	0.1	1	0.6	0.75	0.28	0.36	two cutoff
104	0.2	0.1	1	0.6	0.75	0.28	0.36	two cutoff
105	0.3	0.1	1	0.6	0.75	0.28	0.35	two cutoff
106	0.4	0.1	1	0.6	0.75	0.28	0.35	two cutoff
107	0.5	0.1	1	0.6	0.75	0.28	0.35	two cutoff
108	0.6	0.1	1	0.6	0.75	0.28	0.35	two cutoff
109	0.7	0.1	1	0.6	0.75	0.28	0.35	two cutoff
110	0.8	0.1	1	0.6	0.75	0.28	0.35	two cutoff
111	0.9	0.1	1	0.6	0.75	0.28	0.35	two cutoff
112	0	0.2	1	0.2	0.75	0.54	0.37	two cutoff
113	0.1	0.2	1	0.2	0.75	0.55	0.37	two cutoff
114	0.2	0.2	1	0.2	0.75	0.55	0.36	two cutoff
115	0.3	0.2	1	0.2	0.75	0.56	0.36	two cutoff
116	0.4	0.2	1	0.2	0.75	0.57	0.35	two cutoff

117	0.5	0.2	1	0.2	0.75	0.57	0.35	two cutoff
118	0.6	0.2	1	0.2	0.75	0.58	0.35	two cutoff
119	0.7	0.2	1	0.2	0.75	0.58	0.34	two cutoff
120	0.8	0.2	1	0.2	0.75	0.59	0.34	two cutoff
121	0.9	0.2	1	0.2	0.75	0.60	0.33	two cutoff
122	0	0.3	1	0.3	0.75	0.40	0.74	two cutoff
123	0.1	0.3	1	0.3	0.75	0.41	0.74	two cutoff
124	0.2	0.3	1	0.3	0.75	0.41	0.73	two cutoff
125	0.3	0.3	1	0.3	0.75	0.41	0.72	two cutoff
126	0.4	0.3	1	0.3	0.75	0.43	0.70	two cutoff
127	0.5	0.3	1	0.3	0.75	0.43	0.69	two cutoff
128	0.6	0.3	1	0.3	0.75	0.44	0.68	two cutoff
129	0.7	0.3	1	0.3	0.75	0.44	0.68	two cutoff
130	0.8	0.3	1	0.3	0.75	0.45	0.66	two cutoff
131	0.9	0.3	1	0.3	0.75	0.46	0.65	two cutoff
132	0	0.4	1	0.4	0.75	0.32	1.23	two cutoff
133	0.1	0.4	1	0.4	0.75	0.32	1.24	two cutoff
134	0.2	0.4	1	0.4	0.75	0.33	1.21	two cutoff
135	0.3	0.4	1	0.4	0.75	0.34	1.19	two cutoff
136	0.4	0.4	1	0.4	0.75	0.34	1.17	two cutoff
137	0.5	0.4	1	0.4	0.75	0.35	1.15	two cutoff
138	0.6	0.4	1	0.4	0.75	0.35	1.13	two cutoff
139	0.7	0.4	1	0.4	0.75	0.36	1.10	two cutoff
140	0.8	0.4	1	0.4	0.75	0.37	1.09	two cutoff
141	0.9	0.4	1	0.4	0.75	0.38	1.06	two cutoff
142	0	0.5	1	0.5	0.75	0.27	1.84	two cutoff
143	0.1	0.5	1	0.5	0.75	0.28	1.82	two cutoff
144	0.2	0.5	1	0.5	0.75	0.28	1.81	two cutoff
145	0.3	0.5	1	0.5	0.75	0.28	1.77	two cutoff
146	0.4	0.5	1	0.5	0.75	0.29	1.74	two cutoff
147	0.5	0.5	1	0.5	0.75	0.29	1.72	two cutoff
148	0.6	0.5	1	0.5	0.75	0.30	1.67	two cutoff
149	0.7	0.5	1	0.5	0.75	0.30	1.65	two cutoff
150	0.8	0.5	1	0.5	0.75	0.31	1.61	two cutoff
151	0.9	0.5	1	0.5	0.75	0.32	1.56	two cutoff
152	0	0.1	0.1	0.1	0.75	1.65	0.06	two cutoff
153	0	0.1	0.2	0.1	0.75	1.63	0.06	two cutoff
154	0	0.1	0.3	0.1	0.75	1.61	0.06	two cutoff
155	0	0.1	0.4	0.1	0.75	1.58	0.06	two cutoff
156	0	0.1	0.5	0.1	0.75	1.55	0.06	two cutoff
157	0	0.1	0.6	0.1	0.75	1.49	0.07	two cutoff
158	0	0.1	0.7	0.1	0.75	1.43	0.07	two cutoff
159	0	0.1	0.8	0.1	0.75	1.37	0.07	two cutoff

160	0	0.1	0.9	0.1	0.75	1.01	0.10	two cutoff
161	0	0.1	1	0.1	0.75	0.84	0.12	two cutoff
162	0	0.2	0.1	0.1	0.75	1.46	0.14	two cutoff
163	0	0.2	0.2	0.1	0.75	1.45	0.14	two cutoff
164	0	0.2	0.3	0.1	0.75	1.43	0.14	two cutoff
165	0	0.2	0.4	0.1	0.75	1.41	0.14	two cutoff
166	0	0.2	0.5	0.1	0.75	1.38	0.14	two cutoff
167	0	0.2	0.6	0.1	0.75	1.34	0.15	two cutoff
168	0	0.2	0.7	0.1	0.75	1.24	0.16	two cutoff
169	0	0.2	0.8	0.1	0.75	1.15	0.17	two cutoff
170	0	0.2	0.9	0.1	0.75	0.91	0.22	two cutoff
171	0	0.2	1	0.1	0.75	0.75	0.26	two cutoff
172	0	0.3	0.1	0.1	0.75	1.28	0.24	two cutoff
173	0	0.3	0.2	0.1	0.75	1.27	0.24	two cutoff
174	0	0.3	0.3	0.1	0.75	1.26	0.24	two cutoff
175	0	0.3	0.4	0.1	0.75	1.24	0.24	two cutoff
176	0	0.3	0.5	0.1	0.75	1.22	0.25	two cutoff
177	0	0.3	0.6	0.1	0.75	1.19	0.25	two cutoff
178	0	0.3	0.7	0.1	0.75	1.07	0.28	two cutoff
179	0	0.3	0.8	0.1	0.75	1.02	0.29	two cutoff
180	0	0.3	0.9	0.1	0.75	0.93	0.32	two cutoff
181	0	0.3	1	0.1	0.75	0.68	0.44	two cutoff
182	0	0.4	0.1	0.1	0.75	1.11	0.36	two cutoff
183	0	0.4	0.2	0.1	0.75	1.11	0.36	two cutoff
184	0	0.4	0.3	0.1	0.75	1.10	0.36	two cutoff
185	0	0.4	0.4	0.1	0.75	1.09	0.37	two cutoff
186	0	0.4	0.5	0.1	0.75	1.07	0.38	two cutoff
187	0	0.4	0.6	0.1	0.75	1.05	0.38	two cutoff
188	0	0.4	0.7	0.1	0.75	1.03	0.39	two cutoff
189	0	0.4	0.8	0.1	0.75	0.90	0.44	two cutoff
190	0	0.4	0.9	0.1	0.75	0.78	0.51	two cutoff
191	0	0.4	1	0.1	0.75	0.60	0.66	two cutoff
192	0	0.5	0.1	0.1	0.75	0.98	0.51	two cutoff
193	0	0.5	0.2	0.1	0.75	0.97	0.51	two cutoff
194	0	0.5	0.3	0.1	0.75	0.97	0.52	two cutoff
195	0	0.5	0.4	0.1	0.75	0.96	0.52	two cutoff
196	0	0.5	0.5	0.1	0.75	0.95	0.53	two cutoff
197	0	0.5	0.6	0.1	0.75	0.92	0.54	two cutoff
198	0	0.5	0.7	0.1	0.75	0.86	0.58	two cutoff
199	0	0.5	0.8	0.1	0.75	0.80	0.62	two cutoff
200	0	0.5	0.9	0.1	0.75	0.70	0.72	two cutoff
201	0	0.5	1	0.1	0.75	0.54	0.93	two cutoff
202	0	0.1	0.1	0.2	0.75	1.48	0.07	two cutoff

203	0	0.1	0.2	0.2	0.75	1.46	0.07	two cutoff
204	0	0.1	0.3	0.2	0.75	1.44	0.07	two cutoff
205	0	0.1	0.4	0.2	0.75	1.40	0.07	two cutoff
206	0	0.1	0.5	0.2	0.75	1.30	0.08	two cutoff
207	0	0.1	0.6	0.2	0.75	1.26	0.08	two cutoff
208	0	0.1	0.7	0.2	0.75	1.14	0.09	two cutoff
209	0	0.1	0.8	0.2	0.75	0.99	0.10	two cutoff
210	0	0.1	0.9	0.2	0.75	0.72	0.14	two cutoff
211	0	0.1	1	0.2	0.75	0.59	0.17	two cutoff
212	0	0.1	0.1	0.3	0.75	1.29	0.08	two cutoff
213	0	0.1	0.2	0.3	0.75	1.27	0.08	two cutoff
214	0	0.1	0.3	0.3	0.75	1.24	0.08	two cutoff
215	0	0.1	0.4	0.3	0.75	1.19	0.08	two cutoff
216	0	0.1	0.5	0.3	0.75	1.07	0.09	two cutoff
217	0	0.1	0.6	0.3	0.75	1.04	0.10	two cutoff
218	0	0.1	0.7	0.3	0.75	0.93	0.11	two cutoff
219	0	0.1	0.8	0.3	0.75	0.77	0.13	two cutoff
220	0	0.1	0.9	0.3	0.75	0.57	0.18	two cutoff
221	0	0.1	1	0.3	0.75	0.46	0.22	two cutoff
222	0	0.1	0.1	0.4	0.75	1.12	0.09	two cutoff
223	0	0.1	0.2	0.4	0.75	1.09	0.09	two cutoff
224	0	0.1	0.3	0.4	0.75	1.06	0.09	two cutoff
225	0	0.1	0.4	0.4	0.75	1.02	0.10	two cutoff
226	0	0.1	0.5	0.4	0.75	0.96	0.10	two cutoff
227	0	0.1	0.6	0.4	0.75	0.88	0.11	two cutoff
228	0	0.1	0.7	0.4	0.75	0.77	0.13	two cutoff
229	0	0.1	0.8	0.4	0.75	0.68	0.15	two cutoff
230	0	0.1	0.9	0.4	0.75	0.47	0.21	two cutoff
231	0	0.1	1	0.4	0.75	0.38	0.26	two cutoff
232	0	0.1	0.1	0.5	0.75	0.97	0.10	two cutoff
233	0	0.1	0.2	0.5	0.75	0.95	0.11	two cutoff
234	0	0.1	0.3	0.5	0.75	0.92	0.11	two cutoff
235	0	0.1	0.4	0.5	0.75	0.88	0.11	two cutoff
236	0	0.1	0.5	0.5	0.75	0.82	0.12	two cutoff
237	0	0.1	0.6	0.5	0.75	0.75	0.13	two cutoff
238	0	0.1	0.7	0.5	0.75	0.66	0.15	two cutoff
239	0	0.1	0.8	0.5	0.75	0.52	0.19	two cutoff
240	0	0.1	0.9	0.5	0.75	0.40	0.25	two cutoff
241	0	0.1	1	0.5	0.75	0.32	0.31	two cutoff
242	0	0.2	0.1	0.2	0.75	1.42	0.14	two cutoff
243	0	0.2	0.2	0.2	0.75	1.38	0.15	two cutoff
244	0	0.2	0.3	0.2	0.75	1.34	0.15	two cutoff
245	0	0.2	0.4	0.2	0.75	1.29	0.15	two cutoff

246	0	0.2	0.5	0.2	0.75	1.24	0.16	two cutoff
247	0	0.2	0.6	0.2	0.75	1.15	0.17	two cutoff
248	0	0.2	0.7	0.2	0.75	1.05	0.19	two cutoff
249	0	0.2	0.8	0.2	0.75	0.90	0.22	two cutoff
250	0	0.2	0.9	0.2	0.75	0.66	0.30	two cutoff
251	0	0.2	1	0.2	0.75	0.54	0.37	two cutoff
252	0	0.3	0.1	0.3	0.75	1.23	0.24	two cutoff
253	0	0.3	0.2	0.3	0.75	1.18	0.25	two cutoff
254	0	0.3	0.3	0.3	0.75	1.13	0.27	two cutoff
255	0	0.3	0.4	0.3	0.75	1.07	0.28	two cutoff
256	0	0.3	0.5	0.3	0.75	1.00	0.30	two cutoff
257	0	0.3	0.6	0.3	0.75	0.92	0.33	two cutoff
258	0	0.3	0.7	0.3	0.75	0.82	0.37	two cutoff
259	0	0.3	0.8	0.3	0.75	0.68	0.44	two cutoff
260	0	0.3	0.9	0.3	0.75	0.50	0.60	two cutoff
261	0	0.3	1	0.3	0.75	0.40	0.74	two cutoff
262	0	0.4	0.1	0.4	0.75	1.06	0.38	two cutoff
263	0	0.4	0.2	0.4	0.75	1.01	0.40	two cutoff
264	0	0.4	0.3	0.4	0.75	0.96	0.42	two cutoff
265	0	0.4	0.4	0.4	0.75	0.90	0.45	two cutoff
266	0	0.4	0.5	0.4	0.75	0.83	0.48	two cutoff
267	0	0.4	0.6	0.4	0.75	0.75	0.54	two cutoff
268	0	0.4	0.7	0.4	0.75	0.66	0.60	two cutoff
269	0	0.4	0.8	0.4	0.75	0.55	0.73	two cutoff
270	0	0.4	0.9	0.4	0.75	0.40	1.00	two cutoff
271	0	0.4	1	0.4	0.75	0.32	1.23	two cutoff
272	0	0.5	0.1	0.5	0.75	0.93	0.54	two cutoff
273	0	0.5	0.2	0.5	0.75	0.85	0.59	two cutoff
274	0	0.5	0.3	0.5	0.75	0.82	0.61	two cutoff
275	0	0.5	0.4	0.5	0.75	0.77	0.65	two cutoff
276	0	0.5	0.5	0.5	0.75	0.70	0.72	two cutoff
277	0	0.5	0.6	0.5	0.75	0.64	0.78	two cutoff
278	0	0.5	0.7	0.5	0.75	0.56	0.90	two cutoff
279	0	0.5	0.8	0.5	0.75	0.46	1.09	two cutoff
280	0	0.5	0.9	0.5	0.75	0.33	1.50	two cutoff
281	0	0.5	1	0.5	0.75	0.27	1.84	two cutoff
282	0	0.6	0.1	0.6	0.75	0.81	0.74	two cutoff
283	0	0.6	0.2	0.6	0.75	0.74	0.81	two cutoff
284	0	0.6	0.3	0.6	0.75	0.72	0.84	two cutoff
285	0	0.6	0.4	0.6	0.75	0.67	0.90	two cutoff
286	0	0.6	0.5	0.6	0.75	0.61	0.98	two cutoff
287	0	0.6	0.6	0.6	0.75	0.56	1.08	two cutoff
288	0	0.6	0.7	0.6	0.75	0.48	1.25	two cutoff

289	0	0.6	0.8	0.6	0.75	0.39	1.53	two cutoff
290	0	0.6	0.9	0.6	0.75	0.34	1.76	two cutoff
291	0	0.6	1	0.6	0.75	0.23	2.58	two cutoff

A.4- Table of results the specific uplift force and exit gradient ratio of permeability in x-direction to permeability of y-direction ($K_x/K_y=8$)

Table (A-4): The Results obtained by using the ratio of ($\frac{K_x}{K_y}=8$) with the ratio of head differential ($\frac{H}{B} = 0.75$)

Run No.	x1/B	d1/B	x2/B	d2/B	H/B	(i.g*H)/B	(f uplift/max f)	case
1	0	0.1	1	0.1	0.75	0.93	1.04	two cutoff
2	0	0.2	1	0.1	0.75	0.82	1.12	two cutoff
3	0	0.3	1	0.1	0.75	0.71	1.57	two cutoff
4	0	0.4	1	0.1	0.75	0.62	1.81	two cutoff
5	0	0.5	1	0.1	0.75	0.56	2.14	two cutoff
6	0	0.6	1	0.1	0.75	0.47	2.50	two cutoff
7	0	0.1	1	0.1	0.75	2.21	1.49	two cutoff
8	0	0.1	1	0.2	0.75	0.65	0.87	two cutoff
9	0	0.1	1	0.3	0.75	0.51	0.80	two cutoff
10	0	0.1	1	0.4	0.75	0.42	0.72	two cutoff
11	0	0.1	1	0.5	0.75	0.36	0.69	two cutoff
12	0	0.1	1	0.6	0.75	0.31	0.66	two cutoff
13	0.1	0.1	1	0.1	0.75	0.95	0.91	two cutoff
14	0.2	0.1	1	0.1	0.75	0.97	0.86	two cutoff
15	0.3	0.1	1	0.1	0.75	0.98	0.82	two cutoff
16	0.4	0.1	1	0.1	0.75	0.99	0.78	two cutoff
17	0.5	0.1	1	0.1	0.75	0.99	0.76	two cutoff
18	0.6	0.1	1	0.1	0.75	1.00	0.73	two cutoff
19	0.7	0.1	1	0.1	0.75	1.00	0.70	two cutoff
20	0.8	0.1	1	0.1	0.75	1.01	0.69	two cutoff
21	0.9	0.1	1	0.1	0.75	1.03	0.68	two cutoff
22	0.1	0.2	1	0.1	0.75	0.82	1.09	two cutoff
23	0.2	0.2	1	0.1	0.75	0.83	0.99	two cutoff
24	0.3	0.2	1	0.1	0.75	0.85	0.91	two cutoff
25	0.4	0.2	1	0.1	0.75	0.85	0.85	two cutoff

26	0.5	0.2	1	0.1	0.75	0.85	0.81	two cutoff
27	0.6	0.2	1	0.1	0.75	0.84	0.76	two cutoff
28	0.7	0.2	1	0.1	0.75	0.83	0.72	two cutoff
29	0.8	0.2	1	0.1	0.75	0.80	0.68	two cutoff
30	0.9	0.2	1	0.1	0.75	0.75	0.65	two cutoff
31	0.1	0.3	1	0.1	0.75	0.71	1.27	two cutoff
32	0.2	0.3	1	0.1	0.75	0.71	1.12	two cutoff
33	0.3	0.3	1	0.1	0.75	0.72	1.00	two cutoff
34	0.4	0.3	1	0.1	0.75	0.72	0.91	two cutoff
35	0.5	0.3	1	0.1	0.75	0.71	0.01	two cutoff
36	0.6	0.3	1	0.1	0.75	0.69	0.77	two cutoff
37	0.7	0.3	1	0.1	0.75	0.68	0.71	two cutoff
38	0.8	0.3	1	0.1	0.75	0.63	0.67	two cutoff
39	0.9	0.3	1	0.1	0.75	0.58	0.62	two cutoff
40	0.1	0.4	1	0.1	0.75	0.62	1.45	two cutoff
41	0.2	0.4	1	0.1	0.75	0.61	1.28	two cutoff
42	0.3	0.4	1	0.1	0.75	0.61	1.07	two cutoff
43	0.4	0.4	1	0.1	0.75	0.61	0.95	two cutoff
44	0.5	0.4	1	0.1	0.75	0.60	0.83	two cutoff
45	0.6	0.4	1	0.1	0.75	0.57	0.78	two cutoff
46	0.7	0.4	1	0.1	0.75	0.55	0.71	two cutoff
47	0.8	0.4	1	0.1	0.75	0.52	0.65	two cutoff
48	0.9	0.4	1	0.1	0.75	0.47	0.61	two cutoff
49	0.1	0.5	1	0.1	0.75	0.54	1.62	two cutoff
50	0.2	0.5	1	0.1	0.75	0.53	1.30	two cutoff
51	0.3	0.5	1	0.1	0.75	0.53	1.13	two cutoff
52	0.4	0.5	1	0.1	0.75	0.52	0.99	two cutoff
53	0.5	0.5	1	0.1	0.75	0.51	0.87	two cutoff
54	0.6	0.5	1	0.1	0.75	0.49	0.81	two cutoff
55	0.7	0.5	1	0.1	0.75	0.47	0.71	two cutoff
56	0.8	0.5	1	0.1	0.75	0.44	0.64	two cutoff
57	0.9	0.5	1	0.1	0.75	0.40	0.59	two cutoff
58	0.1	0.6	1	0.1	0.75	0.47	1.77	two cutoff
59	0.2	0.6	1	0.1	0.75	0.47	1.38	two cutoff
60	0.3	0.6	1	0.1	0.75	0.46	1.18	two cutoff
61	0.4	0.6	1	0.1	0.75	0.45	1.01	two cutoff
62	0.5	0.6	1	0.1	0.75	0.44	0.92	two cutoff
63	0.6	0.6	1	0.1	0.75	0.43	0.79	two cutoff
64	0.7	0.6	1	0.1	0.75	0.40	0.71	two cutoff
65	0.8	0.6	1	0.1	0.75	0.37	0.65	two cutoff

66	0.9	0.6	1	0.1	0.75	0.35	0.59	two cutoff
67	0.1	0.1	1	0.2	0.75	0.66	0.78	two cutoff
68	0.2	0.1	1	0.2	0.75	0.66	0.75	two cutoff
69	0.3	0.1	1	0.2	0.75	0.68	0.72	two cutoff
70	0.4	0.1	1	0.2	0.75	0.68	0.70	two cutoff
71	0.5	0.1	1	0.2	0.75	0.69	0.67	two cutoff
72	0.6	0.1	1	0.2	0.75	0.69	0.66	two cutoff
73	0.7	0.1	1	0.2	0.75	0.70	0.65	two cutoff
74	0.8	0.1	1	0.2	0.75	0.70	0.64	two cutoff
75	0.9	0.1	1	0.2	0.75	0.71	0.62	two cutoff
76	0.1	0.1	1	0.3	0.75	0.51	0.71	two cutoff
77	0.2	0.1	1	0.3	0.75	0.52	0.68	two cutoff
78	0.3	0.1	1	0.3	0.75	0.52	0.66	two cutoff
79	0.4	0.1	1	0.3	0.75	0.53	0.64	two cutoff
80	0.5	0.1	1	0.3	0.75	0.53	0.64	two cutoff
81	0.6	0.1	1	0.3	0.75	0.53	0.62	two cutoff
82	0.7	0.1	1	0.3	0.75	0.53	0.61	two cutoff
83	0.8	0.1	1	0.3	0.75	0.54	0.60	two cutoff
84	0.9	0.1	1	0.3	0.75	0.54	0.59	two cutoff
85	0.1	0.1	1	0.4	0.75	0.42	0.66	two cutoff
86	0.2	0.1	1	0.4	0.75	0.43	0.64	two cutoff
87	0.3	0.1	1	0.4	0.75	0.43	0.63	two cutoff
88	0.4	0.1	1	0.4	0.75	0.43	0.61	two cutoff
89	0.5	0.1	1	0.4	0.75	0.43	0.60	two cutoff
90	0.6	0.1	1	0.4	0.75	0.43	0.59	two cutoff
91	0.7	0.1	1	0.4	0.75	0.44	0.59	two cutoff
92	0.8	0.1	1	0.4	0.75	0.44	0.58	two cutoff
93	0.9	0.1	1	0.4	0.75	0.44	0.57	two cutoff
94	0.1	0.1	1	0.5	0.75	0.36	0.63	two cutoff
95	0.2	0.1	1	0.5	0.75	0.36	0.62	two cutoff
96	0.3	0.1	1	0.5	0.75	0.36	0.60	two cutoff
97	0.4	0.1	1	0.5	0.75	0.37	0.59	two cutoff
98	0.5	0.1	1	0.5	0.75	0.37	0.59	two cutoff
99	0.6	0.1	1	0.5	0.75	0.37	0.58	two cutoff
100	0.7	0.1	1	0.5	0.75	0.37	0.57	two cutoff
101	0.8	0.1	1	0.5	0.75	0.37	0.57	two cutoff
102	0.9	0.1	1	0.5	0.75	0.37	0.56	two cutoff
103	0.1	0.1	1	0.6	0.75	0.31	0.61	two cutoff
104	0.2	0.1	1	0.6	0.75	0.31	0.59	two cutoff
105	0.3	0.1	1	0.6	0.75	0.32	0.59	two cutoff

106	0.4	0.1	1	0.6	0.75	0.32	0.58	two cutoff
107	0.5	0.1	1	0.6	0.75	0.32	0.58	two cutoff
108	0.6	0.1	1	0.6	0.75	0.32	0.57	two cutoff
109	0.7	0.1	1	0.6	0.75	0.32	0.56	two cutoff
110	0.8	0.1	1	0.6	0.75	0.32	0.55	two cutoff
111	0.9	0.1	1	0.6	0.75	0.32	0.55	two cutoff
112	0	0.2	1	0.2	0.75	0.59	1.04	two cutoff
113	0.1	0.2	1	0.2	0.75	0.60	0.93	two cutoff
114	0.2	0.2	1	0.2	0.75	0.61	0.84	two cutoff
115	0.3	0.2	1	0.2	0.75	0.62	0.80	two cutoff
116	0.4	0.2	1	0.2	0.75	0.63	0.75	two cutoff
117	0.5	0.2	1	0.2	0.75	0.64	0.70	two cutoff
118	0.6	0.2	1	0.2	0.75	0.65	0.67	two cutoff
119	0.7	0.2	1	0.2	0.75	0.66	0.67	two cutoff
120	0.8	0.2	1	0.2	0.75	0.67	0.64	two cutoff
121	0.9	0.2	1	0.2	0.75	0.68	0.63	two cutoff
122	0	0.3	1	0.3	0.75	0.44	1.04	two cutoff
123	0.1	0.3	1	0.3	0.75	0.45	0.93	two cutoff
124	0.2	0.3	1	0.3	0.75	0.46	0.84	two cutoff
125	0.3	0.3	1	0.3	0.75	0.47	0.78	two cutoff
126	0.4	0.3	1	0.3	0.75	0.48	0.76	two cutoff
127	0.5	0.3	1	0.3	0.75	0.48	0.70	two cutoff
128	0.6	0.3	1	0.3	0.75	0.49	0.67	two cutoff
129	0.7	0.3	1	0.3	0.75	0.50	0.64	two cutoff
130	0.8	0.3	1	0.3	0.75	0.51	0.62	two cutoff
131	0.9	0.3	1	0.3	0.75	0.52	0.60	two cutoff
132	0	0.4	1	0.4	0.75	0.36	1.04	two cutoff
133	0.1	0.4	1	0.4	0.75	0.36	0.96	two cutoff
134	0.2	0.4	1	0.4	0.75	0.37	0.83	two cutoff
135	0.3	0.4	1	0.4	0.75	0.38	0.78	two cutoff
136	0.4	0.4	1	0.4	0.75	0.38	0.73	two cutoff
137	0.5	0.4	1	0.4	0.75	0.39	0.69	two cutoff
138	0.6	0.4	1	0.4	0.75	0.40	0.65	two cutoff
139	0.7	0.4	1	0.4	0.75	0.41	0.63	two cutoff
140	0.8	0.4	1	0.4	0.75	0.42	0.60	two cutoff
141	0.9	0.4	1	0.4	0.75	0.42	0.58	two cutoff
142	0	0.5	1	0.5	0.75	0.30	1.04	two cutoff
143	0.1	0.5	1	0.5	0.75	0.31	0.95	two cutoff
144	0.2	0.5	1	0.5	0.75	0.31	0.83	two cutoff
145	0.3	0.5	1	0.5	0.75	0.32	0.74	two cutoff

146	0.4	0.5	1	0.5	0.75	0.32	0.72	two cutoff
147	0.5	0.5	1	0.5	0.75	0.33	0.68	two cutoff
148	0.6	0.5	1	0.5	0.75	0.34	0.65	two cutoff
149	0.7	0.5	1	0.5	0.75	0.34	0.62	two cutoff
150	0.8	0.5	1	0.5	0.75	0.35	0.59	two cutoff
151	0.9	0.5	1	0.5	0.75	0.36	0.57	two cutoff
152	0	0.1	0.1	0.1	0.75	1.86	6.89	two cutoff
153	0	0.1	0.2	0.1	0.75	1.82	3.85	two cutoff
154	0	0.1	0.3	0.1	0.75	1.79	2.73	two cutoff
155	0	0.1	0.4	0.1	0.75	1.74	2.11	two cutoff
156	0	0.1	0.5	0.1	0.75	1.69	1.79	two cutoff
157	0	0.1	0.6	0.1	0.75	1.60	1.51	two cutoff
158	0	0.1	0.7	0.1	0.75	1.51	1.36	two cutoff
159	0	0.1	0.8	0.1	0.75	1.33	1.22	two cutoff
160	0	0.1	0.9	0.1	0.75	1.15	1.08	two cutoff
161	0	0.1	1	0.1	0.75	0.93	1.04	two cutoff
162	0	0.2	0.1	0.1	0.75	1.57	9.63	two cutoff
163	0	0.2	0.2	0.1	0.75	1.54	6.83	two cutoff
164	0	0.2	0.3	0.1	0.75	1.53	3.72	two cutoff
165	0	0.2	0.4	0.1	0.75	1.49	3.02	two cutoff
166	0	0.2	0.5	0.1	0.75	1.45	2.35	two cutoff
167	0	0.2	0.6	0.1	0.75	1.37	2.00	two cutoff
168	0	0.2	0.7	0.1	0.75	1.24	1.77	two cutoff
169	0	0.2	0.8	0.1	0.75	1.15	1.54	two cutoff
170	0	0.2	0.9	0.1	0.75	1.01	1.48	two cutoff
171	0	0.2	1	0.1	0.75	0.82	1.29	two cutoff
172	0	0.3	0.1	0.1	0.75	1.30	11.84	two cutoff
173	0	0.3	0.2	0.1	0.75	1.29	6.19	two cutoff
174	0	0.3	0.3	0.1	0.75	1.28	4.76	two cutoff
175	0	0.3	0.4	0.1	0.75	1.27	3.80	two cutoff
176	0	0.3	0.5	0.1	0.75	1.23	2.98	two cutoff
177	0	0.3	0.6	0.1	0.75	1.17	2.61	two cutoff
178	0	0.3	0.7	0.1	0.75	1.11	2.18	two cutoff
179	0	0.3	0.8	0.1	0.75	0.99	1.91	two cutoff
180	0	0.3	0.9	0.1	0.75	0.79	1.68	two cutoff
181	0	0.3	1	0.1	0.75	0.71	1.57	two cutoff
182	0	0.4	0.1	0.1	0.75	1.10	13.79	two cutoff
183	0	0.4	0.2	0.1	0.75	1.09	7.93	two cutoff
184	0	0.4	0.3	0.1	0.75	1.08	5.78	two cutoff
185	0	0.4	0.4	0.1	0.75	1.07	4.63	two cutoff

186	0	0.4	0.5	0.1	0.75	1.05	3.64	two cutoff
187	0	0.4	0.6	0.1	0.75	0.96	2.85	two cutoff
188	0	0.4	0.7	0.1	0.75	0.90	2.46	two cutoff
189	0	0.4	0.8	0.1	0.75	0.85	2.30	two cutoff
190	0	0.4	0.9	0.1	0.75	0.78	2.06	two cutoff
191	0	0.4	1	0.1	0.75	0.61	1.87	two cutoff
192	0	0.5	0.1	0.1	0.75	0.94	15.52	two cutoff
193	0	0.5	0.2	0.1	0.75	0.94	10.12	two cutoff
194	0	0.5	0.3	0.1	0.75	0.93	6.77	two cutoff
195	0	0.5	0.4	0.1	0.75	0.92	5.26	two cutoff
196	0	0.5	0.5	0.1	0.75	0.90	4.52	two cutoff
197	0	0.5	0.6	0.1	0.75	0.87	3.62	two cutoff
198	0	0.5	0.7	0.1	0.75	0.80	3.07	two cutoff
199	0	0.5	0.8	0.1	0.75	0.74	2.70	two cutoff
200	0	0.5	0.9	0.1	0.75	0.66	2.39	two cutoff
201	0	0.5	1	0.1	0.75	0.54	2.19	two cutoff
202	0	0.1	0.1	0.2	0.75	1.58	5.88	two cutoff
203	0	0.1	0.2	0.2	0.75	1.55	3.75	two cutoff
204	0	0.1	0.3	0.2	0.75	1.51	2.31	two cutoff
205	0	0.1	0.4	0.2	0.75	1.47	2.00	two cutoff
206	0	0.1	0.5	0.2	0.75	1.38	1.51	two cutoff
207	0	0.1	0.6	0.2	0.75	1.24	1.34	two cutoff
208	0	0.1	0.7	0.2	0.75	1.09	1.17	two cutoff
209	0	0.1	0.8	0.2	0.75	0.97	1.02	two cutoff
210	0	0.1	0.9	0.2	0.75	0.74	0.96	two cutoff
211	0	0.1	1	0.2	0.75	0.65	0.87	two cutoff
212	0	0.1	0.1	0.3	0.75	1.31	5.61	two cutoff
213	0	0.1	0.2	0.3	0.75	1.28	3.01	two cutoff
214	0	0.1	0.3	0.3	0.75	1.22	2.29	two cutoff
215	0	0.1	0.4	0.3	0.75	1.18	1.64	two cutoff
216	0	0.1	0.5	0.3	0.75	1.11	1.35	two cutoff
217	0	0.1	0.6	0.3	0.75	1.01	1.21	two cutoff
218	0	0.1	0.7	0.3	0.75	0.90	1.02	two cutoff
219	0	0.1	0.8	0.3	0.75	0.76	0.90	two cutoff
220	0	0.1	0.9	0.3	0.75	0.70	0.85	two cutoff
221	0	0.1	1	0.3	0.75	0.51	0.78	two cutoff
222	0	0.1	0.1	0.4	0.75	1.09	5.48	two cutoff
223	0	0.1	0.2	0.4	0.75	1.06	3.47	two cutoff
224	0	0.1	0.3	0.4	0.75	1.03	2.00	two cutoff
225	0	0.1	0.4	0.4	0.75	0.99	1.54	two cutoff

226	0	0.1	0.5	0.4	0.75	0.92	1.27	two cutoff
227	0	0.1	0.6	0.4	0.75	0.84	1.08	two cutoff
228	0	0.1	0.7	0.4	0.75	0.25	2.83	two cutoff
229	0	0.1	0.8	0.4	0.75	0.62	0.84	two cutoff
230	0	0.1	0.9	0.4	0.75	0.50	0.75	two cutoff
231	0	0.1	1	0.4	0.75	0.42	0.72	two cutoff
232	0	0.1	0.1	0.5	0.75	0.93	5.39	two cutoff
233	0	0.1	0.2	0.5	0.75	0.90	3.91	two cutoff
234	0	0.1	0.3	0.5	0.75	0.87	1.94	two cutoff
235	0	0.1	0.4	0.5	0.75	0.83	1.49	two cutoff
236	0	0.1	0.5	0.5	0.75	0.80	1.29	two cutoff
237	0	0.1	0.6	0.5	0.75	0.71	1.03	two cutoff
238	0	0.1	0.7	0.5	0.75	0.64	0.90	two cutoff
239	0	0.1	0.8	0.5	0.75	0.53	0.80	two cutoff
240	0	0.1	0.9	0.5	0.75	0.40	0.74	two cutoff
241	0	0.1	1	0.5	0.75	0.36	0.68	two cutoff
242	0	0.2	0.1	0.2	0.75	1.52	7.62	two cutoff
243	0	0.2	0.2	0.2	0.75	1.47	5.99	two cutoff
244	0	0.2	0.3	0.2	0.75	1.40	2.96	two cutoff
245	0	0.2	0.4	0.2	0.75	1.29	2.36	two cutoff
246	0	0.2	0.5	0.2	0.75	1.26	1.89	two cutoff
247	0	0.2	0.6	0.2	0.75	1.17	1.52	two cutoff
248	0	0.2	0.7	0.2	0.75	1.04	1.40	two cutoff
249	0	0.2	0.8	0.2	0.75	1.04	1.40	two cutoff
250	0	0.2	0.9	0.2	0.75	0.83	1.21	two cutoff
251	0	0.2	1	0.2	0.75	0.59	1.04	two cutoff
252	0	0.3	0.1	0.3	0.75	1.25	8.09	two cutoff
253	0	0.3	0.2	0.3	0.75	1.19	6.28	two cutoff
254	0	0.3	0.3	0.3	0.75	1.13	3.09	two cutoff
255	0	0.3	0.4	0.3	0.75	1.06	2.38	two cutoff
256	0	0.3	0.5	0.3	0.75	0.97	1.88	two cutoff
257	0	0.3	0.6	0.3	0.75	0.89	1.63	two cutoff
258	0	0.3	0.7	0.3	0.75	0.79	1.41	two cutoff
259	0	0.3	0.8	0.3	0.75	0.66	1.24	two cutoff
260	0	0.3	0.9	0.3	0.75	0.55	1.15	two cutoff
261	0	0.3	1	0.3	0.75	0.44	1.04	two cutoff
262	0	0.4	0.1	0.4	0.75	1.04	8.40	two cutoff
263	0	0.4	0.2	0.4	0.75	0.99	4.59	two cutoff
264	0	0.4	0.3	0.4	0.75	0.93	3.17	two cutoff
265	0	0.4	0.4	0.4	0.75	0.85	2.33	two cutoff

266	0	0.4	0.5	0.4	0.75	0.80	1.97	two cutoff
267	0	0.4	0.6	0.4	0.75	0.73	1.56	two cutoff
268	0	0.4	0.7	0.4	0.75	0.64	1.42	two cutoff
269	0	0.4	0.8	0.4	0.75	0.55	1.21	two cutoff
270	0	0.4	0.9	0.4	0.75	0.40	1.10	two cutoff
271	0	0.4	1	0.4	0.75	0.36	1.04	two cutoff
272	0	0.5	0.1	0.5	0.75	0.89	8.62	two cutoff
273	0	0.5	0.2	0.5	0.75	0.86	6.58	two cutoff
274	0	0.5	0.3	0.5	0.75	0.81	4.03	two cutoff
275	0	0.5	0.4	0.5	0.75	0.74	2.46	two cutoff
276	0	0.5	0.5	0.5	0.75	0.70	2.00	two cutoff
277	0	0.5	0.6	0.5	0.75	0.61	1.66	two cutoff
278	0	0.5	0.7	0.5	0.75	0.54	1.43	two cutoff
279	0	0.5	0.8	0.5	0.75	0.47	1.24	two cutoff
280	0	0.5	0.9	0.5	0.75	0.38	1.14	two cutoff
281	0	0.5	1	0.5	0.75	0.30	1.04	two cutoff
282	0	0.6	0.1	0.6	0.75	0.78	8.78	two cutoff
283	0	0.6	0.2	0.6	0.75	0.74	5.78	two cutoff
284	0	0.6	0.3	0.6	0.75	0.68	3.49	two cutoff
285	0	0.6	0.4	0.6	0.75	0.64	2.48	two cutoff
286	0	0.6	0.5	0.6	0.75	0.60	2.00	two cutoff
287	0	0.6	0.6	0.6	0.75	0.53	1.67	two cutoff
288	0	0.6	0.7	0.6	0.75	0.48	1.50	two cutoff
289	0	0.6	0.8	0.6	0.75	0.39	1.25	two cutoff
290	0	0.6	0.9	0.6	0.75	0.32	1.15	two cutoff
291	0	0.6	1	0.6	0.75	0.26	1.04	two cutoff

A.5- the flow net under dam at different cases by finite difference programming

A.5.1- for ratio of permeability in x direction to permeability in y direction ($K_x/K_y=1$) for $H/B = (0.5)$:

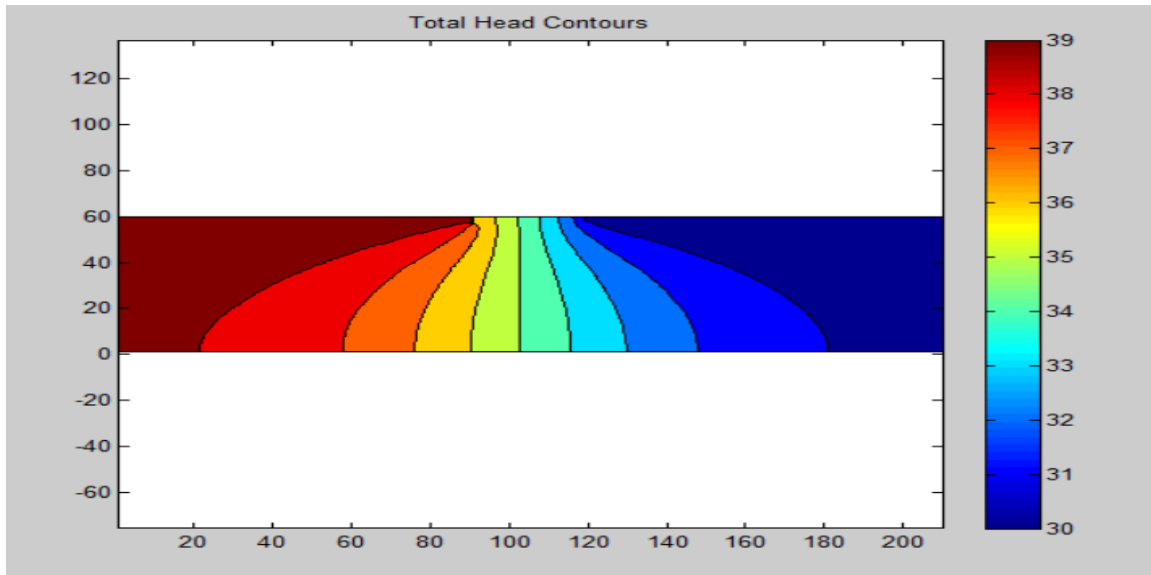


Figure (A-1): The flow net under dam at With one cutoff , $d_1/B=0.1, x_1/B=0, x_2/B=0, d_2/B=0$

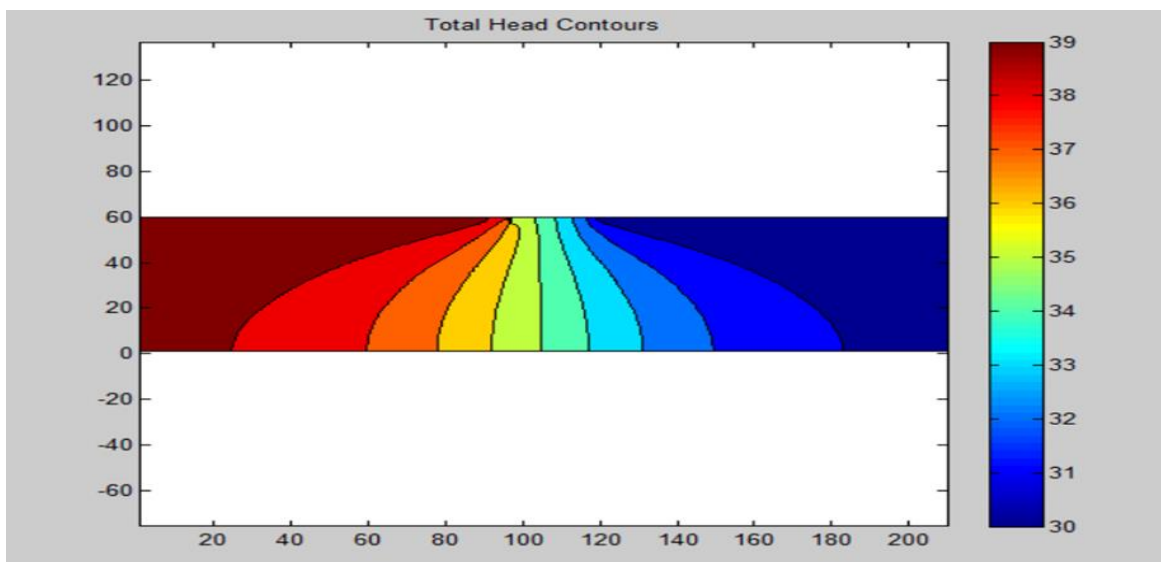


Figure (A-2): The flow net under dam at With one cutoff $d_1/B=0.1, x_1/B=0.2, x_2/B=0, d_2/B=0$

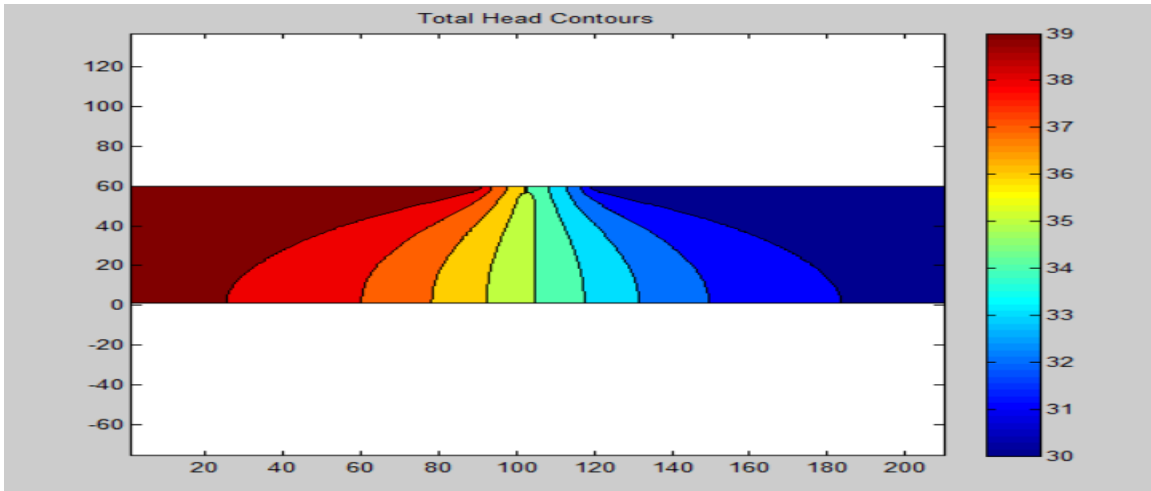


Figure (A-3): The flow net under dam at With one cutoff $d1/B=0.1, x1/B=0.4, x2/B=0, d2/B=0$

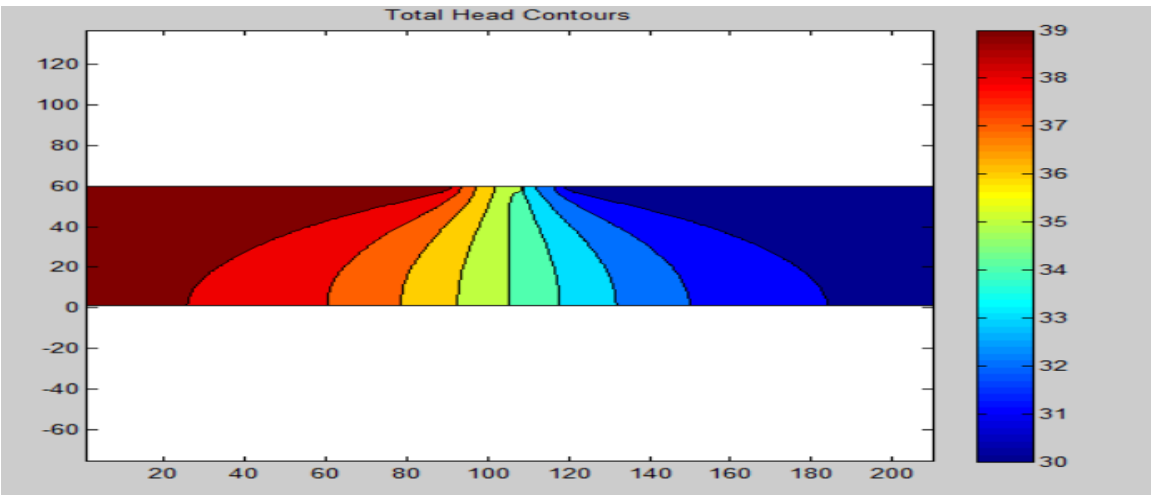


Figure (A-4): The flow net under dam at With one cutoff $d1/B=0.1, x1/B=0.6, x2/B=0, d2/B=0$

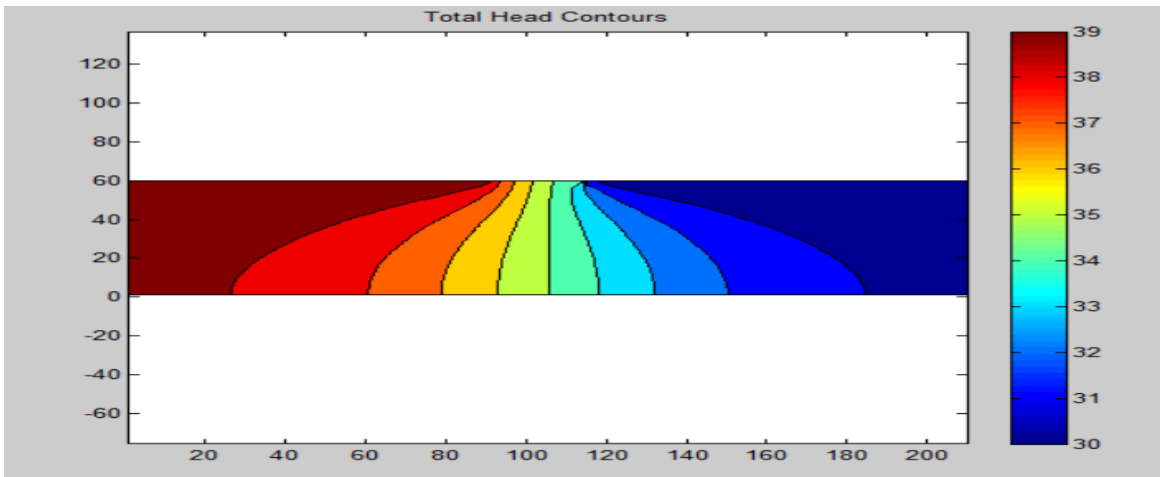


Figure (A-5): The flow net under dam at With one cutoff $d1/B=0.1, x1/B=0.8, x2/B=0, d2/B=0$

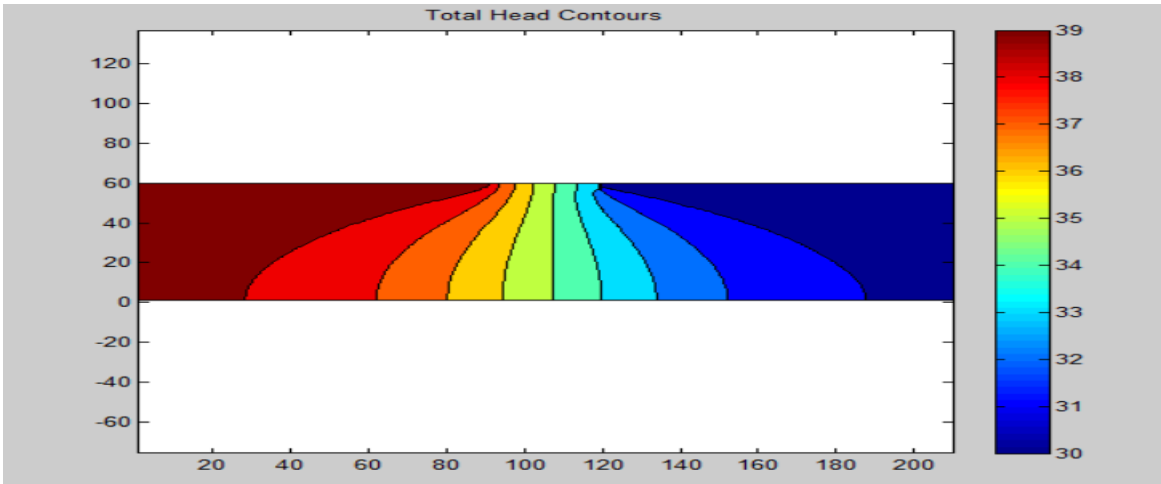


Figure (A-6): The flow net under dam at With one cutoff $d1/B=0.1, x1/B=1, x2/B=0, d2/B=0$

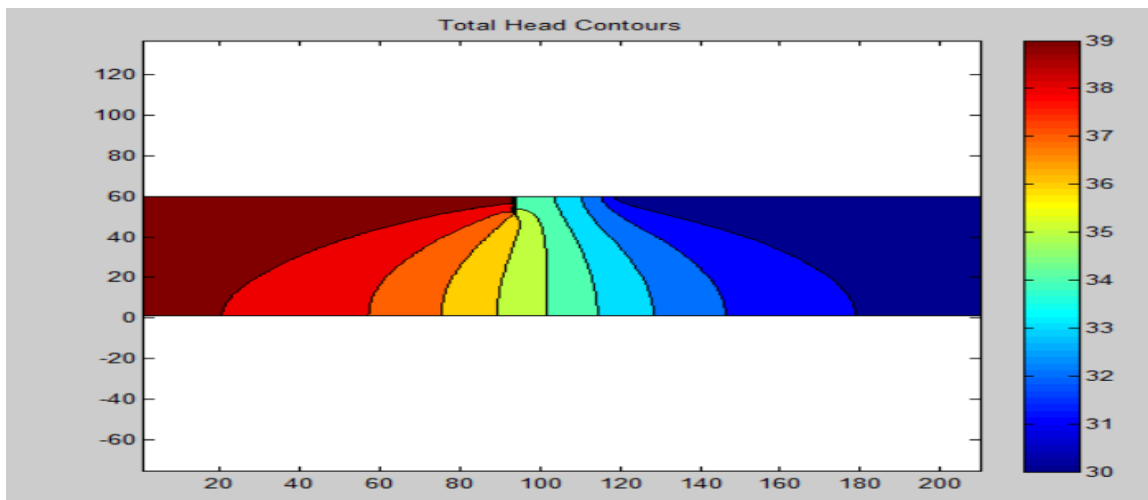


Figure (A-7): The flow net under dam at With one cutoff $d1/B=0.3, x1/B=0.1, x2/B=0, d2/B=0$

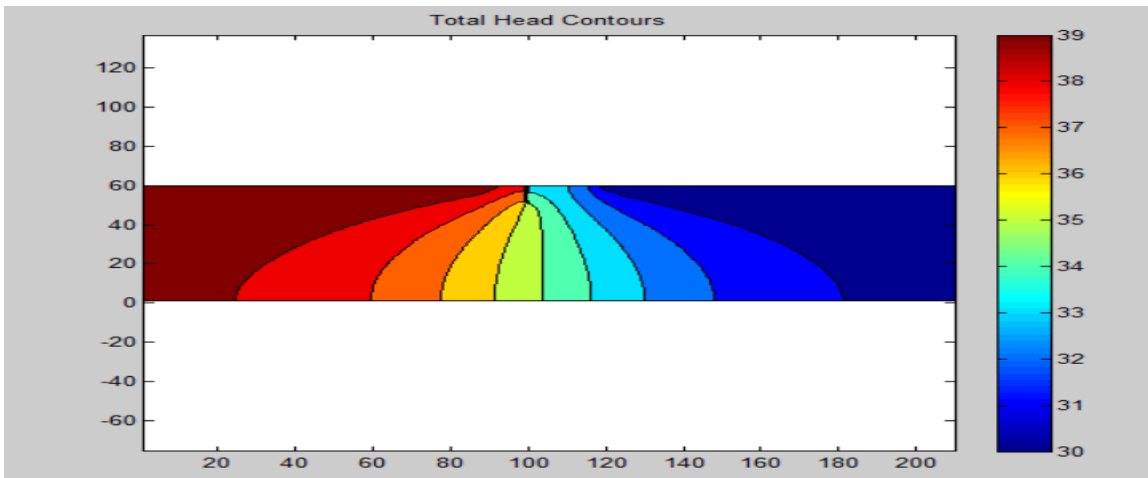


Figure (A-8): The flow net under dam at With one cutoff $d1/B=0.3, x1/B=0.3, x2/B=0, d2/B=0$

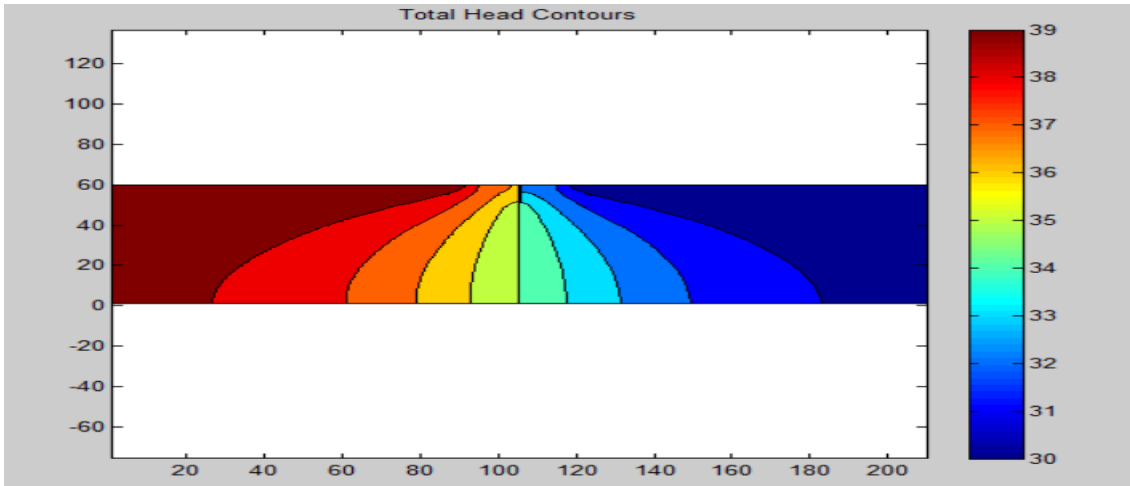


Figure (A-9): The flow net under dam at With one cutoff $d1/B=0.3$, $x1/B=0.5$, $x2/B=0$, $d2/B=0$

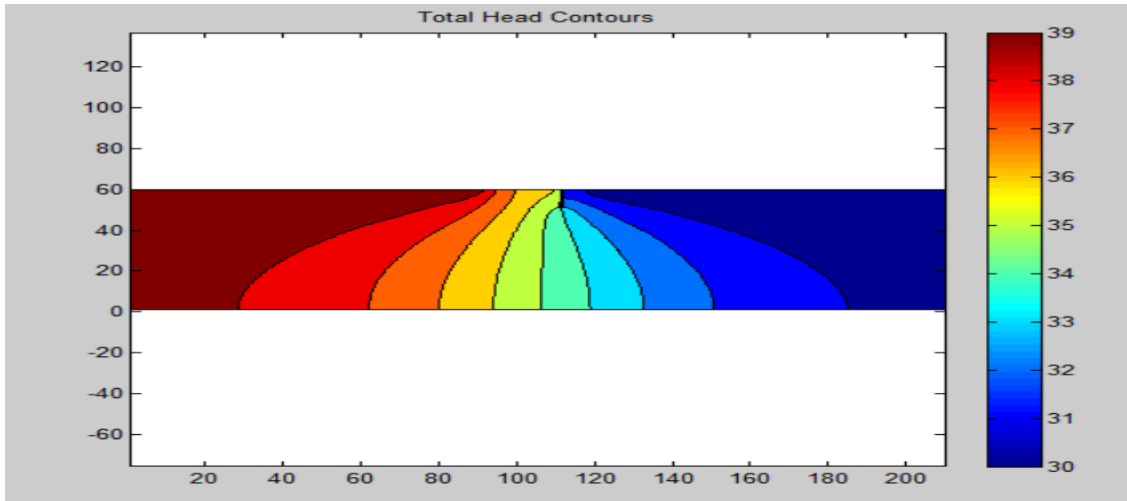


Figure (A-10): The flow net under dam at With one cutoff $d1/B=0.3$, $x1/B=0.7$, $x2/B=0$, $d2/B=0$

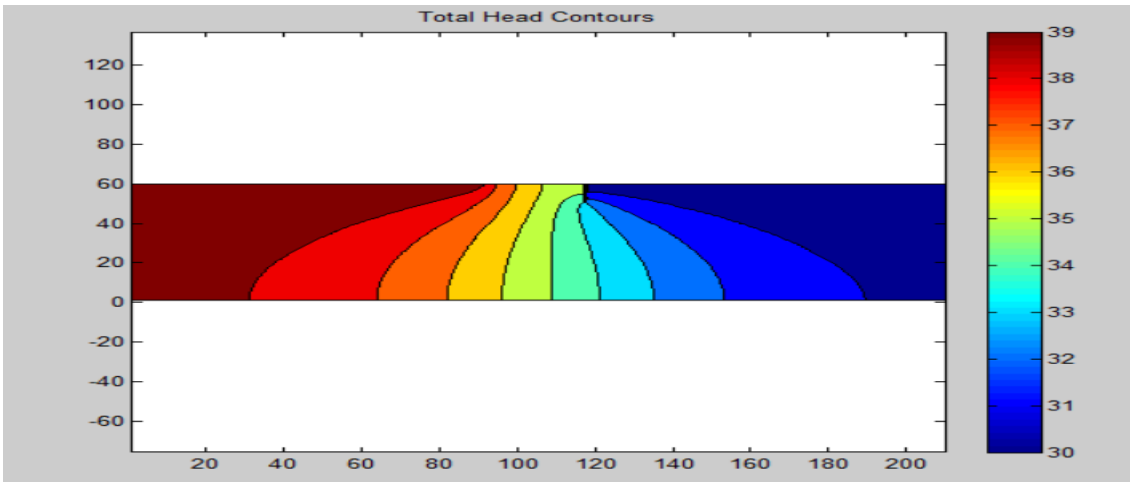


Figure (A-11): the flow net under dam at With one cutoff $d1/B=0.3$, $x1/B=0.9$, $x2/B=0$, $d2/B=0$

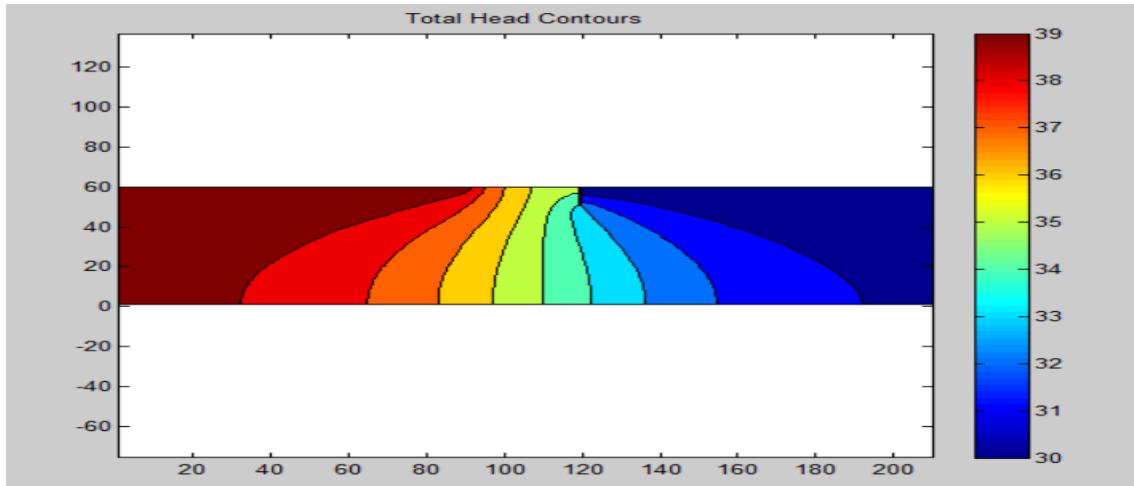


Figure (A-12): The flow net under dam at With one cutoff $d1/B=0.3$, $x1/B=1$, $x2/B=0$, $d2/B=0$

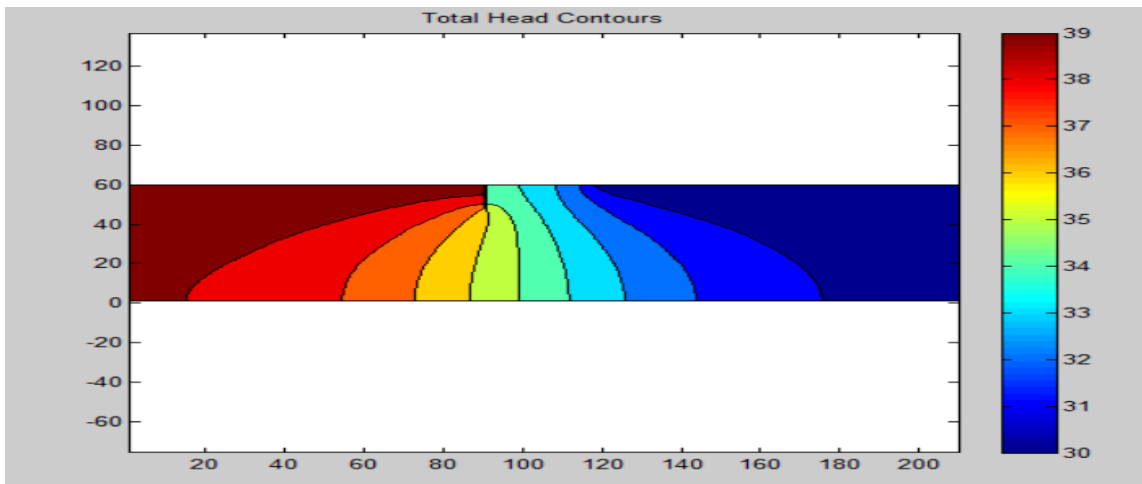


Figure (A-13): The flow net under dam at With one cutoff $d1/B=0.4$, $x1/B=0$, $x2/B=0$, $d2/B=0$

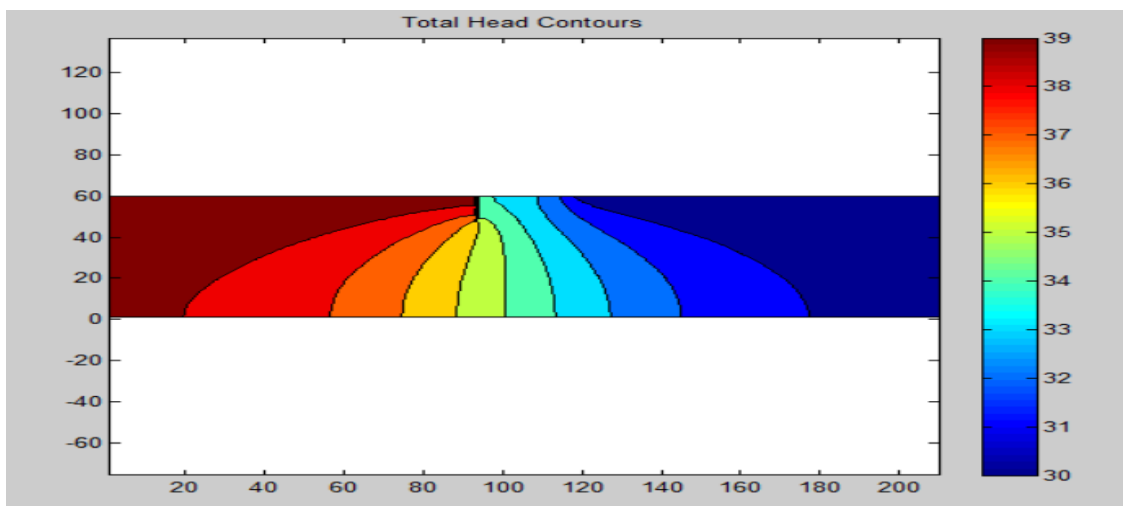


Figure (A-14): The flow net under dam at With one cutoff $d1/B=0.4$, $x1/B=0.2$, $x2/B=0$, $d2/B=0$

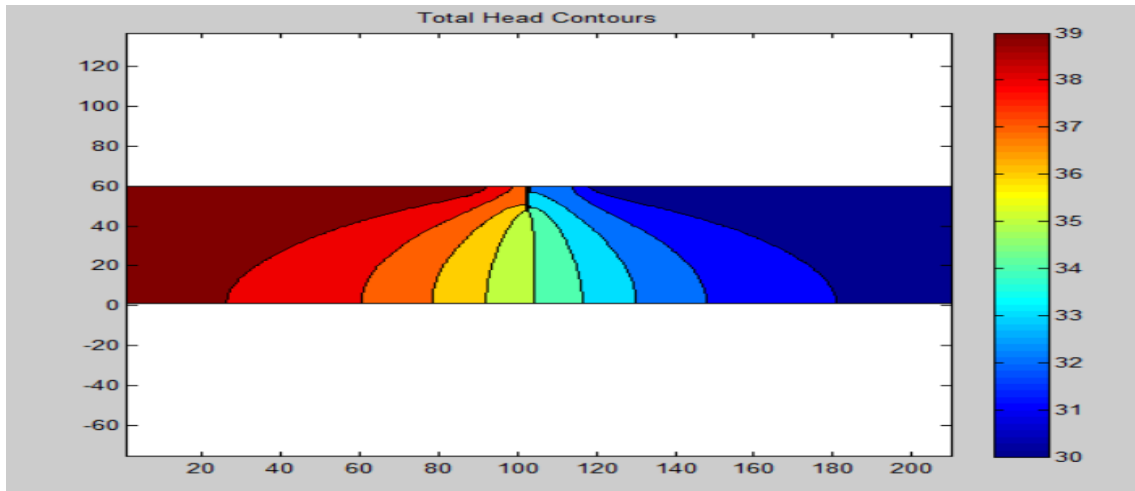


Figure (A-15): The flow net under dam at With one cutoff $d_1/B=0.4$, $x_1/B=0.4$, $x_2/B=0$, $d_2/B=0$

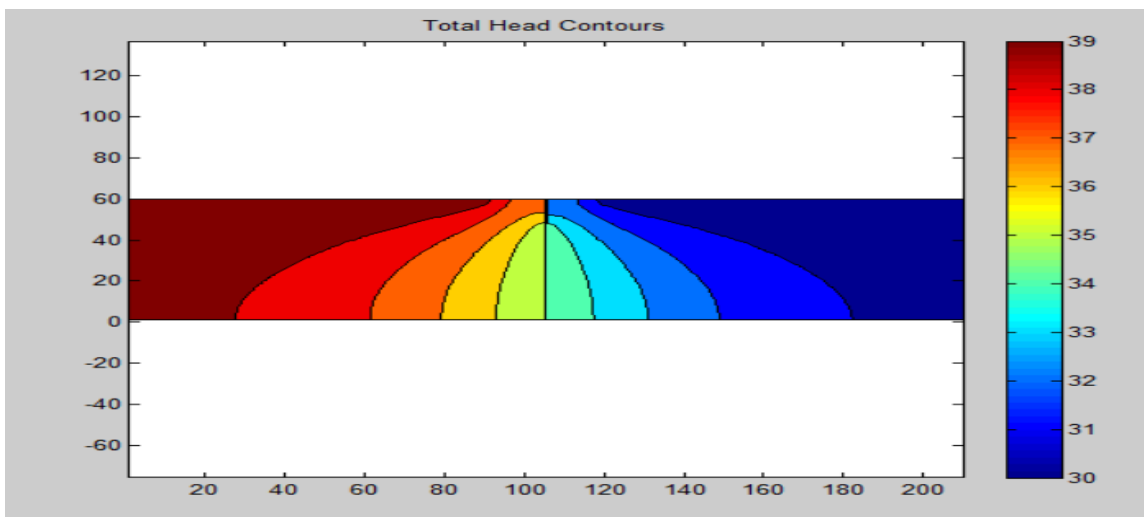


Figure (A-16): The flow net under dam at With one cutoff $d_1/B=0.4$, $x_1/B=0.5$, $x_2/B=0$, $d_2/B=0$

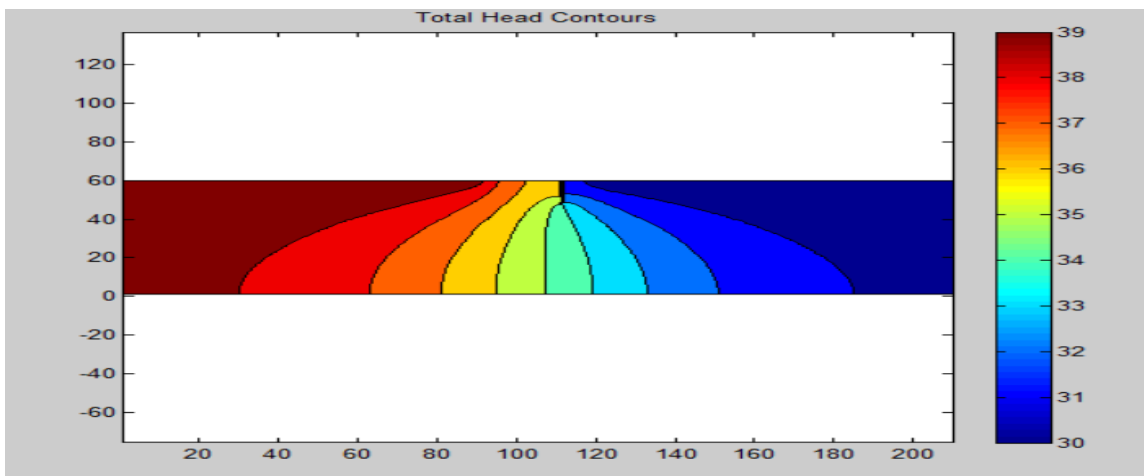


Figure (A-17): The flow net under dam at With one cutoff $d_1/B=0.4$, $x_1/B=0.7$, $x_2/B=0$, $d_2/B=0$

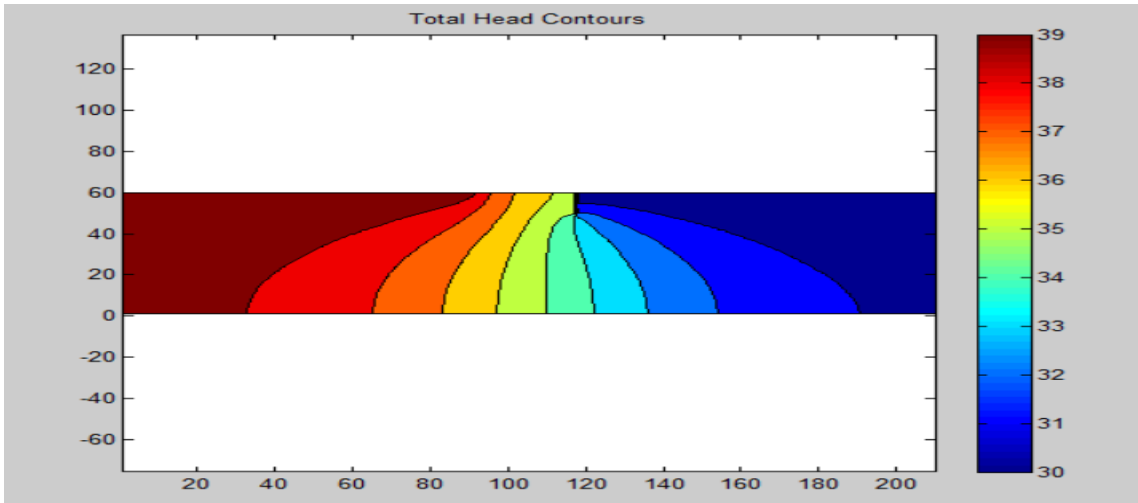


Figure (A-18): The flow net under dam at With one cutoff $d_1/B=0.4$, $x_1/B=0.9$, $x_2/B=0$, $d_2/B=0$

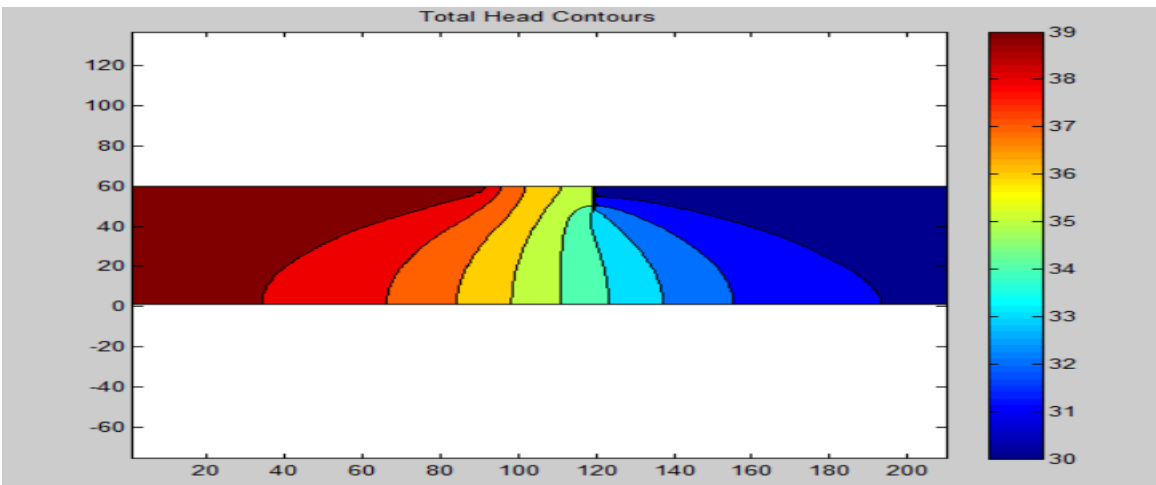


Figure (A-19): The flow net under dam at With one cutoff $d_1/B=0.4$, $x_1/B=1$, $x_2/B=0$, $d_2/B=0$

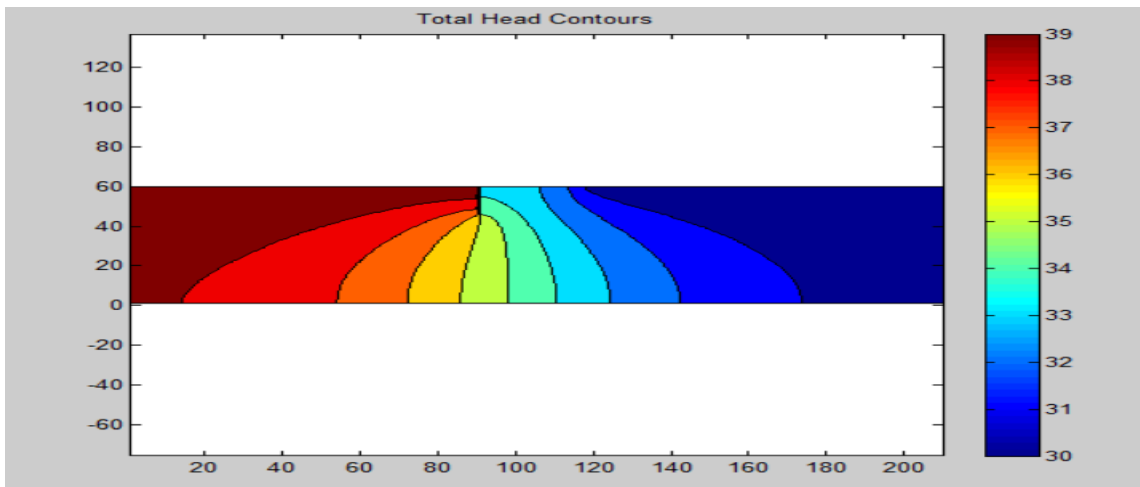


Figure (A-20): The flow net under dam at With one cutoff $d_1/B=0.5$, $x_1/B=0$, $x_2/B=0$, $d_2/B=0$

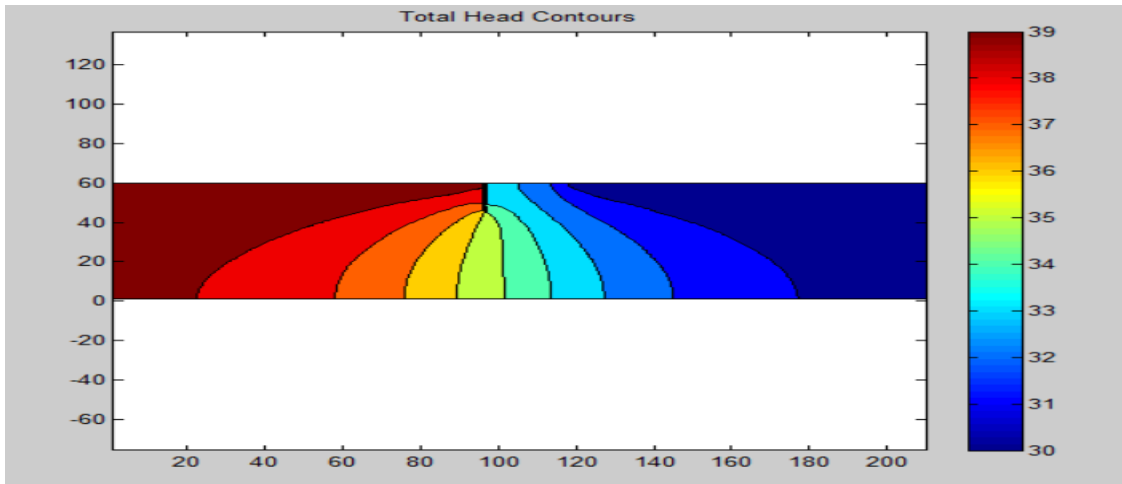


Figure (A-21): The flow net under dam at With one cutoff $d_1/B=0.5$, $x_1/B=0.2$, $x_2/B=0$, $d_2/B=0$

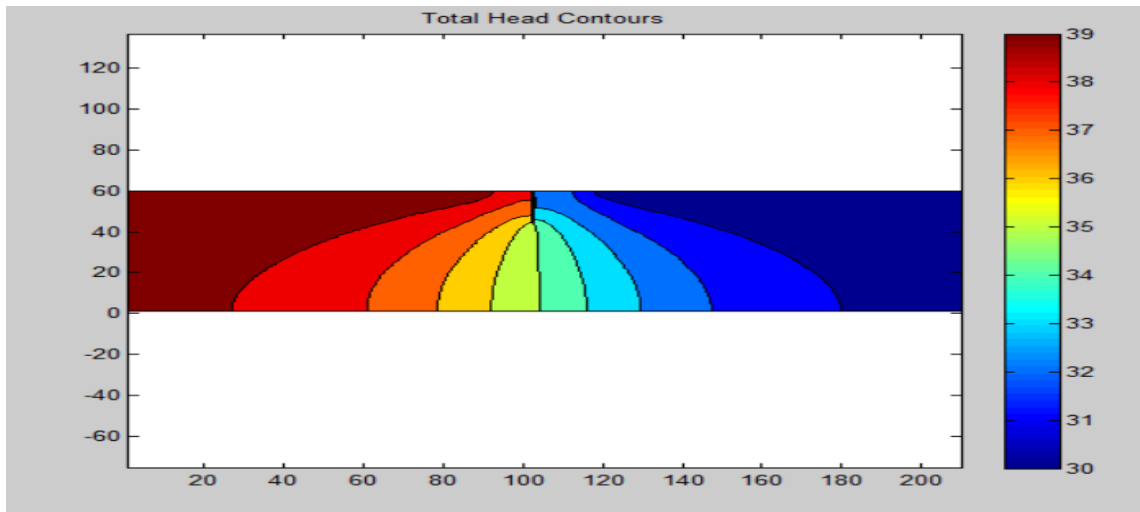


Figure (A-22): The flow net under dam at With one cutoff $d_1/B=0.5$, $x_1/B=0.4$, $x_2/B=0$, $d_2/B=0$

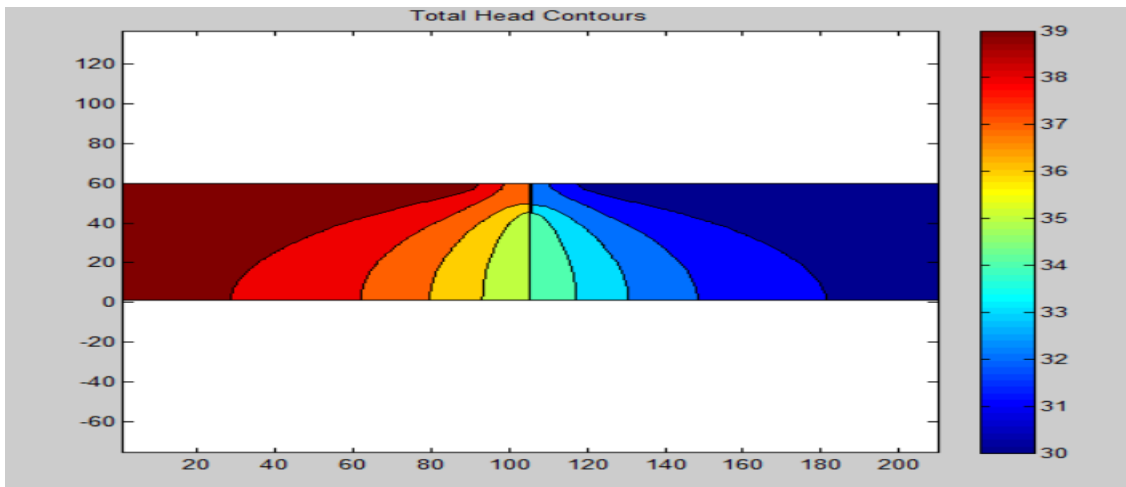


Figure (A-23): The flow net under dam at With one cutoff $d_1/B=0.5$, $x_1/B=0.6$, $x_2/B=0$, $d_2/B=0$

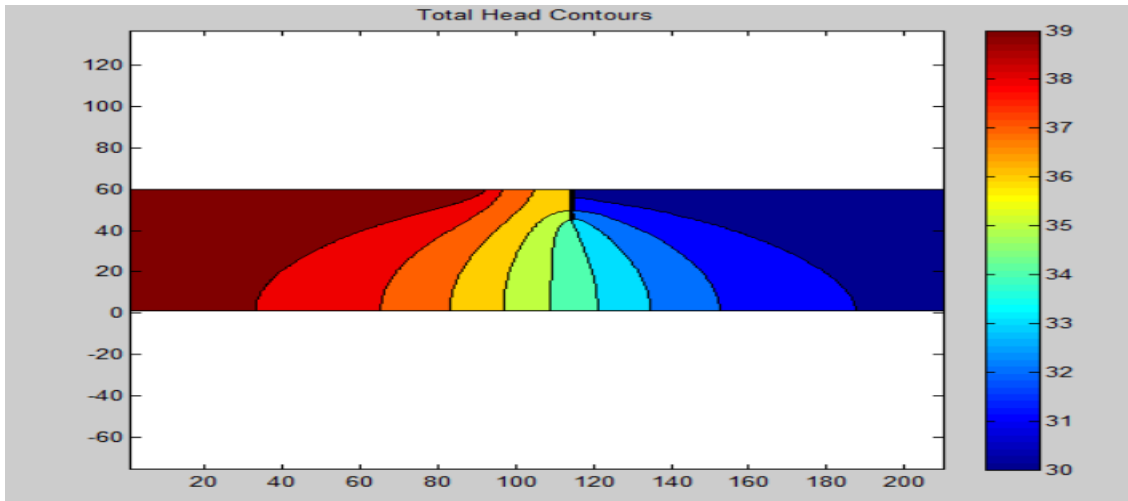


Figure (A-24): The flow net under dam at With one cutoff $d_1/B=0.5$, $x_1/B=0.8$, $x_2/B=0$, $d_2/B=0$

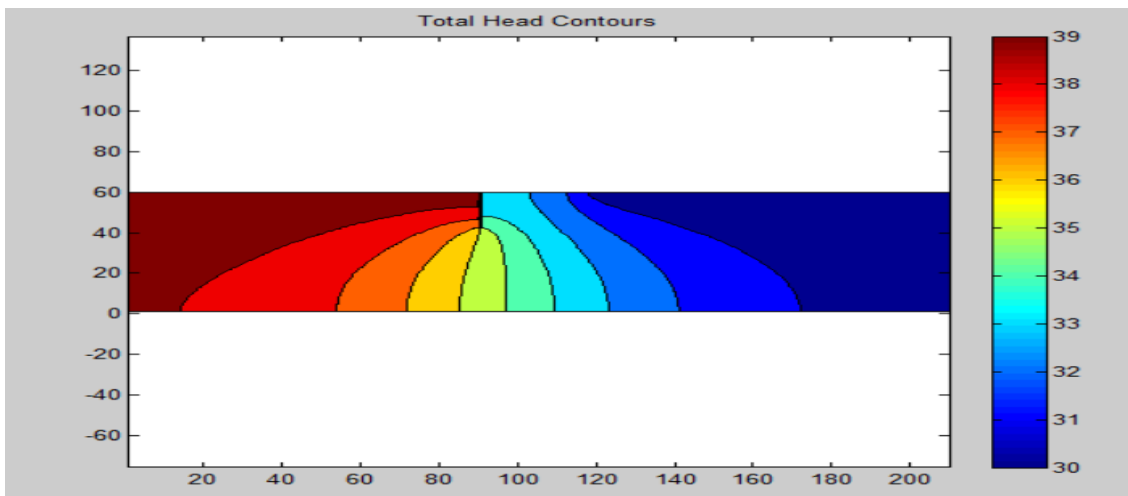


Figure (A-25): The flow net under dam at With one cutoff $d_1/B=0.6$, $x_1/B=0$, $x_2/B=0$, $d_2/B=0$

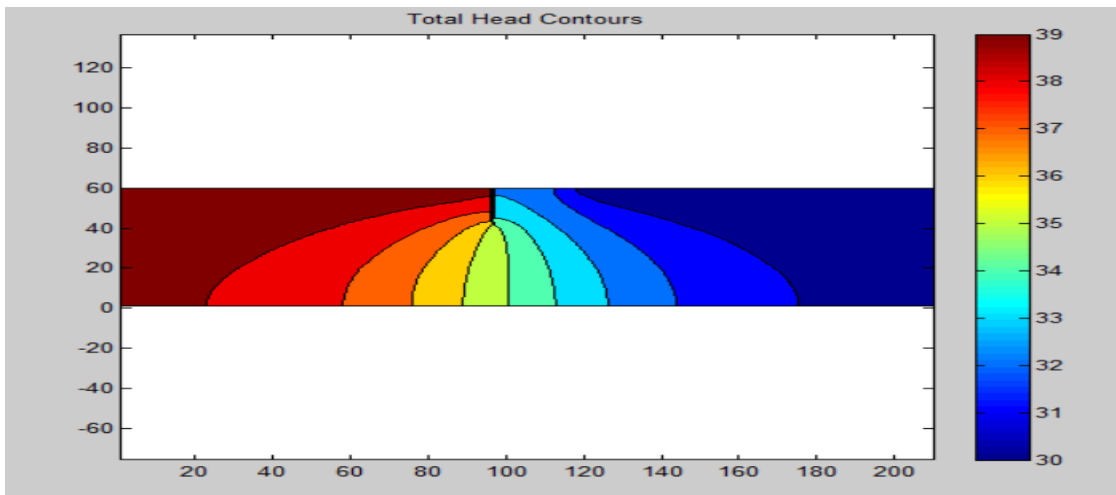


Figure (A-26): The flow net under dam at With one cutoff $d_1/B=0.6$, $x_1/B=0.2$, $x_2/B=0$, $d_2/B=0$

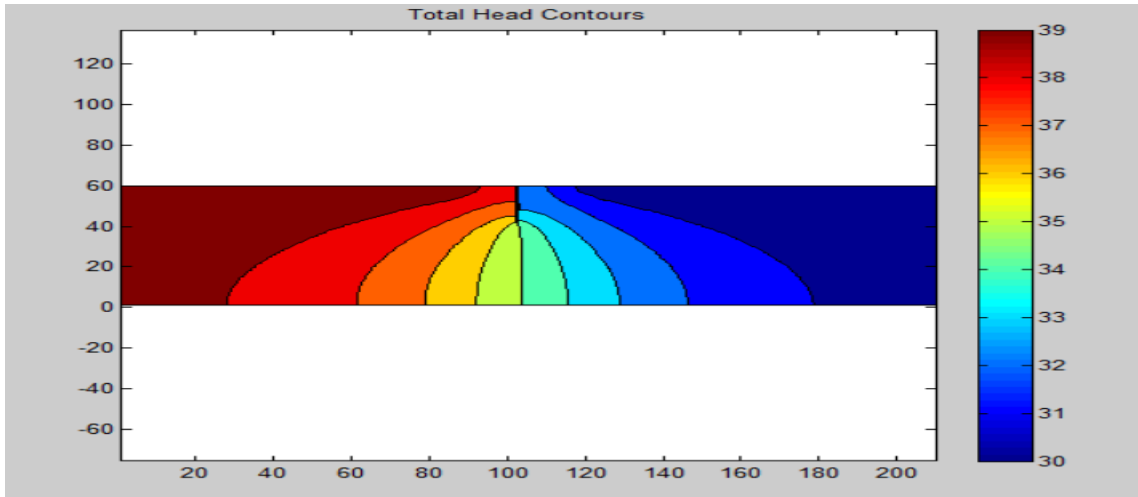


Figure (A-27): The flow net under dam at With one cutoff $d_1/B=0.6$, $x_1/B=0.4$, $x_2/B=0$, $d_2/B=0$

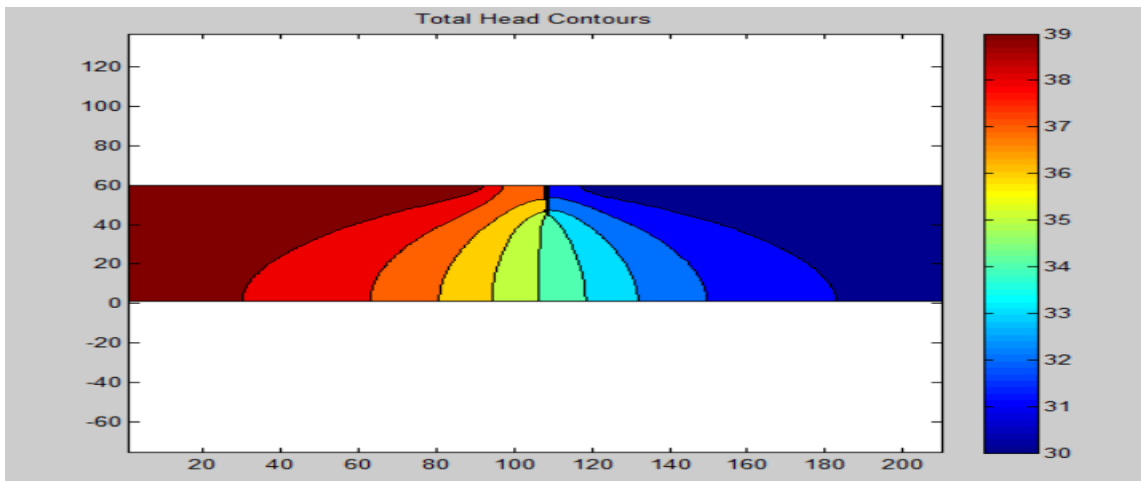


Figure (A-28): The flow net under dam at With one cutoff $d_1/B=0.6$, $x_1/B=0.6$, $x_2/B=0$, $d_2/B=0$

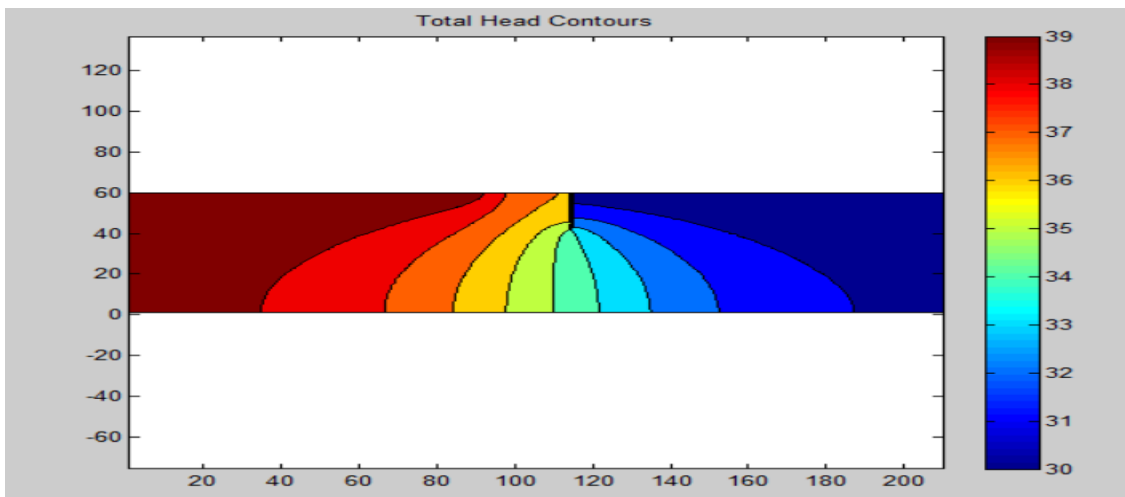


Figure (A-29): The flow net under dam at With one cutoff $d_1/B=0.6$, $x_1/B=0.8$, $x_2/B=0$, $d_2/B=0$

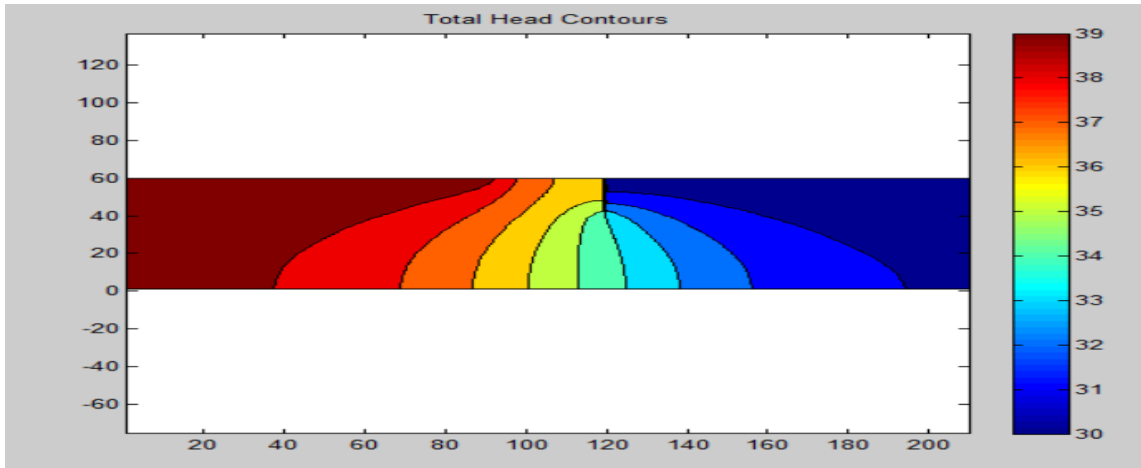


Figure (A-30): The flow net under dam at With one cutoff $d1/B=0.6$, $x1/B=1$, $x2/B=0$, $d2/B=0$

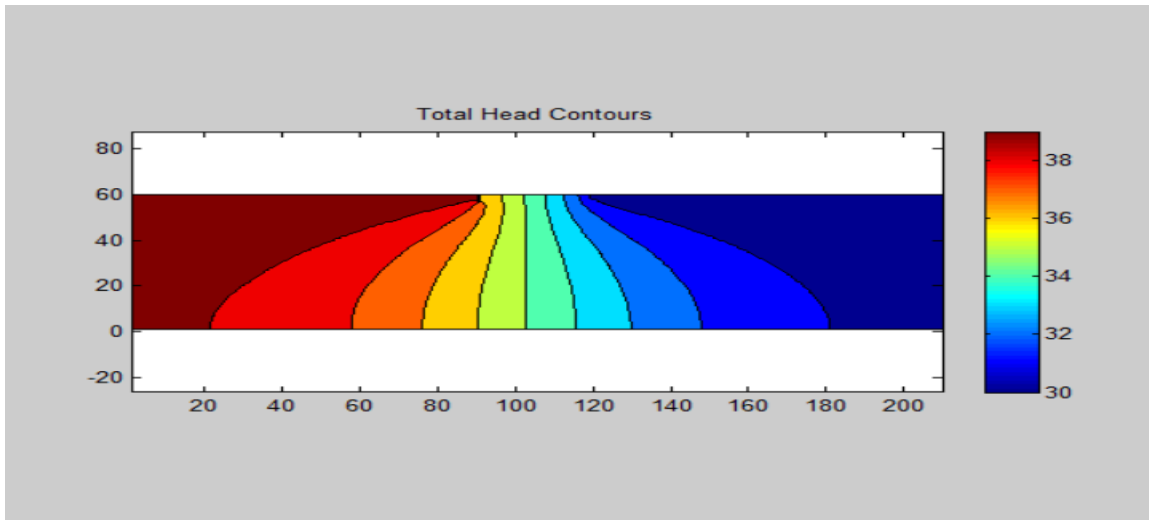


Figure (A-31): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0$, $x2/B=1$, $d2/B=0.1$

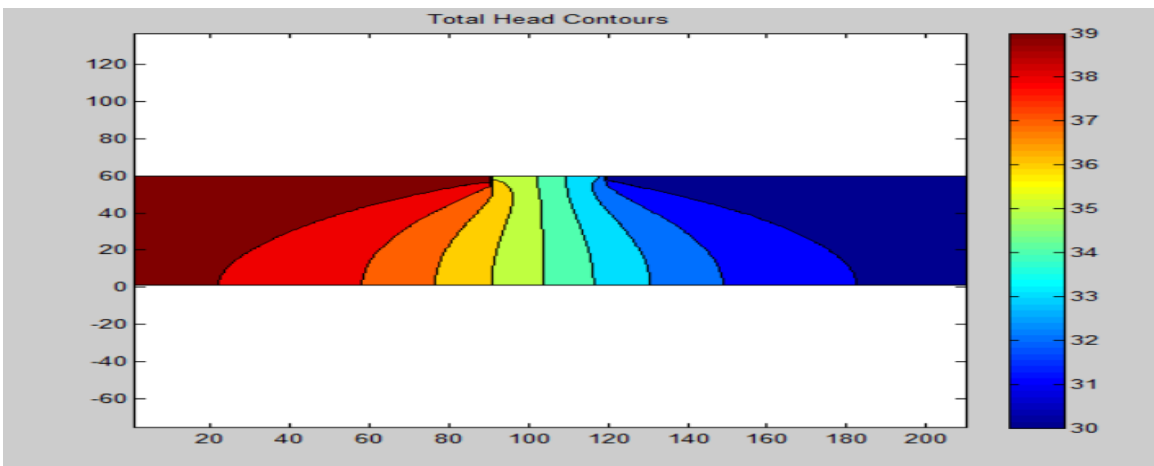


Figure (A-32): The flow net under dam at With two cutoff $d1/B=0.2$, $x1/B=0$, $x2/B=1$, $d2/B=0.1$

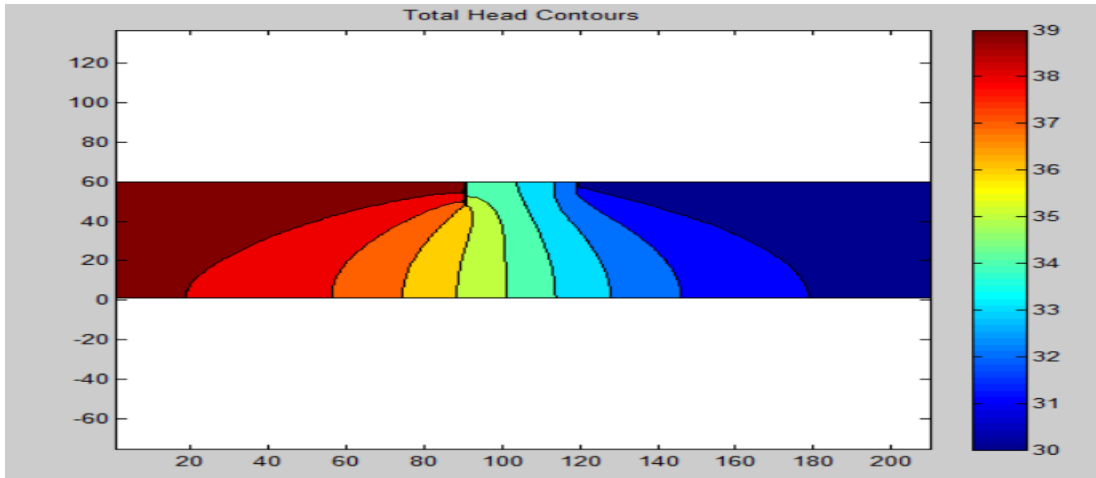


Figure (A-33): The flow net under dam at With two cutoff $d_1/B=0.4$, $x_1/B=0$, $x_2/B=1$, $d_2/B=0.1$

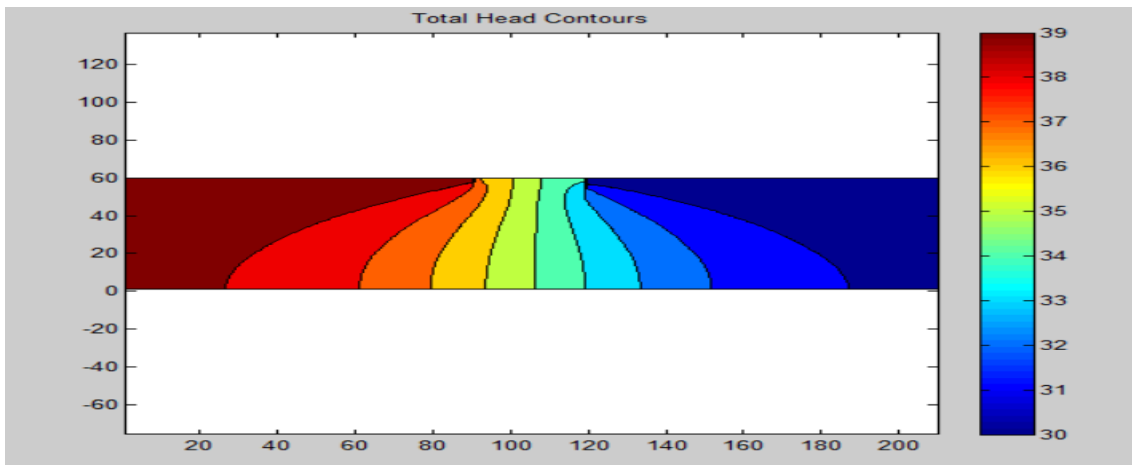


Figure (A-34): The flow net under dam at With two cutoff $d_1/B=0.1$, $x_1/B=0$, $x_2/B=1$, $d_2/B=0.2$

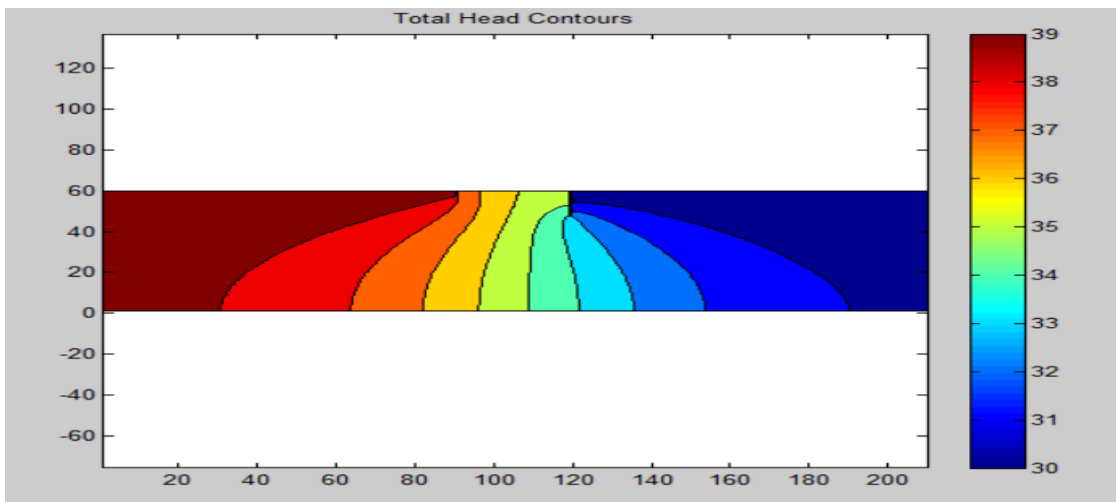


Figure (A-35): The flow net under dam at With two cutoff $d_1/B=0.1$, $x_1/B=0$, $x_2/B=1$, $d_2/B=0.4$

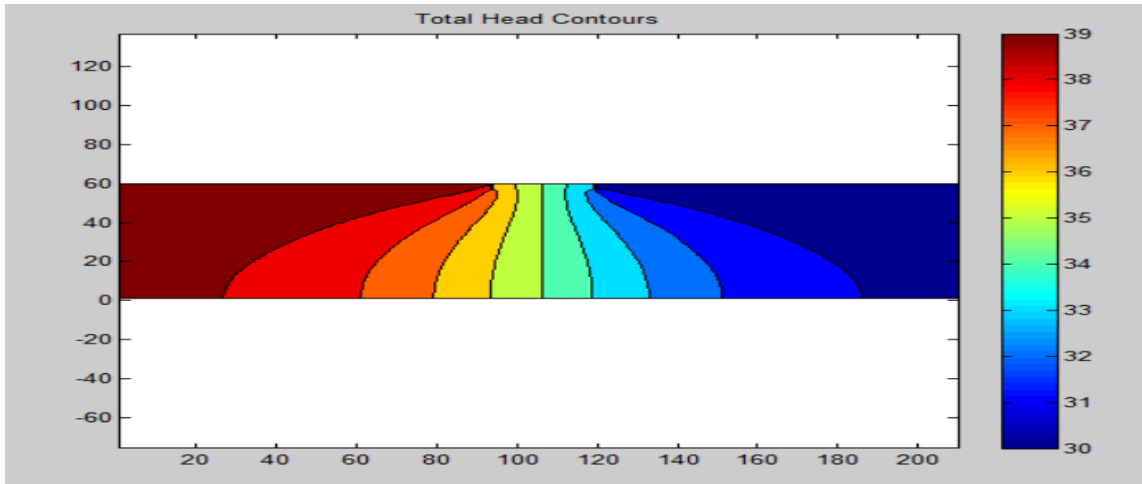


Figure (A-36): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0.1$, $x2/B=1$, $d2/B=0.1$

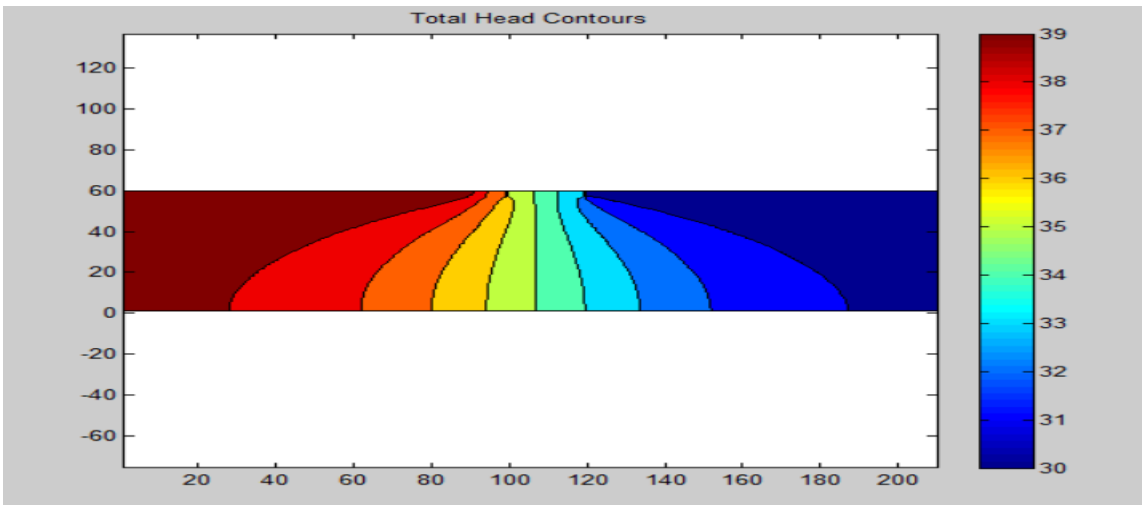


Figure (A-37): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0.3$, $x2/B=1$, $d2/B=0.1$

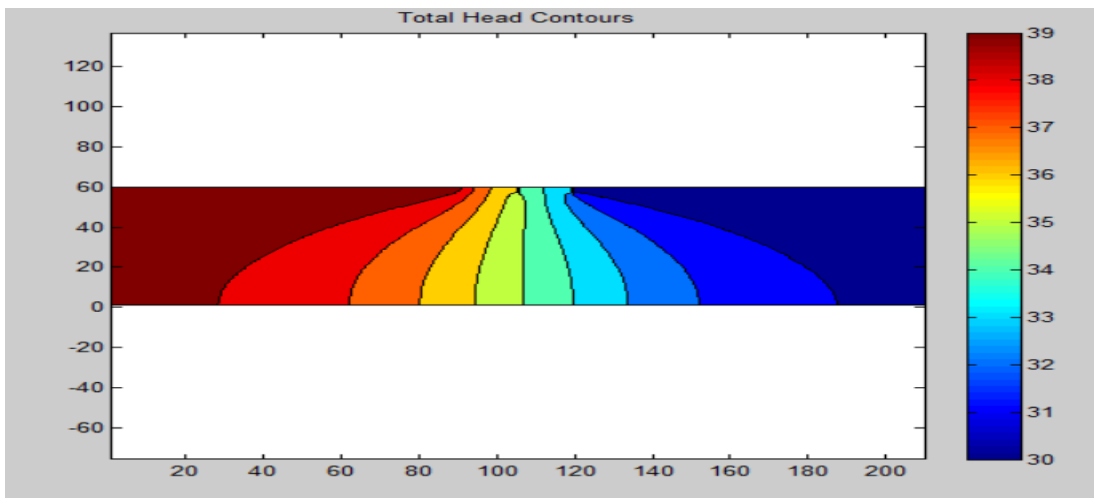


Figure (A-38): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0.5$, $x2/B=1$, $d2/B=0.1$

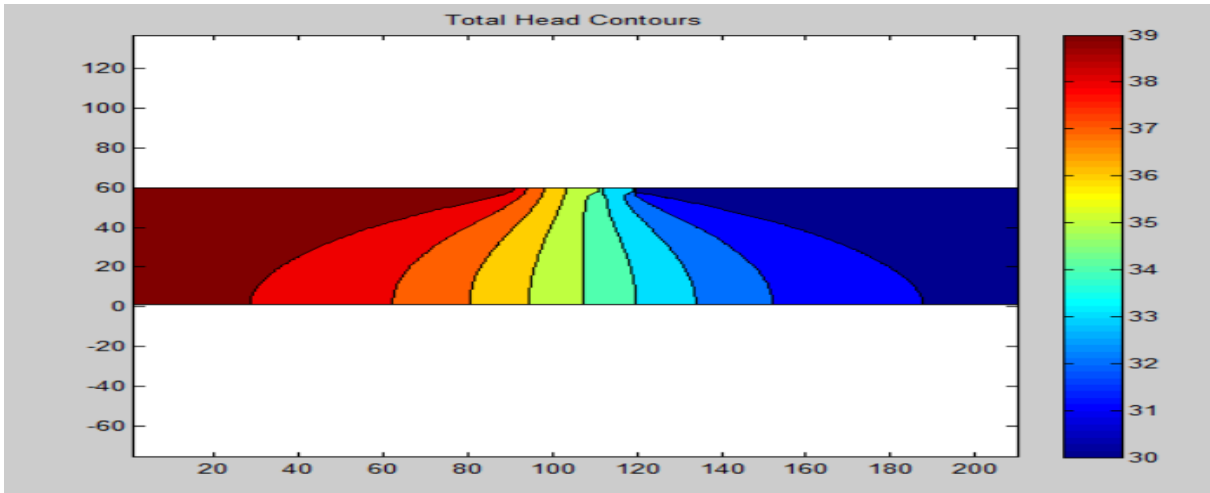


Figure (A-39): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0.7$, $x2/B=1$, $d2/B=0.1$

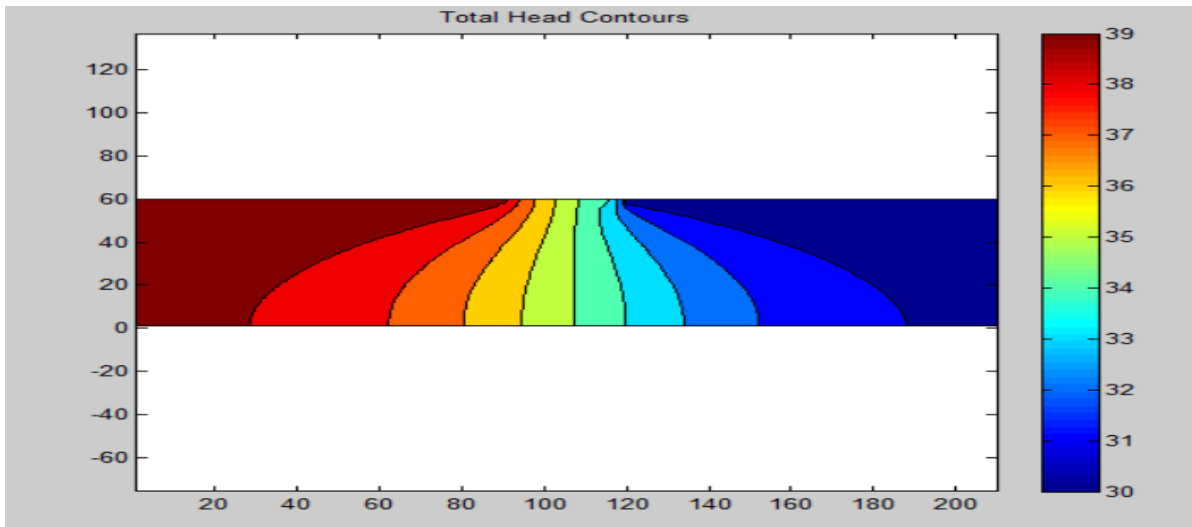


Figure (A-40): The flow net under dam at With two cutoff $d1/B=0.1$, $x1/B=0.9$, $x2/B=1$, $d2/B=0.1$

A.5.2- for ratio of permeability in x direction to permeability in y direction ($K_x/K_y=2$) for ($H/B=0.5$):

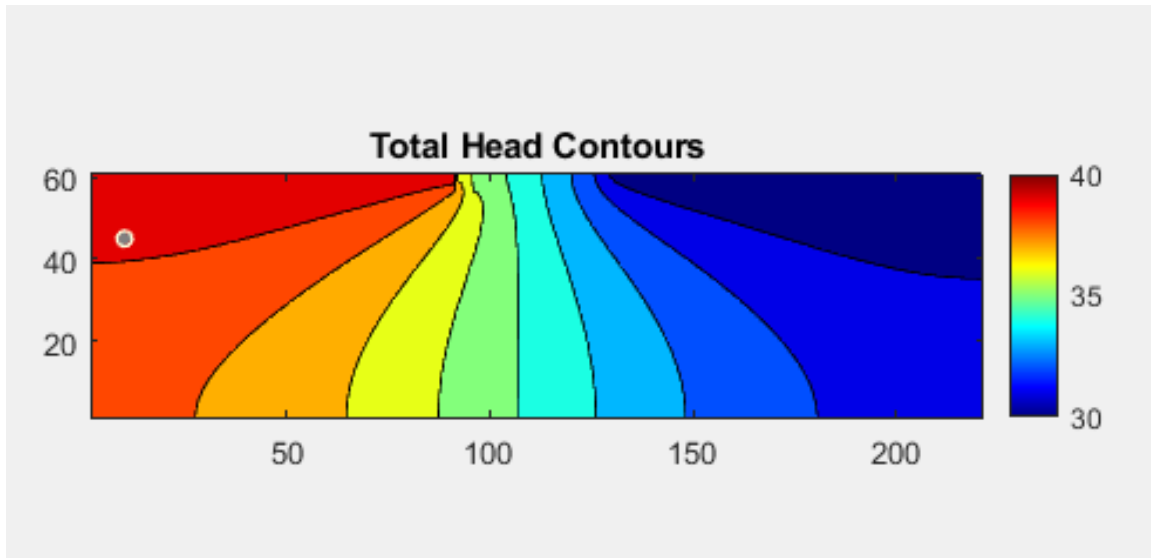


Figure (A-41): The flow net under dam at With two cutoff $d_1/B=0.1$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

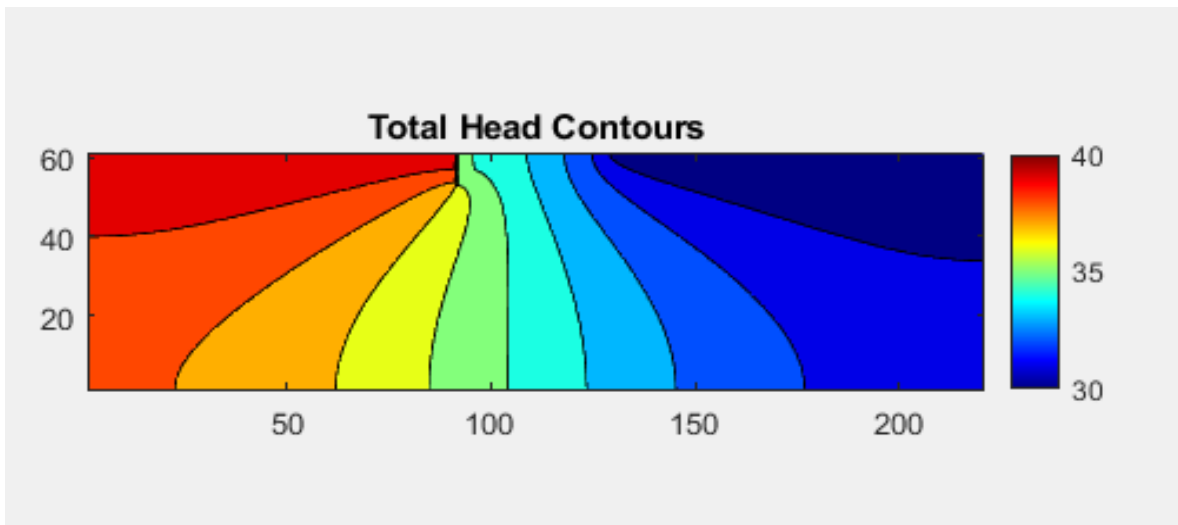


Figure (A-42): The flow net under dam at With two cutoff $d_1/B=0.2$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

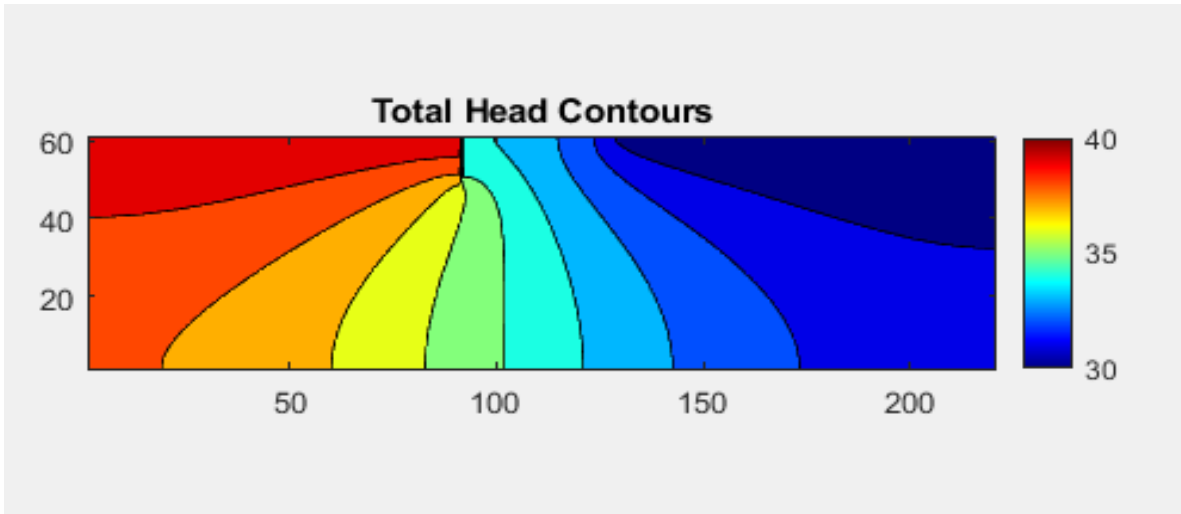


Figure (A-43): The flow net under dam at With two cutoff $d1/B=0.3$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

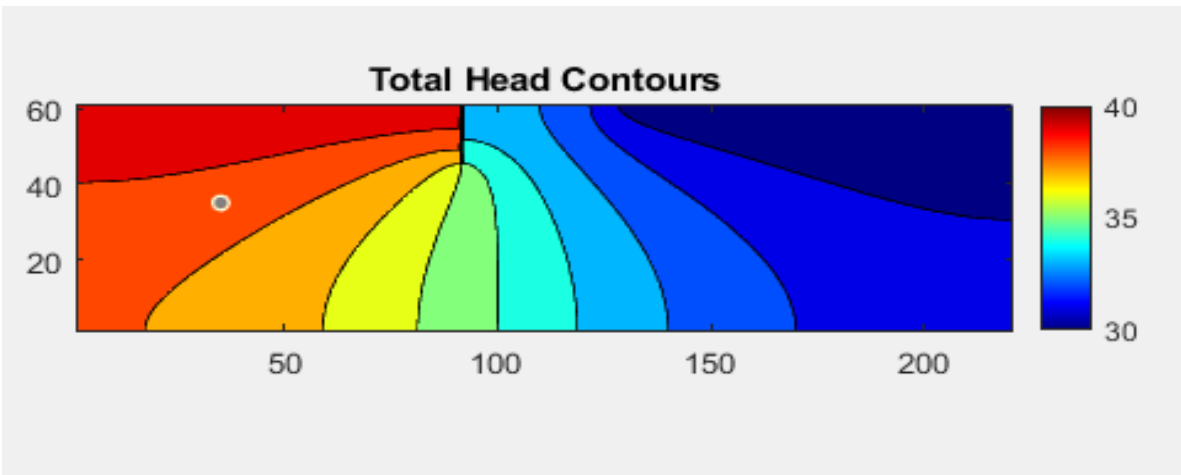


Figure (A-44): The flow net under dam at With two cutoff $d1/B=0.4$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

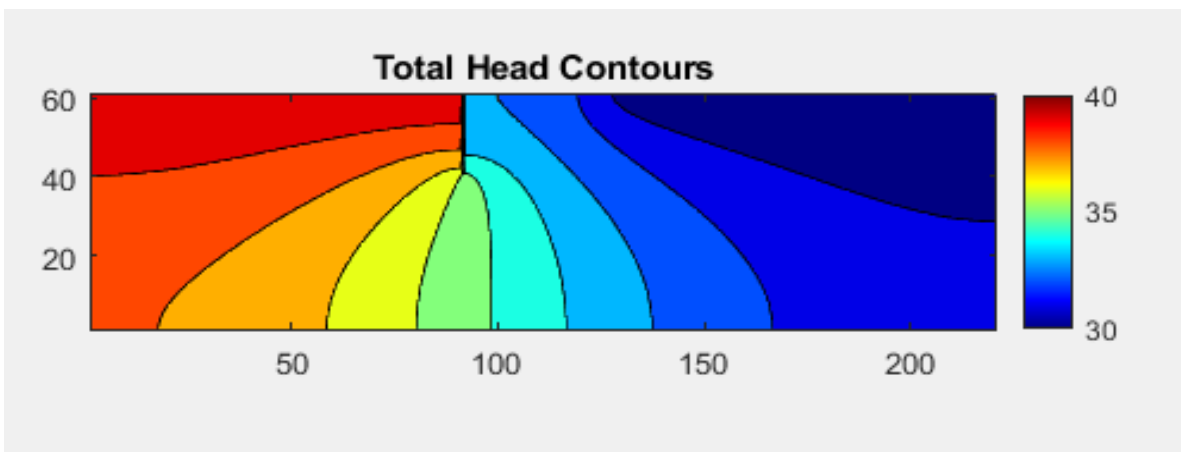


Figure (A-45): The flow net under dam at With two cutoff $d1/B=0.5$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

A.5.3- for ratio of permeability in x direction to permeability in y direction ($K_x/K_y=4$) for ($H/B=0.5$):

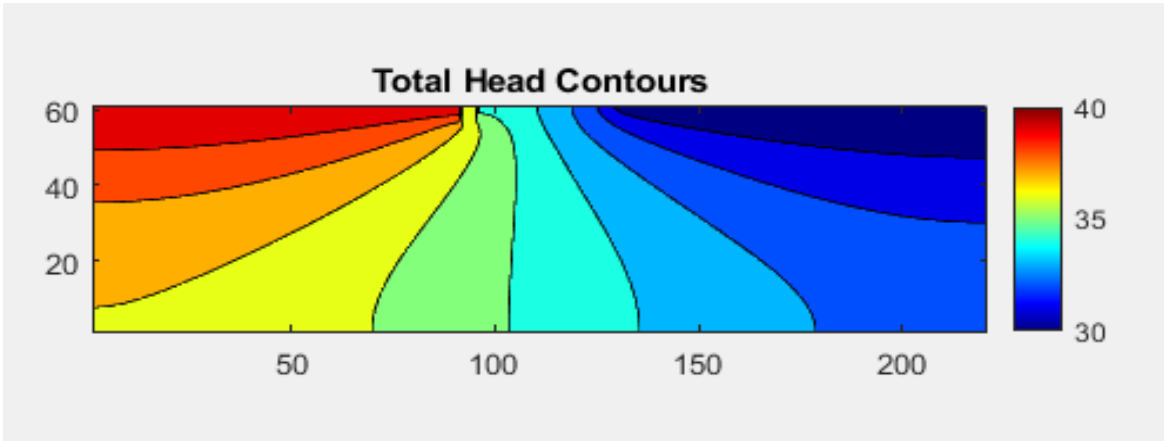


Figure (A-46): The flow net under dam at with two cutoff $d_1/B=0.1$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

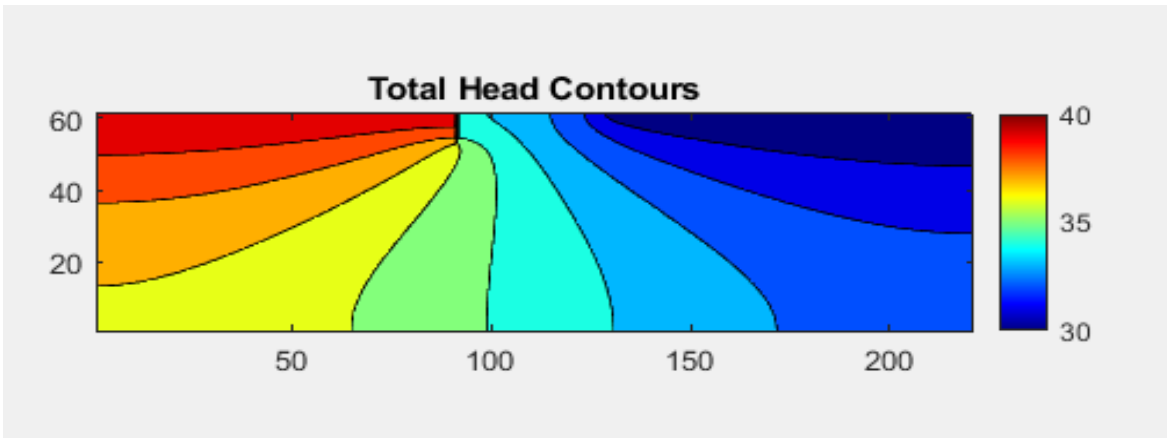


Figure (A-47): The flow net under dam at with two cutoff $d_1/B=0.2$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

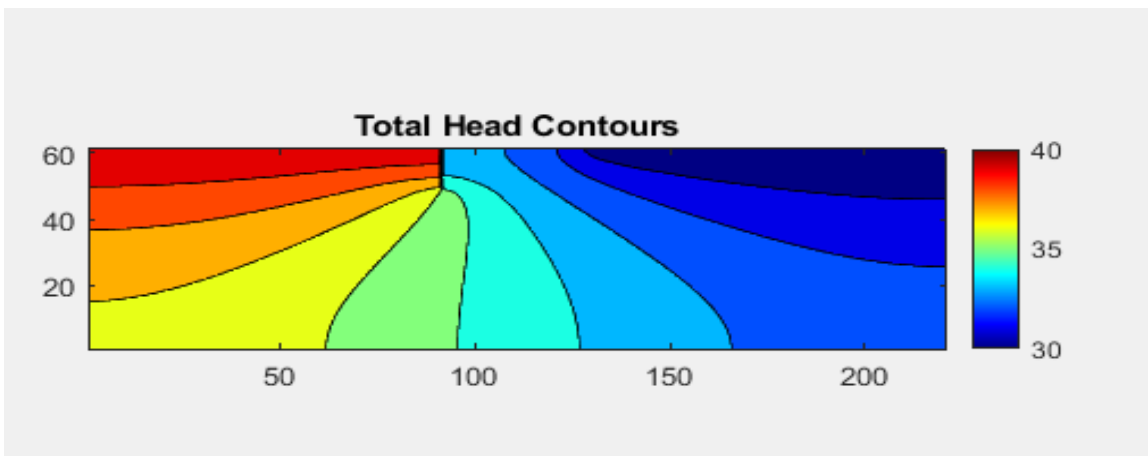


Figure (A-48): The flow net under dam at with two cutoff $d1/B=0.3$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

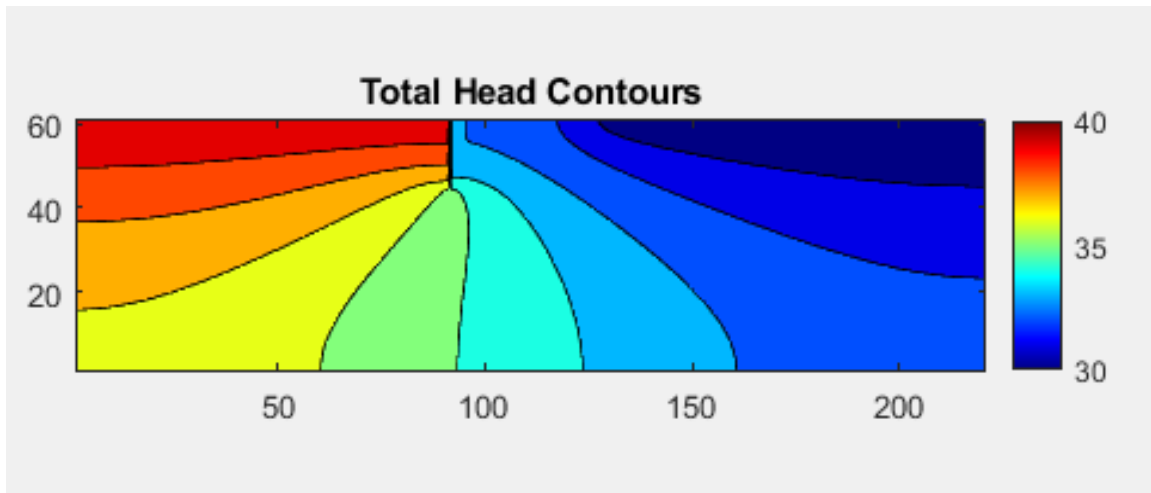


Figure (A-49): The flow net under dam at with two cutoff $d1/B=0.4$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

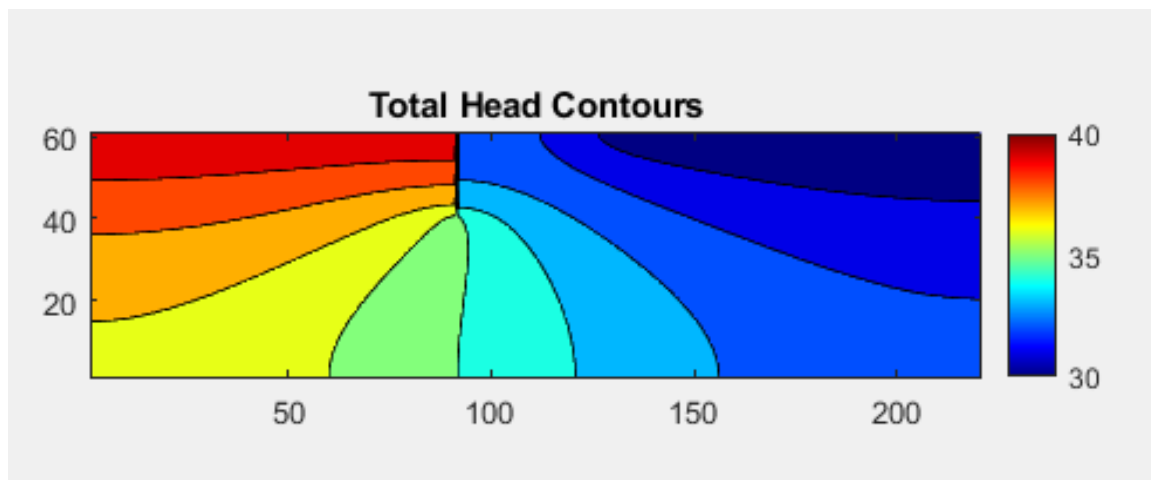


Figure (A-50): The flow net under dam at with two cutoff $d1/B=0.5$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

A.5.4- for ratio of permeability in x direction to permeability in y direction ($K_x/K_y=8$) for ($H/B=0.5$):

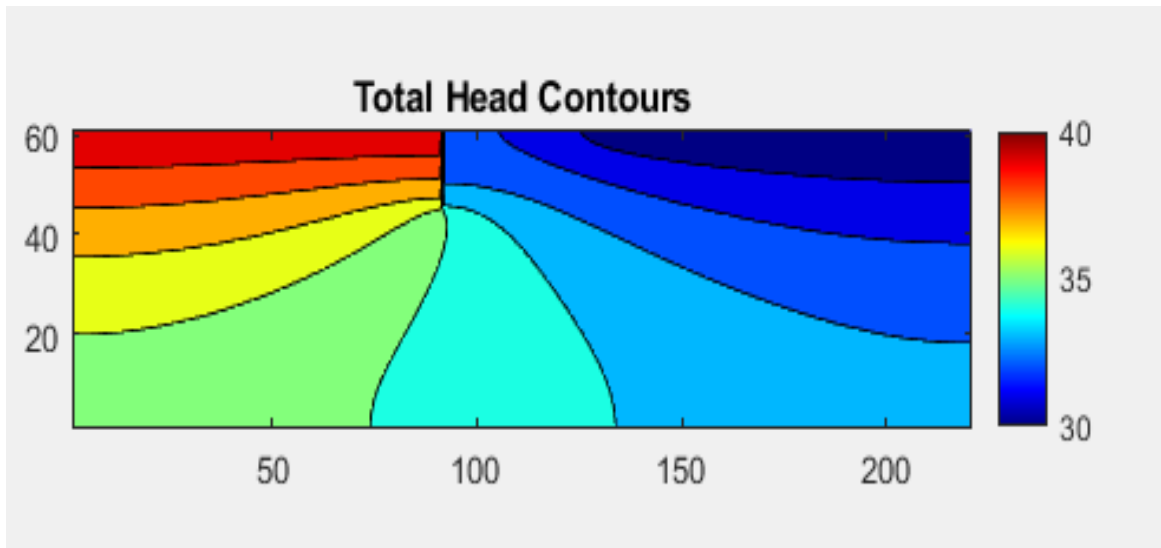


Figure (A-51): The flow net under dam at with two cutoff $d_1/B=0.1$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

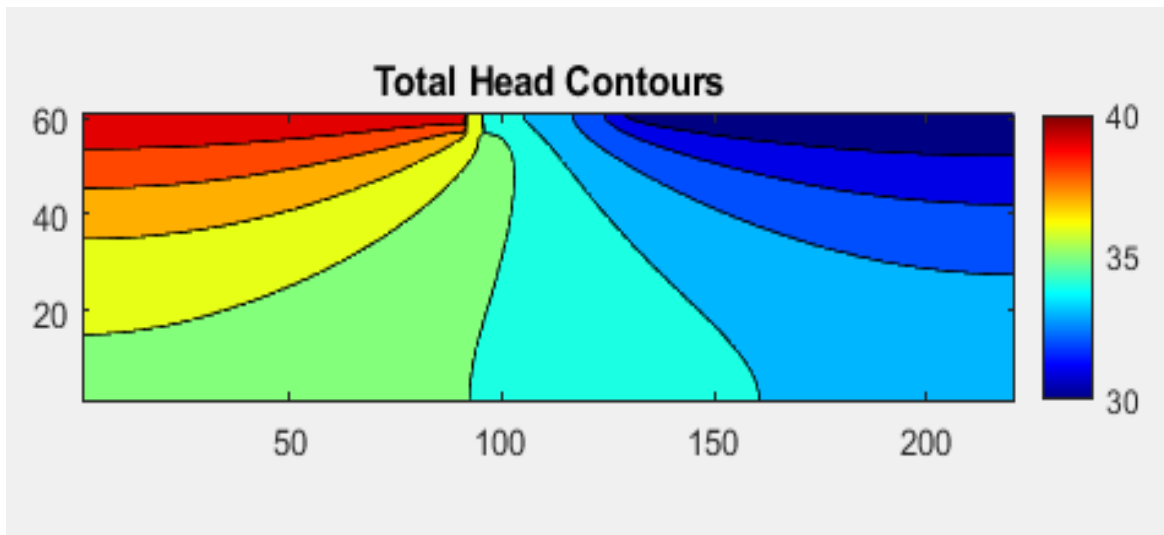


Figure (A-52): The flow net under dam at With two cutoff $d_1/B=0.2$, $x_1/B=0$, $x_2/B=0.1$, $d_2/B=0.1$

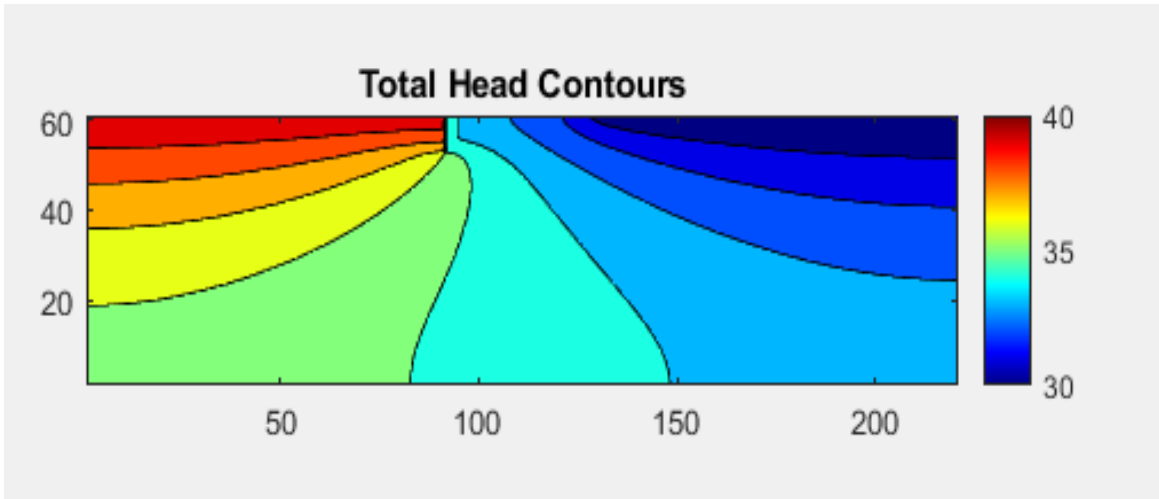


Figure (A-53): The flow net under dam at with two cutoff $d1/B=0.3$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

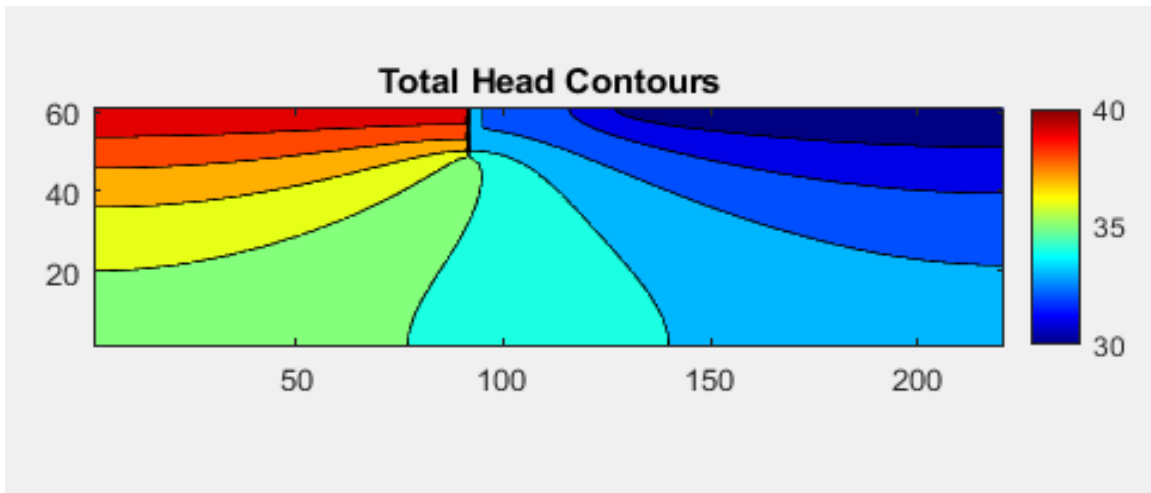


Figure (A-54): The flow net under dam at with two cutoff $d1/B=0.4$, $x1/B=0$, $x2/B=0.1$, $d2/B=0.1$

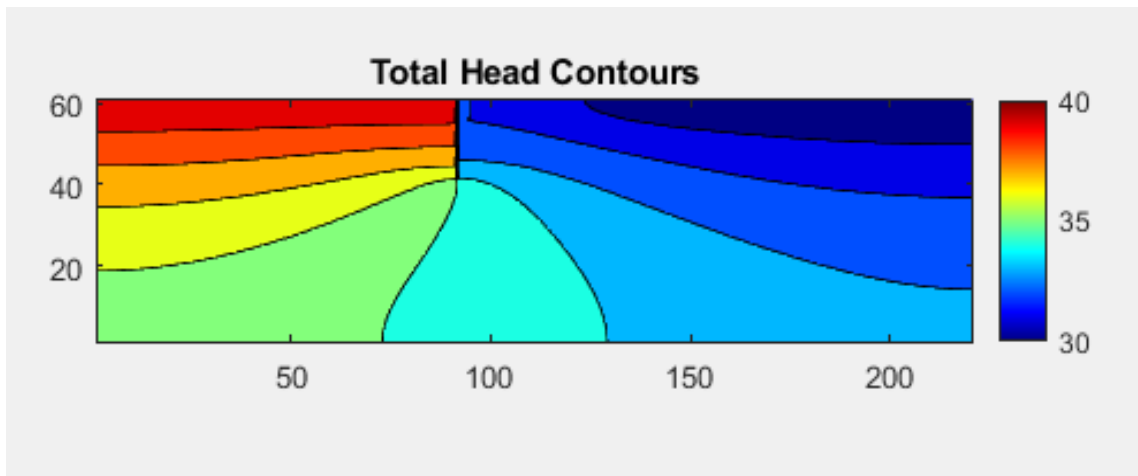


Figure (A-55): The flow net under dam at with two cutoff $d1/B=0.5$, $x1/B=0$, $x2/B=0.1$, $d2/B=0$

المستخلص

تعتبر المنشآت الهيدروليكية مثل السدود الخرسانية على أنها منشآت أساسية لها دور حيوي في خزن وتوفير المياه اضافة الى انها منشآت تحتاج الى كلفة عالية لأنشائها. تتعرض هذه الهياكل إلى مشكلة التسرب والتي تعتبر ظاهرة خطيرة قد تولد ضغطاً مرتفعاً ، مما قد يؤدي إلى عمل السد بشكل غير صحيح ، بالإضافة إلى تدرج الخروج الذي قد يتسبب في حدوث الظاهرة الانبوبية إذا تجاوزت القيمة الآمنة.

يوضح هذا البحث تطبيق خوارزمية وراثية جديدة الخوارزمية الجينية بتقنية برمجة الفروق المحدودة. تم تمثيل هذا الموديل بكود داخل الماتلاب لإيجاد افضل تصميم لمنشأ هيدروليكي امن. حيث دالة الهدف كانت هي اقل كلفة للمنشأ اما القيود احدهما معامل الامان ضد الظاهرة الانبوبية والاخر معامل الامان ضد ضغط الرفع

هذا الموديل يتضمن جزأين , الاول: وهو موديل الفروق المحددة الذي يتم من خلاله تحليل ظاهرة التسرب عددياً بعد ان يتم التحقق منه من خلال مطابقته مع برنامج الجيوستوديو للحصول على ضغط الرفع وميل الخروج وارتفاع الماء الكلي والتصريف وكلفة المنشأ. وايضاً يتم مقارنه برنامج الجيوستوديو مع الحساب اليدوي لمثال في كتاب ميكانيك التربة وكانت نتائج المطابقة جيدة. الثاني : وهي الخوارزمية الجينية يتم تطبيقها مع الفروق المحددة للحصول على افضل موقع وعمق للجدار القاطع الذي يحقق معامل الامان ضد الظاهرة الانبوبية وضد ضغط الرفع.

تم تحليل ظاهرة التسرب داخل موديل الفروق المحددة لمعرفة تأثير كل من عدد وعمق وموقع الجدار ودرجة التماثل للتربة وعدم التماثل على قيمة ضغط الرفع وميل الخروج. حيث بينت النتائج انه كلما زاد ارتفاع الماء في مقدم المنشأ له تأثير كبير على زيادة قيمة ضغط الرفع والميل ,حيث تم ملاحظة اقل قيمة لميل الخروج عندما يكون الجدار القاطع في نهاية الارضية وبأكبر عمق للجدار القاطع بينما اقل ضغط رفع تم ملاحظته عندما يكون الجدار القاطع في مقدمة الارضية وبأقل عمق.

في موديل الخوارزمية الجينية_الفروق المحددة تم استخدام طول ارضية وعمق لأساس التربة معروف وبقيم مختلفة لكل من (نسبة ارتفاع الماء الى الارضية , ونوع التربة, عمق الجدار يتراوح (0الى0.6) من طول الارضية , وموقع الجدار (0الى1) من طول الارضية).

من الواضح ان النتائج التي تم الحصول عليها من هذا الموديل تعتبر الحل الامثل التي تحقق كلاً من معاملي الامان ضد ظاهرة الانايبب وضغط الرفع وبأقل كلفة , حيث افضل موقع للجدار الاول يتراوح بين (0 و0.33) من طول الارضية أي ضمن الثلث الاول اما بالنسبة للجدار الثاني يتراوح بين (1 و0.875) من الارضية لمختلف انواع التربة . بالنسبة لعمق الجدار الاول يتراوح بين (0 و0.35) من الارضية لمعامل النفاذية (1,2,4) ويتراوح بين (0 و0.6) من الارضية لمعامل نفاذية 8 , اما عمق الجدار الثاني يتراوح بين (0.15 و0.5) وبين(0.1 و0.6) وبين(0.2 و0.7) وبين(0.3 و0.7) من طول الارضية لكل من معامل النفاذية (1,2,4,8).

حيث ان هذا الموديل قلل الكلفة بمقدار (81,72,66,53)% لمعامل النفاذية (1,2,4,8) على التوالي مقارنة بالحلول الاخرى التي تأخذ وقتاً وجهداً اضافياً.



جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة كربلاء
كلية الهندسة
قسم الهندسة المدنية

التصميم الامثل للمنشآت الهيدروليكية باستخدام طريقة المحاكاة – الامثلية

الرسالة مقدمة الى

قسم الهندسة المدنية / كلية الهندسة في جامعة كربلاء كجزء من متطلبات نيل درجة
الماجستير في علوم الهندسة المدنية (البنى التحتية)

من قبل

دعاء هادي خشان

(بكالوريوس الهندسة المدنية 2014)

بإشراف

الاستاذ الدكتور

واقد حميد حسن